Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.2-Cosine/82-4.2.0-a-cos-^m-b-trg-^n

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [294]. This is test number [82].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (294)	0.00 (0)
Mathematica	100.00 (294)	0.00 (0)
Fricas	67.01 (197)	32.99 (97)
Maple	66.67 (196)	33.33 (98)
Maxima	31.29 (92)	68.71 (202)
Mupad	27.21 (80)	72.79 (214)
Giac	11.56 (34)	88.44 (260)
Sympy	6.12 (18)	93.88 (276)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

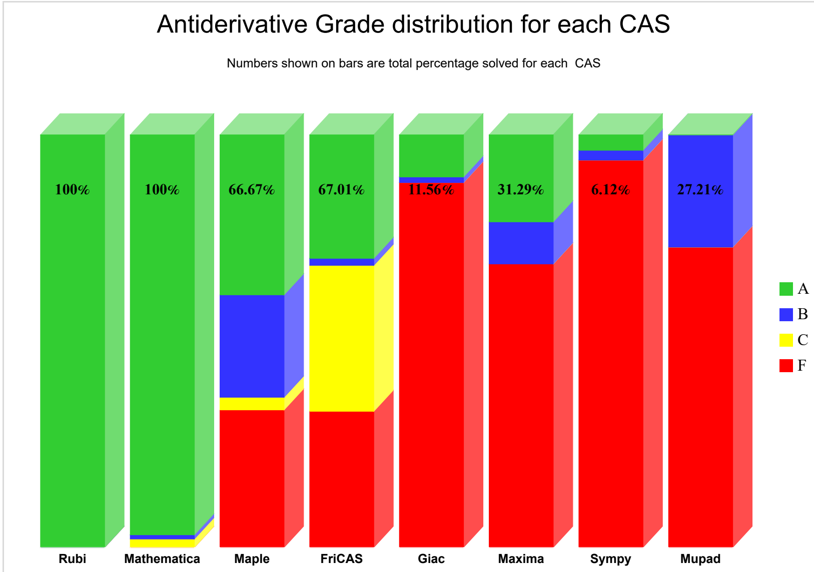
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

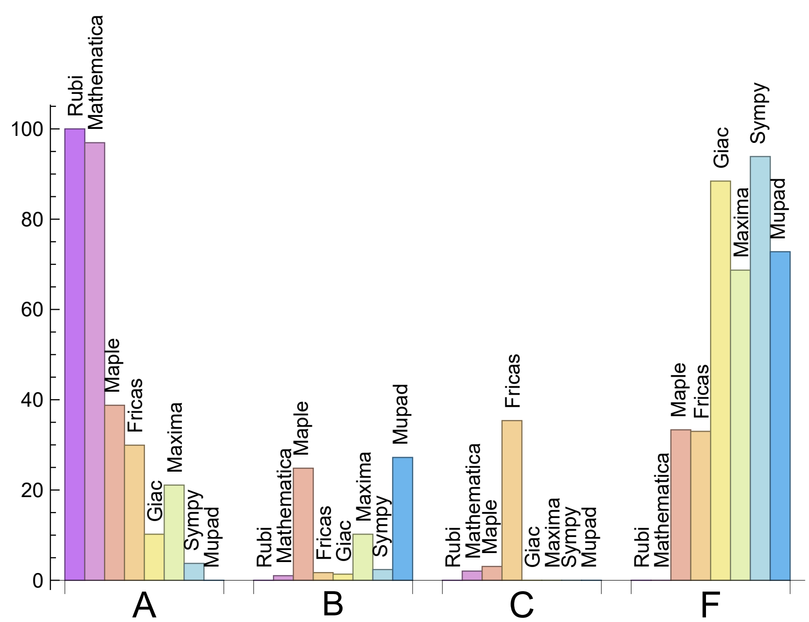
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	96.94	1.02	2.04	0.00
Maple	38.78	24.83	3.06	33.33
Fricas	29.93	1.70	35.37	32.99
Maxima	21.09	10.20	0.00	68.71
Giac	10.20	1.36	0.00	88.44
Sympy	3.74	2.38	0.00	93.88
Mupad	N/A	27.21	0.00	72.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	98	100.00 %	0.00 %	0.00 %
Fricas	97	93.81 %	0.00 %	6.19 %
Giac	260	97.69 %	0.38 %	1.92 %
Maxima	202	99.50 %	0.00 %	0.50 %
Sympy	276	46.74 %	32.25 %	21.01 %
Mupad	214	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

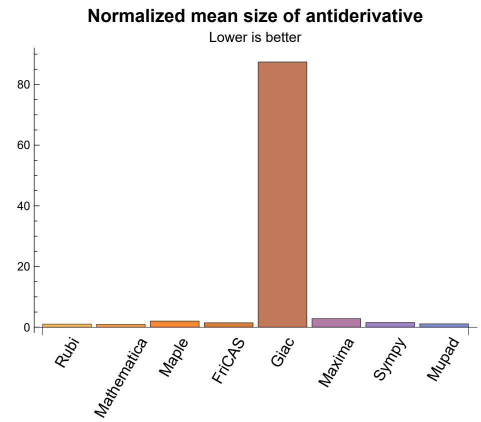
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

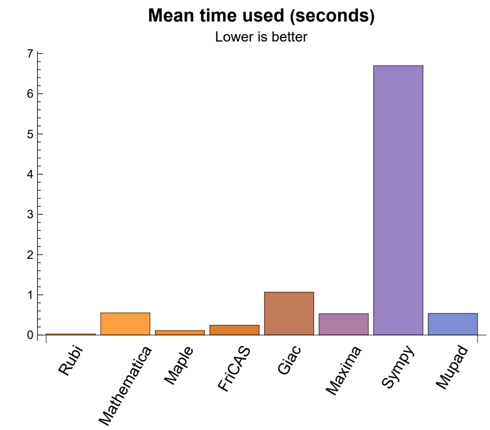
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.03	68.33	1.00	69.00	1.00
Mathematica	0.55	59.73	0.92	56.00	0.91
Maple	0.11	141.83	2.00	121.00	1.66
Maxima	0.53	221.09	2.81	48.50	0.86
Fricas	0.24	91.11	1.45	91.00	1.21
Sympy	6.70	60.28	1.52	46.00	1.43
Giac	1.07	6304.65	87.39	33.00	0.74
Mupad	0.54	54.98	1.09	43.50	0.87

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
20
40
60
80
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{284, 285, 286, 287, 292\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

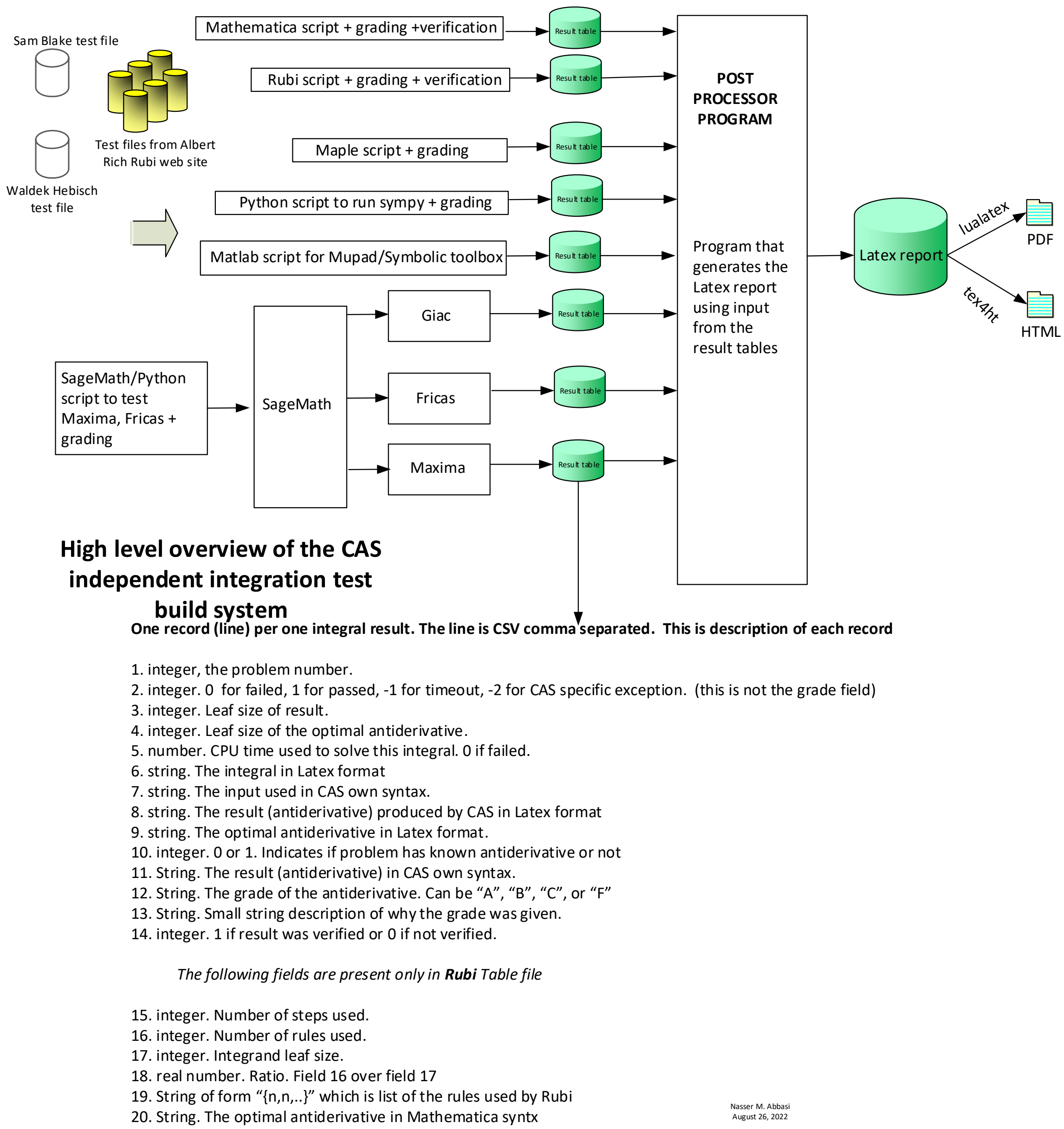
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.8 Mupad . . . 24

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 288, 289, 290, 291, 293, 294 }

B grade: { 1, 42, 43 }

C grade: { 276, 284, 285, 286, 287, 292 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 17, 39, 40, 41, 44, 51, 52, 53, 54, 55, 56, 66, 68, 78, 80, 90, 92, 102, 103, 104, 105, 115, 116, 117, 127, 128, 129, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278 }

B grade: { 9, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 42, 43, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 91, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 110, 111, 112, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 274 }

C grade: { 13, 21, 45, 46, 47, 48, 49, 50, 109 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 57, 58, 59, 60, 61, 62, 63, 64, 65, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 51, 52, 53, 54, 55, 56, 140, 141, 142, 143, 144, 146, 150, 151, 152, 153, 154, 156, 160, 161, 162, 163, 164, 165, 167, 171, 172, 173, 174, 175, 176, 182, 183, 184, 185, 186, 192, 193, 194, 195, 196, 261, 262, 265, 266, 267, 270, 271, 272, 275, 276 }

B grade: { 42, 43, 44, 145, 147, 148, 149, 155, 157, 158, 159, 166, 168, 169, 170, 177, 178, 179, 180, 181, 187, 188, 189, 190, 191, 197, 198, 199, 200, 201 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 43, 44, 51, 52, 53, 54, 55, 56, 64, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 261, 262, 265, 266, 267, 270 }

B grade: { 42, 271, 272, 275, 276 }

C grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 45, 46, 47, 48, 49, 50, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 263, 264, 268, 269, 273, 274, 277, 278 }

F grade: { 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 57, 58, 59, 60, 61, 62, 63, 65, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.6 Sympy

A grade: { 1, 3, 5, 7, 41, 144, 153, 154, 175, 176, 186 }

B grade: { 2, 4, 6, 8, 64, 142, 143 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 145, 146, 147, 148, 149, 150, 151, 152, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 39, 40, 41, 43, 44, 51, 52, 53, 54, 55, 56, 143, 261, 262, 265, 266, 267, 270, 271, 272, 275, 276 }

B grade: { 64, 141, 151, 162 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 42, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 268, 269, 273, 274, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 21, 25, 26, 27, 28, 29, 30, 37, 41, 54, 55, 56, 64, 107, 108, 109, 140, 141, 142, 143, 144, 146, 148, 150, 151, 152, 153, 154, 156, 158, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 178, 180, 182, 183, 184, 185, 186, 188, 190, 192, 193, 194, 195, 196, 198, 200, 261, 262, 265, 270 }

C grade: { }

F grade: { 17, 18, 19, 20, 22, 23, 24, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 145, 147, 149, 155, 157, 159, 166, 168, 170, 177, 179, 181, 187, 189, 191, 197, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	10	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	1.00	1.00
time (sec)	N/A	0.003	0.056	0.085	0.287	0.351	0.038	0.453	0.249

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	22	22	46	18	18
N.S.	1	1.00	0.92	1.08	0.88	0.88	1.84	0.72	0.72
time (sec)	N/A	0.006	0.019	0.038	0.272	0.356	0.068	0.490	0.228

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	22	21	36	22	24
N.S.	1	1.00	1.00	0.85	0.85	0.81	1.38	0.85	0.92
time (sec)	N/A	0.008	0.006	0.073	0.268	0.354	0.093	0.467	0.071

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	33	38	33	36	95	32	31
N.S.	1	1.00	0.72	0.83	0.72	0.78	2.07	0.70	0.67
time (sec)	N/A	0.014	0.026	0.080	0.278	0.390	0.156	0.463	0.188

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	32	34	33	58	34	31
N.S.	1	1.00	1.07	0.78	0.83	0.80	1.41	0.83	0.76
time (sec)	N/A	0.011	0.010	0.069	0.277	0.354	0.231	0.460	0.095

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	43	48	48	46	139	46	42
N.S.	1	1.00	0.64	0.72	0.72	0.69	2.07	0.69	0.63
time (sec)	N/A	0.022	0.023	0.092	0.276	0.378	0.371	0.448	0.222

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	42	44	43	78	44	43
N.S.	1	1.00	1.09	0.78	0.81	0.80	1.44	0.81	0.80
time (sec)	N/A	0.011	0.009	0.065	0.268	0.365	0.551	0.469	0.090

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	55	58	59	56	184	60	53
N.S.	1	1.00	0.62	0.66	0.67	0.64	2.09	0.68	0.60
time (sec)	N/A	0.031	0.034	0.112	0.273	0.360	0.847	0.451	0.279

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	199	0	80	0	0	42
N.S.	1	1.00	0.78	3.06	0.00	1.23	0.00	0.00	0.65
time (sec)	N/A	0.021	0.176	0.089	0.000	0.121	0.000	0.000	0.294

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	202	0	74	0	0	42
N.S.	1	1.00	0.95	4.81	0.00	1.76	0.00	0.00	1.00
time (sec)	N/A	0.014	0.031	0.033	0.000	0.117	0.000	0.000	0.166

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	179	0	68	0	0	35
N.S.	1	1.00	0.86	4.26	0.00	1.62	0.00	0.00	0.83
time (sec)	N/A	0.013	0.026	0.032	0.000	0.092	0.000	0.000	0.078

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	133	0	57	0	0	15
N.S.	1	1.00	1.00	8.31	0.00	3.56	0.00	0.00	0.94
time (sec)	N/A	0.007	0.015	0.100	0.000	0.113	0.000	0.000	0.116

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	0	51	0	0	15
N.S.	1	1.00	1.00	1.12	0.00	3.19	0.00	0.00	0.94
time (sec)	N/A	0.007	0.019	0.029	0.000	0.105	0.000	0.000	0.097

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	182	0	93	0	0	42
N.S.	1	1.00	1.00	4.79	0.00	2.45	0.00	0.00	1.11
time (sec)	N/A	0.013	0.037	0.037	0.000	0.111	0.000	0.000	0.246

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	36	213	0	92	0	0	42
N.S.	1	1.00	0.86	5.07	0.00	2.19	0.00	0.00	1.00
time (sec)	N/A	0.013	0.038	0.040	0.000	0.097	0.000	0.000	0.265

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	358	0	110	0	0	42
N.S.	1	1.00	0.91	5.51	0.00	1.69	0.00	0.00	0.65
time (sec)	N/A	0.020	0.062	0.048	0.000	0.105	0.000	0.000	0.309

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	76	210	0	95	0	0	-1
N.S.	1	1.00	0.78	2.14	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.058	0.054	0.000	0.105	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	213	0	91	0	0	-1
N.S.	1	1.00	0.89	3.04	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.054	0.036	0.000	0.097	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	58	190	0	77	0	0	-1
N.S.	1	1.00	0.83	2.71	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.029	0.035	0.000	0.098	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	142	0	63	0	0	-1
N.S.	1	1.00	1.00	3.74	0.00	1.66	0.00	0.00	-0.03
time (sec)	N/A	0.014	0.014	0.073	0.000	0.091	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	0	60	0	0	33
N.S.	1	1.00	1.00	1.42	0.00	1.58	0.00	0.00	0.87
time (sec)	N/A	0.014	0.017	0.043	0.000	0.089	0.000	0.000	0.146

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	198	0	104	0	0	-1
N.S.	1	1.00	0.74	2.91	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.021	0.041	0.000	0.095	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	103	0	0	-1
N.S.	1	1.00	0.71	3.35	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.039	0.041	0.000	0.111	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	68	366	0	121	0	0	-1
N.S.	1	1.00	0.68	3.66	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.061	0.056	0.000	0.100	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.009	0.040	0.017	0.000	0.000	0.000	0.000	0.193

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.009	0.020	0.014	0.000	0.000	0.000	0.000	0.191

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.009	0.020	0.015	0.000	0.000	0.000	0.000	0.182

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.79
time (sec)	N/A	0.010	0.017	0.015	0.000	0.000	0.000	0.000	0.210

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.010	0.017	0.015	0.000	0.000	0.000	0.000	0.203

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	0	0	0	0	0	42
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.82
time (sec)	N/A	0.009	0.017	0.016	0.000	0.000	0.000	0.000	0.227

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.031	0.014	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.024	0.015	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.023	0.013	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.026	0.012	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.026	0.011	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.026	0.013	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	64	0	0	0	0	0	57
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.89
time (sec)	N/A	0.011	0.029	0.020	0.000	0.000	0.000	0.000	0.535

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.027	0.020	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	31	40	0	34	-1
N.S.	1	1.00	0.68	0.60	0.58	0.75	0.00	0.64	-0.02
time (sec)	N/A	0.026	0.013	0.054	0.519	0.372	0.000	0.433	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	17	26	0	17	-1
N.S.	1	1.00	0.76	0.71	0.50	0.76	0.00	0.50	-0.03
time (sec)	N/A	0.015	0.007	0.033	0.515	0.369	0.000	0.464	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	6	15	15	9	46
N.S.	1	1.00	1.00	1.15	0.46	1.15	1.15	0.69	3.54
time (sec)	N/A	0.007	0.003	0.031	0.513	0.349	0.180	0.530	0.214

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	48	38	65	0	0	-1
N.S.	1	1.00	2.88	3.00	2.38	4.06	0.00	0.00	-0.06
time (sec)	N/A	0.008	0.018	0.046	0.529	0.368	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	91	70	304	40	0	57	-1
N.S.	1	1.00	2.17	1.67	7.24	0.95	0.00	1.36	-0.02
time (sec)	N/A	0.014	0.038	0.048	0.540	0.372	0.000	0.560	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	72	89	933	49	0	35	-1
N.S.	1	1.00	1.18	1.46	15.30	0.80	0.00	0.57	-0.02
time (sec)	N/A	0.025	0.093	0.054	0.799	0.394	0.000	0.453	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	61	114	0	92	0	0	-1
N.S.	1	1.00	0.52	0.97	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.071	0.266	0.000	0.143	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	50	198	0	66	0	0	-1
N.S.	1	1.00	0.75	2.96	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.045	0.085	0.000	0.123	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	37	76	0	59	0	0	-1
N.S.	1	1.00	0.84	1.73	0.00	1.34	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.016	0.089	0.000	0.093	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	191	0	71	0	0	-1
N.S.	1	1.00	0.74	4.55	0.00	1.69	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.014	0.149	0.000	0.093	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	44	87	0	74	0	0	-1
N.S.	1	1.00	0.62	1.23	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.039	0.233	0.000	0.102	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	57	223	0	92	0	0	-1
N.S.	1	1.00	0.49	1.91	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.067	0.337	0.000	0.121	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	57	85	68	0	57	-1
N.S.	1	1.00	0.40	0.43	0.64	0.52	0.00	0.43	-0.01
time (sec)	N/A	0.035	0.076	0.413	0.495	0.368	0.000	0.443	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	41	55	42	0	25	-1
N.S.	1	1.00	0.49	0.53	0.71	0.54	0.00	0.32	-0.01
time (sec)	N/A	0.025	0.044	0.148	0.495	0.369	0.000	0.462	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	22	22	21	0	13	-1
N.S.	1	1.00	0.69	0.61	0.61	0.58	0.00	0.36	-0.03
time (sec)	N/A	0.011	0.009	0.079	0.514	0.391	0.000	0.440	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	6	18	0	6	6
N.S.	1	1.00	1.00	0.93	0.40	1.20	0.00	0.40	0.40
time (sec)	N/A	0.011	0.004	0.076	0.500	0.374	0.000	0.455	0.231

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	30	29	22	33	0	22	36
N.S.	1	1.00	0.45	0.43	0.33	0.49	0.00	0.33	0.54
time (sec)	N/A	0.015	0.017	0.094	0.497	0.373	0.000	0.446	0.541

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	47	41	34	45	0	34	306
N.S.	1	1.00	0.40	0.35	0.29	0.38	0.00	0.29	2.62
time (sec)	N/A	0.021	0.029	0.168	0.488	0.393	0.000	0.457	3.744

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.041	0.053	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.106	0.053	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.076	0.044	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.045	0.047	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.049	0.043	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.067	0.040	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.067	0.039	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	0	15	61	300	40
N.S.	1	1.00	1.00	0.00	0.00	0.62	2.54	12.50	1.67
time (sec)	N/A	0.013	0.018	0.046	0.000	0.368	0.441	1.203	0.404

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	74	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.036	0.053	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	83	234	0	98	0	0	-1
N.S.	1	1.00	0.67	1.90	0.00	0.80	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.128	0.136	0.000	0.121	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	221	0	101	0	0	-1
N.S.	1	1.00	0.77	2.28	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.096	0.086	0.000	0.113	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	208	0	88	0	0	-1
N.S.	1	1.00	0.77	2.19	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.090	0.084	0.000	0.106	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	62	211	0	88	0	0	-1
N.S.	1	1.00	0.90	3.06	0.00	1.28	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.040	0.083	0.000	0.114	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	188	0	76	0	0	-1
N.S.	1	1.00	0.91	2.81	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.039	0.080	0.000	0.106	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	142	0	63	0	0	-1
N.S.	1	1.00	1.00	3.74	0.00	1.66	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.016	0.177	0.000	0.102	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	142	0	57	0	0	-1
N.S.	1	1.00	1.00	3.64	0.00	1.46	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.021	0.096	0.000	0.101	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	48	196	0	101	0	0	-1
N.S.	1	1.00	0.76	3.11	0.00	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.037	0.095	0.000	0.097	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	49	239	0	100	0	0	-1
N.S.	1	1.00	0.70	3.41	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.055	0.104	0.000	0.099	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	69	363	0	118	0	0	-1
N.S.	1	1.00	0.73	3.82	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.142	0.129	0.000	0.112	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	69	396	0	112	0	0	-1
N.S.	1	1.00	0.70	4.04	0.00	1.14	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.122	0.110	0.000	0.097	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	79	412	0	128	0	0	-1
N.S.	1	1.00	0.64	3.35	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.134	0.144	0.000	0.109	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	83	236	0	102	0	0	-1
N.S.	1	1.00	0.66	1.87	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.065	0.082	0.000	0.123	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	223	0	103	0	0	-1
N.S.	1	1.00	0.79	2.35	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.064	0.082	0.000	0.111	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	73	210	0	91	0	0	-1
N.S.	1	1.00	0.74	2.14	0.00	0.93	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.063	0.080	0.000	0.098	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	65	213	0	89	0	0	-1
N.S.	1	1.00	0.97	3.18	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.057	0.079	0.000	0.115	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	58	190	0	77	0	0	-1
N.S.	1	1.00	0.83	2.71	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.011	0.070	0.000	0.101	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	144	0	63	0	0	-1
N.S.	1	1.00	1.00	3.69	0.00	1.62	0.00	0.00	-0.03
time (sec)	N/A	0.021	0.011	0.085	0.000	0.101	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	57	0	0	-1
N.S.	1	1.00	1.00	3.51	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.015	0.085	0.000	0.119	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	50	198	0	102	0	0	-1
N.S.	1	1.00	0.76	3.00	0.00	1.55	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.035	0.091	0.000	0.117	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	101	0	0	-1
N.S.	1	1.00	0.71	3.35	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.043	0.091	0.000	0.096	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	69	364	0	119	0	0	-1
N.S.	1	1.00	0.70	3.71	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.075	0.112	0.000	0.117	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	398	0	115	0	0	-1
N.S.	1	1.00	0.69	3.98	0.00	1.15	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.085	0.099	0.000	0.118	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	79	414	0	132	0	0	-1
N.S.	1	1.00	0.63	3.29	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.115	0.131	0.000	0.119	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	83	236	0	108	0	0	-1
N.S.	1	1.00	0.66	1.89	0.00	0.86	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.087	0.085	0.000	0.123	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	75	223	0	107	0	0	-1
N.S.	1	1.00	0.77	2.28	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.067	0.084	0.000	0.150	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	76	210	0	95	0	0	-1
N.S.	1	1.00	0.78	2.16	0.00	0.98	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.061	0.079	0.000	0.118	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	213	0	91	0	0	-1
N.S.	1	1.00	0.89	3.04	0.00	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.012	0.069	0.000	0.120	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	190	0	79	0	0	-1
N.S.	1	1.00	0.82	2.64	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.016	0.079	0.000	0.105	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	63	0	0	-1
N.S.	1	1.00	1.00	3.51	0.00	1.54	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.011	0.091	0.000	0.096	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	144	0	57	0	0	-1
N.S.	1	1.00	0.93	3.51	0.00	1.39	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.029	0.089	0.000	0.102	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	198	0	104	0	0	-1
N.S.	1	1.00	0.74	2.91	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.036	0.095	0.000	0.096	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	103	0	0	-1
N.S.	1	1.00	0.71	3.35	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.045	0.093	0.000	0.103	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	366	0	123	0	0	-1
N.S.	1	1.00	0.69	3.66	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.097	0.117	0.000	0.126	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	398	0	119	0	0	-1
N.S.	1	1.00	0.69	3.98	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.116	0.098	0.000	0.112	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	79	414	0	138	0	0	-1
N.S.	1	1.00	0.62	3.23	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.156	0.131	0.000	0.104	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	76	210	0	95	0	0	-1
N.S.	1	1.00	0.78	2.14	0.00	0.97	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.014	0.072	0.000	0.109	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	73	233	0	101	0	0	-1
N.S.	1	1.00	0.58	1.86	0.00	0.81	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.086	0.090	0.000	0.126	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	71	220	0	104	0	0	-1
N.S.	1	1.00	0.71	2.20	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.072	0.084	0.000	0.113	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	63	207	0	91	0	0	-1
N.S.	1	1.00	0.65	2.13	0.00	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.064	0.083	0.000	0.100	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	210	0	91	0	0	-1
N.S.	1	1.00	0.81	2.92	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.030	0.036	0.084	0.000	0.113	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	187	0	79	0	0	58
N.S.	1	1.00	0.74	2.71	0.00	1.14	0.00	0.00	0.84
time (sec)	N/A	0.028	0.029	0.081	0.000	0.099	0.000	0.000	0.140

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	141	0	66	0	0	33
N.S.	1	1.00	1.00	3.44	0.00	1.61	0.00	0.00	0.80
time (sec)	N/A	0.016	0.011	0.086	0.000	0.098	0.000	0.000	0.148

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	54	0	60	0	0	33
N.S.	1	1.00	1.00	1.42	0.00	1.58	0.00	0.00	0.87
time (sec)	N/A	0.014	0.011	0.077	0.000	0.095	0.000	0.000	0.192

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	195	0	104	0	0	-1
N.S.	1	1.00	0.72	3.00	0.00	1.60	0.00	0.00	-0.02
time (sec)	N/A	0.031	0.027	0.083	0.000	0.127	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	48	238	0	103	0	0	-1
N.S.	1	1.00	0.72	3.55	0.00	1.54	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.042	0.089	0.000	0.143	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	65	366	0	121	0	0	-1
N.S.	1	1.00	0.67	3.77	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.059	0.121	0.000	0.109	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	395	0	115	0	0	-1
N.S.	1	1.00	0.66	4.16	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.091	0.102	0.000	0.108	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	77	411	0	131	0	0	-1
N.S.	1	1.00	0.62	3.29	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.065	0.187	0.132	0.000	0.122	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	76	236	0	101	0	0	-1
N.S.	1	1.00	0.59	1.84	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.056	0.094	0.000	0.121	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	74	223	0	104	0	0	-1
N.S.	1	1.00	0.74	2.23	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.037	0.086	0.000	0.116	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	66	210	0	91	0	0	-1
N.S.	1	1.00	0.66	2.10	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.034	0.084	0.000	0.112	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	213	0	91	0	0	-1
N.S.	1	1.00	0.85	2.96	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.024	0.084	0.000	0.112	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	190	0	79	0	0	-1
N.S.	1	1.00	0.75	2.64	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.025	0.083	0.000	0.112	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	66	0	0	-1
N.S.	1	1.00	1.00	3.51	0.00	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.011	0.095	0.000	0.116	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	60	0	0	-1
N.S.	1	1.00	1.00	3.51	0.00	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.014	0.080	0.000	0.098	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	198	0	104	0	0	-1
N.S.	1	1.00	0.74	2.91	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.012	0.074	0.000	0.092	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	51	241	0	103	0	0	-1
N.S.	1	1.00	0.74	3.49	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.017	0.085	0.000	0.111	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	68	366	0	121	0	0	-1
N.S.	1	1.00	0.69	3.73	0.00	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.035	0.111	0.000	0.112	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	66	398	0	115	0	0	-1
N.S.	1	1.00	0.68	4.10	0.00	1.19	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.030	0.098	0.000	0.152	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	80	414	0	131	0	0	-1
N.S.	1	1.00	0.63	3.29	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.027	0.132	0.000	0.121	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	76	236	0	101	0	0	-1
N.S.	1	1.00	0.59	1.84	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.036	0.092	0.000	0.120	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	74	223	0	104	0	0	-1
N.S.	1	1.00	0.74	2.23	0.00	1.04	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.037	0.086	0.000	0.118	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	66	210	0	91	0	0	-1
N.S.	1	1.00	0.66	2.10	0.00	0.91	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.029	0.086	0.000	0.125	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	213	0	91	0	0	-1
N.S.	1	1.00	0.85	2.96	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.023	0.087	0.000	0.110	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	54	190	0	79	0	0	-1
N.S.	1	1.00	0.75	2.64	0.00	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.020	0.083	0.000	0.108	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	41	144	0	66	0	0	-1
N.S.	1	1.00	1.00	3.51	0.00	1.61	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.010	0.090	0.000	0.098	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	38	144	0	60	0	0	-1
N.S.	1	1.00	0.93	3.51	0.00	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.034	0.086	0.000	0.098	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	198	0	104	0	0	-1
N.S.	1	1.00	0.74	2.91	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.016	0.084	0.000	0.113	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	51	241	0	103	0	0	-1
N.S.	1	1.00	0.71	3.35	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.013	0.073	0.000	0.098	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	68	366	0	121	0	0	-1
N.S.	1	1.00	0.70	3.77	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.045	0.019	0.108	0.000	0.110	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	66	398	0	115	0	0	-1
N.S.	1	1.00	0.67	4.06	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.051	0.041	0.093	0.000	0.121	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	80	414	0	131	0	0	-1
N.S.	1	1.00	0.64	3.31	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.028	0.132	0.000	0.112	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	68	366	0	121	0	0	-1
N.S.	1	1.00	0.68	3.66	0.00	1.21	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.013	0.099	0.000	0.108	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	55	62	49	176	0	0	75
N.S.	1	1.00	0.56	0.63	0.50	1.80	0.00	0.00	0.77
time (sec)	N/A	0.020	0.056	0.813	0.598	0.434	0.000	0.000	1.257

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	40	42	39	0	71047	57
N.S.	1	1.00	0.64	0.57	0.60	0.56	0.00	1014.96	0.81
time (sec)	N/A	0.013	0.056	0.155	0.587	0.383	0.000	6.746	0.739

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	42	25	150	121	0	62
N.S.	1	1.00	0.71	0.67	0.40	2.38	1.92	0.00	0.98
time (sec)	N/A	0.011	0.034	0.148	0.585	0.429	41.865	0.000	0.689

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	13	28	60	31	44
N.S.	1	1.00	1.00	0.91	0.41	0.88	1.88	0.97	1.38
time (sec)	N/A	0.005	0.017	0.132	0.591	0.460	1.832	0.744	0.389

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	26	94	22	0	20
N.S.	1	1.00	1.00	1.17	1.08	3.92	0.92	0.00	0.83
time (sec)	N/A	0.002	0.007	0.103	0.530	0.464	0.866	0.000	0.096

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	65	113	0	0	-1
N.S.	1	1.00	1.00	1.27	1.97	3.42	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.008	0.143	0.601	0.440	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	54	28	0	0	59
N.S.	1	1.00	1.00	0.91	1.69	0.88	0.00	0.00	1.84
time (sec)	N/A	0.009	0.012	0.136	0.573	0.388	0.000	0.000	0.702

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	52	104	661	201	0	0	-1
N.S.	1	1.00	0.72	1.44	9.18	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.028	0.186	0.592	0.416	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	42	294	41	0	0	128
N.S.	1	1.00	0.64	0.60	4.20	0.59	0.00	0.00	1.83
time (sec)	N/A	0.013	0.052	0.148	0.607	0.358	0.000	0.000	1.855

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	66	121	1656	227	0	0	-1
N.S.	1	1.00	0.62	1.13	15.48	2.12	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.065	0.232	0.673	0.431	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	55	62	53	183	0	0	76
N.S.	1	1.00	0.54	0.61	0.52	1.81	0.00	0.00	0.75
time (sec)	N/A	0.021	0.053	0.159	0.576	0.401	0.000	0.000	1.048

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	40	45	43	0	71048	58
N.S.	1	1.00	0.62	0.56	0.62	0.60	0.00	986.78	0.81
time (sec)	N/A	0.012	0.073	0.127	0.581	0.365	0.000	6.596	0.704

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	45	42	28	153	0	0	63
N.S.	1	1.00	0.69	0.65	0.43	2.35	0.00	0.00	0.97
time (sec)	N/A	0.010	0.035	0.130	0.610	0.424	0.000	0.000	0.528

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	13	29	46	0	29
N.S.	1	1.00	0.97	0.88	0.39	0.88	1.39	0.00	0.88
time (sec)	N/A	0.005	0.026	0.121	0.565	0.376	20.247	0.000	0.240

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	28	26	95	22	0	21
N.S.	1	1.00	0.96	1.12	1.04	3.80	0.88	0.00	0.84
time (sec)	N/A	0.002	0.010	0.089	0.513	0.434	12.928	0.000	0.090

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	42	68	114	0	0	-1
N.S.	1	1.00	0.97	1.24	2.00	3.35	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.010	0.114	0.633	0.414	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	54	29	0	0	60
N.S.	1	1.00	0.97	0.88	1.64	0.88	0.00	0.00	1.82
time (sec)	N/A	0.009	0.012	0.116	0.568	0.362	0.000	0.000	0.504

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	104	691	204	0	0	-1
N.S.	1	1.00	0.70	1.41	9.34	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.031	0.150	0.591	0.474	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	45	42	299	42	0	0	129
N.S.	1	1.00	0.62	0.58	4.15	0.58	0.00	0.00	1.79
time (sec)	N/A	0.013	0.033	0.125	0.591	0.414	0.000	0.000	1.140

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	67	121	1742	234	0	0	-1
N.S.	1	1.00	0.61	1.10	15.84	2.13	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.052	0.199	0.690	0.409	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	58	52	77	61	0	0	73
N.S.	1	1.00	0.50	0.45	0.66	0.53	0.00	0.00	0.63
time (sec)	N/A	0.017	0.054	0.151	0.577	0.376	0.000	0.000	1.355

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	55	62	59	193	0	0	78
N.S.	1	1.00	0.51	0.58	0.55	1.80	0.00	0.00	0.73
time (sec)	N/A	0.020	0.053	0.158	0.569	0.399	0.000	0.000	1.026

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	40	49	47	0	71047	60
N.S.	1	1.00	0.59	0.53	0.64	0.62	0.00	934.83	0.79
time (sec)	N/A	0.013	0.087	0.126	0.571	0.400	0.000	7.544	0.621

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	42	32	159	0	0	40
N.S.	1	1.00	0.65	0.61	0.46	2.30	0.00	0.00	0.58
time (sec)	N/A	0.011	0.045	0.130	0.558	0.419	0.000	0.000	0.444

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	13	31	0	0	31
N.S.	1	1.00	0.91	0.83	0.37	0.89	0.00	0.00	0.89
time (sec)	N/A	0.006	0.034	0.112	0.588	0.349	0.000	0.000	0.323

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	26	97	0	0	23
N.S.	1	1.00	0.89	1.04	0.96	3.59	0.00	0.00	0.85
time (sec)	N/A	0.002	0.008	0.094	0.520	0.396	0.000	0.000	0.090

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	42	72	116	0	0	-1
N.S.	1	1.00	0.92	1.17	2.00	3.22	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.014	0.115	0.587	0.429	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	54	31	0	0	62
N.S.	1	1.00	0.91	0.83	1.54	0.89	0.00	0.00	1.77
time (sec)	N/A	0.009	0.014	0.116	0.641	0.371	0.000	0.000	0.424

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	52	104	747	210	0	0	-1
N.S.	1	1.00	0.67	1.33	9.58	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.035	0.151	0.602	0.426	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	42	311	46	0	0	131
N.S.	1	1.00	0.59	0.55	4.09	0.61	0.00	0.00	1.72
time (sec)	N/A	0.014	0.034	0.125	0.611	0.384	0.000	0.000	1.178

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	66	121	1914	244	0	0	-1
N.S.	1	1.00	0.57	1.04	16.50	2.10	0.00	0.00	-0.01
time (sec)	N/A	0.024	0.064	0.202	0.664	0.423	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	58	52	68	54	0	0	73
N.S.	1	1.00	0.54	0.49	0.64	0.50	0.00	0.00	0.68
time (sec)	N/A	0.016	0.053	0.155	0.582	0.371	0.000	0.000	1.178

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	55	62	49	182	0	0	78
N.S.	1	1.00	0.56	0.63	0.50	1.86	0.00	0.00	0.80
time (sec)	N/A	0.018	0.043	0.172	0.600	0.407	0.000	0.000	1.061

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	40	42	42	0	0	60
N.S.	1	1.00	0.64	0.57	0.60	0.60	0.00	0.00	0.86
time (sec)	N/A	0.012	0.044	0.133	0.574	0.387	0.000	0.000	0.638

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	42	25	157	0	0	65
N.S.	1	1.00	0.71	0.67	0.40	2.49	0.00	0.00	1.03
time (sec)	N/A	0.010	0.034	0.137	0.581	0.408	0.000	0.000	0.653

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	13	31	46	0	47
N.S.	1	1.00	1.00	0.91	0.41	0.97	1.44	0.00	1.47
time (sec)	N/A	0.005	0.017	0.125	0.563	0.383	24.207	0.000	0.363

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	28	26	97	22	0	37
N.S.	1	1.00	1.00	1.17	1.08	4.04	0.92	0.00	1.54
time (sec)	N/A	0.002	0.007	0.098	0.517	0.423	1.091	0.000	0.263

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	65	116	0	0	-1
N.S.	1	1.00	1.00	1.27	1.97	3.52	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.008	0.129	0.558	0.438	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	29	59	31	0	0	62
N.S.	1	1.00	1.00	0.91	1.84	0.97	0.00	0.00	1.94
time (sec)	N/A	0.008	0.014	0.126	0.560	0.421	0.000	0.000	0.542

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	52	104	661	207	0	0	-1
N.S.	1	1.00	0.72	1.44	9.18	2.88	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.022	0.159	0.600	0.393	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	45	42	294	44	0	0	131
N.S.	1	1.00	0.64	0.60	4.20	0.63	0.00	0.00	1.87
time (sec)	N/A	0.013	0.040	0.137	0.582	0.383	0.000	0.000	1.374

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	66	121	1656	233	0	0	-1
N.S.	1	1.00	0.62	1.13	15.48	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.036	0.206	0.579	0.420	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	55	62	49	182	0	0	78
N.S.	1	1.00	0.51	0.58	0.46	1.70	0.00	0.00	0.73
time (sec)	N/A	0.021	0.034	0.158	0.592	0.432	0.000	0.000	0.984

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	40	42	42	0	0	60
N.S.	1	1.00	0.59	0.53	0.55	0.55	0.00	0.00	0.79
time (sec)	N/A	0.013	0.032	0.124	0.580	0.381	0.000	0.000	0.695

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	45	42	25	157	0	0	65
N.S.	1	1.00	0.65	0.61	0.36	2.28	0.00	0.00	0.94
time (sec)	N/A	0.011	0.026	0.124	0.580	0.397	0.000	0.000	0.594

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	13	31	0	0	47
N.S.	1	1.00	0.91	0.83	0.37	0.89	0.00	0.00	1.34
time (sec)	N/A	0.006	0.023	0.114	0.588	0.356	0.000	0.000	0.323

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	26	97	22	0	37
N.S.	1	1.00	0.89	1.04	0.96	3.59	0.81	0.00	1.37
time (sec)	N/A	0.002	0.008	0.095	0.505	0.401	14.535	0.000	0.277

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	42	65	116	0	0	-1
N.S.	1	1.00	0.92	1.17	1.81	3.22	0.00	0.00	-0.03
time (sec)	N/A	0.005	0.009	0.127	0.578	0.406	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	67	31	0	0	62
N.S.	1	1.00	0.91	0.83	1.91	0.89	0.00	0.00	1.77
time (sec)	N/A	0.009	0.009	0.125	0.579	0.395	0.000	0.000	0.511

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	52	104	670	207	0	0	-1
N.S.	1	1.00	0.67	1.33	8.59	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.014	0.019	0.148	0.584	0.423	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	42	311	44	0	0	131
N.S.	1	1.00	0.59	0.55	4.09	0.58	0.00	0.00	1.72
time (sec)	N/A	0.013	0.016	0.123	0.570	0.376	0.000	0.000	1.264

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	66	121	1679	233	0	0	-1
N.S.	1	1.00	0.57	1.04	14.47	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.026	0.191	0.607	0.439	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	58	62	49	182	0	0	78
N.S.	1	1.00	0.54	0.58	0.46	1.70	0.00	0.00	0.73
time (sec)	N/A	0.021	0.045	0.159	0.576	0.444	0.000	0.000	0.999

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	48	40	42	42	0	0	60
N.S.	1	1.00	0.63	0.53	0.55	0.55	0.00	0.00	0.79
time (sec)	N/A	0.013	0.027	0.122	0.601	0.400	0.000	0.000	0.602

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	48	42	25	157	0	0	65
N.S.	1	1.00	0.70	0.61	0.36	2.28	0.00	0.00	0.94
time (sec)	N/A	0.011	0.027	0.126	0.579	0.415	0.000	0.000	0.606

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	29	13	31	0	0	47
N.S.	1	1.00	1.00	0.83	0.37	0.89	0.00	0.00	1.34
time (sec)	N/A	0.005	0.014	0.115	0.579	0.394	0.000	0.000	0.410

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	24	28	26	97	0	0	37
N.S.	1	1.00	0.89	1.04	0.96	3.59	0.00	0.00	1.37
time (sec)	N/A	0.002	0.008	0.095	0.568	0.391	0.000	0.000	0.283

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	33	42	65	116	0	0	-1
N.S.	1	1.00	0.92	1.17	1.81	3.22	0.00	0.00	-0.03
time (sec)	N/A	0.006	0.013	0.117	0.595	0.418	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	29	67	31	0	0	87
N.S.	1	1.00	0.91	0.83	1.91	0.89	0.00	0.00	2.49
time (sec)	N/A	0.009	0.010	0.123	0.560	0.400	0.000	0.000	1.073

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	55	104	688	207	0	0	-1
N.S.	1	1.00	0.71	1.33	8.82	2.65	0.00	0.00	-0.01
time (sec)	N/A	0.015	0.027	0.155	0.593	0.424	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	42	343	44	0	0	131
N.S.	1	1.00	0.59	0.55	4.51	0.58	0.00	0.00	1.72
time (sec)	N/A	0.014	0.018	0.124	0.574	0.384	0.000	0.000	1.292

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	66	121	1729	233	0	0	-1
N.S.	1	1.00	0.57	1.04	14.91	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.030	0.191	0.632	0.414	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.069	0.050	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.037	0.105	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.033	0.043	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.023	0.003	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.030	0.048	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.039	0.063	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.028	0.071	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.073	0.039	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.037	0.098	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.034	0.044	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.024	0.002	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.030	0.046	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.038	0.061	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.029	0.072	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.112	0.040	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.048	0.101	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.045	0.044	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.003	0.005	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.004	0.046	0.000	0.000	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	56	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.007	0.060	0.000	0.000	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.037	0.073	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.072	0.037	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.016	0.046	0.094	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.004	0.038	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.003	0.003	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.029	0.050	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.032	0.064	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.068	0.074	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.068	0.037	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	63	0	0	0	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.045	0.092	0.000	0.000	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.004	0.037	0.000	0.000	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.011	0.003	0.001	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.029	0.049	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.035	0.062	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.024	0.065	0.075	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.067	0.036	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.004	0.089	0.000	0.000	0.000	0.000	0.000

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.006	0.039	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	53	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.012	0.003	0.003	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	56	0	0	0	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.020	0.007	0.049	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.039	0.063	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.068	0.076	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	77	0	0	0	0	0	-1
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.053	0.069	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	72	0	0	0	0	0	-1
N.S.	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.047	0.112	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	70	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.034	0.072	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	64	0	0	0	0	0	-1
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.013	0.028	0.013	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	63	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.035	0.074	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	0	0	0	0	0	-1
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.040	0.054	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.028	0.030	0.057	0.000	0.000	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	72	0	0	0	0	0	-1
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.035	0.059	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.083	0.053	0.000	0.000	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.069	0.051	0.000	0.000	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.056	0.051	0.000	0.000	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.037	0.046	0.000	0.000	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.019	0.051	0.044	0.000	0.000	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.054	0.043	0.000	0.000	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.020	0.054	0.044	0.000	0.000	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.021	0.056	0.046	0.000	0.000	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	89	0	0	0	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	6.758	0.108	0.000	0.000	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	13	0	13	15
N.S.	1	1.00	1.00	0.93	0.87	0.87	0.00	0.87	1.00
time (sec)	N/A	0.016	0.014	0.083	0.283	0.362	0.000	0.432	0.304

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	23	0	13	15
N.S.	1	1.00	1.00	0.82	0.76	1.35	0.00	0.76	0.88
time (sec)	N/A	0.016	0.016	0.038	0.285	0.364	0.000	0.475	0.213

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	53	88	0	67	0	0	-1
N.S.	1	1.00	0.79	1.31	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.066	0.102	0.000	0.099	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	61	142	0	83	0	0	-1
N.S.	1	1.00	0.91	2.12	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.088	0.093	0.000	0.108	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	13	22	0	14	16
N.S.	1	1.00	0.86	0.67	0.62	1.05	0.00	0.67	0.76
time (sec)	N/A	0.016	0.015	0.086	0.286	0.348	0.000	0.465	0.335

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	27	26	25	23	0	24	-1
N.S.	1	1.00	0.82	0.79	0.76	0.70	0.00	0.73	-0.03
time (sec)	N/A	0.022	0.037	0.104	0.280	0.360	0.000	0.456	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	29	26	25	33	0	26	-1
N.S.	1	1.00	0.83	0.74	0.71	0.94	0.00	0.74	-0.03
time (sec)	N/A	0.022	0.040	0.074	0.285	0.383	0.000	0.450	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	100	0	78	0	0	-1
N.S.	1	1.00	0.68	1.09	0.00	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.088	0.103	0.000	0.111	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	63	152	0	95	0	0	-1
N.S.	1	1.00	0.68	1.65	0.00	1.03	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.255	0.098	0.000	0.109	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	6	6	0	6	8
N.S.	1	1.00	1.00	0.70	0.60	0.60	0.00	0.60	0.80
time (sec)	N/A	0.011	0.006	0.034	0.283	0.364	0.000	0.447	0.203

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	47	24	41	95	0	42	-1
N.S.	1	1.00	1.47	0.75	1.28	2.97	0.00	1.31	-0.03
time (sec)	N/A	0.019	0.016	0.072	0.536	0.431	0.000	0.435	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	50	43	41	97	0	42	-1
N.S.	1	1.00	1.61	1.39	1.32	3.13	0.00	1.35	-0.03
time (sec)	N/A	0.019	0.020	0.077	0.535	0.425	0.000	0.520	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	123	0	82	0	0	-1
N.S.	1	1.00	0.80	2.02	0.00	1.34	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.093	0.118	0.000	0.096	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	177	0	96	0	0	-1
N.S.	1	1.00	0.87	2.85	0.00	1.55	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.091	0.097	0.000	0.106	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	73	73	65	131	0	66	-1
N.S.	1	1.00	1.18	1.18	1.05	2.11	0.00	1.06	-0.02
time (sec)	N/A	0.036	0.069	0.099	0.515	0.399	0.000	0.433	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	33	71	63	141	0	64	-1
N.S.	1	1.00	0.53	1.15	1.02	2.27	0.00	1.03	-0.02
time (sec)	N/A	0.030	0.018	0.094	0.499	0.402	0.000	0.466	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	64	168	0	98	0	0	-1
N.S.	1	1.00	0.70	1.83	0.00	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.244	0.131	0.000	0.096	0.000	0.000	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	76	160	0	114	0	0	-1
N.S.	1	1.00	0.83	1.74	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.148	0.112	0.000	0.101	0.000	0.000	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	105	0	0	0	0	0	-1
N.S.	1	1.00	1.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	41.819	0.111	0.000	0.000	0.000	0.000	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	70	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	10.110	0.086	0.000	0.000	0.000	0.000	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	10.143	0.077	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	68	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	10.173	0.072	0.000	0.000	0.000	0.000	0.000

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	0	0	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	10.185	0.076	0.000	0.000	0.000	0.000	0.000

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	312	0	0	0	0	0	-1
N.S.	1	1.00	3.67	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	1.207	0.094	0.000	0.000	0.000	0.000	0.000

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	314	0	0	0	0	0	-1
N.S.	1	1.00	3.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.175	0.077	0.000	0.000	0.000	0.000	0.000

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	314	0	0	0	0	0	-1
N.S.	1	1.00	3.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.173	0.093	0.000	0.000	0.000	0.000	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	316	0	0	0	0	0	-1
N.S.	1	1.00	3.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.173	0.084	0.000	0.000	0.000	0.000	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	76	94	0	0	0	0	0	-1
N.S.	1	0.97	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	9.828	0.109	0.000	0.000	0.000	0.000	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	94	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	6.157	0.104	0.000	0.000	0.000	0.000	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	94	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.075	5.433	0.101	0.000	0.000	0.000	0.000	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	96	0	0	0	0	0	-1
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	5.153	0.110	0.000	0.000	0.000	0.000	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	225	0	0	0	0	0	-1
N.S.	1	1.00	2.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.068	17.423	0.089	0.000	0.000	0.000	0.000	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	116	0	0	0	0	0	-1
N.S.	1	1.00	1.49	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	8.691	0.084	0.000	0.000	0.000	0.000	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	125	0	0	0	0	0	-1
N.S.	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	6.700	0.084	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [140] had the largest ratio of [23]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	6	0.167
2	A	2	2	1.00	8	0.250
3	A	2	1	1.00	8	0.125
4	A	3	2	1.00	8	0.250
5	A	2	1	1.00	8	0.125
6	A	4	2	1.00	8	0.250
7	A	2	1	1.00	8	0.125
8	A	5	2	1.00	8	0.250
9	A	3	2	1.00	10	0.200
10	A	2	2	1.00	10	0.200
11	A	2	2	1.00	10	0.200
12	A	1	1	1.00	10	0.100
13	A	1	1	1.00	10	0.100
14	A	2	2	1.00	10	0.200
15	A	2	2	1.00	10	0.200
16	A	3	2	1.00	10	0.200
17	A	4	3	1.00	12	0.250
18	A	3	3	1.00	12	0.250
19	A	3	3	1.00	12	0.250
20	A	2	2	1.00	12	0.167
21	A	2	2	1.00	12	0.167
22	A	3	3	1.00	12	0.250
23	A	3	3	1.00	12	0.250
24	A	4	3	1.00	12	0.250
25	A	1	1	1.00	10	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	1	1	1.00	10	0.100
27	A	1	1	1.00	10	0.100
28	A	1	1	1.00	10	0.100
29	A	1	1	1.00	10	0.100
30	A	1	1	1.00	10	0.100
31	A	1	1	1.00	12	0.083
32	A	1	1	1.00	12	0.083
33	A	1	1	1.00	12	0.083
34	A	1	1	1.00	12	0.083
35	A	1	1	1.00	12	0.083
36	A	1	1	1.00	12	0.083
37	A	1	1	1.00	8	0.125
38	A	1	1	1.00	10	0.100
39	A	4	3	1.00	10	0.300
40	A	3	3	1.00	10	0.300
41	A	2	2	1.00	10	0.200
42	A	2	2	1.00	10	0.200
43	A	3	3	1.00	10	0.300
44	A	4	3	1.00	10	0.300
45	A	6	3	1.00	10	0.300
46	A	4	3	1.00	10	0.300
47	A	3	3	1.00	10	0.300
48	A	3	3	1.00	10	0.300
49	A	4	3	1.00	10	0.300
50	A	6	3	1.00	10	0.300
51	A	7	3	1.00	10	0.300
52	A	5	3	1.00	10	0.300
53	A	3	3	1.00	10	0.300
54	A	3	3	1.00	10	0.300
55	A	3	2	1.00	10	0.200
56	A	3	2	1.00	10	0.200
57	A	2	2	1.00	12	0.167
58	A	2	2	1.00	14	0.143
59	A	2	2	1.00	14	0.143
60	A	2	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	2	2	1.00	14	0.143
62	A	2	2	1.00	14	0.143
63	A	2	2	1.00	14	0.143
64	A	2	2	1.00	14	0.143
65	A	2	2	1.00	14	0.143
66	A	6	4	1.00	21	0.190
67	A	5	4	1.00	21	0.190
68	A	5	4	1.00	21	0.190
69	A	4	4	1.00	21	0.190
70	A	4	4	1.00	19	0.210
71	A	2	2	1.00	12	0.167
72	A	3	3	1.00	19	0.158
73	A	4	4	1.00	21	0.190
74	A	4	4	1.00	21	0.190
75	A	5	4	1.00	21	0.190
76	A	5	4	1.00	21	0.190
77	A	6	4	1.00	21	0.190
78	A	6	4	1.00	21	0.190
79	A	5	4	1.00	21	0.190
80	A	5	4	1.00	21	0.190
81	A	4	4	1.00	19	0.210
82	A	3	3	1.00	12	0.250
83	A	3	3	1.00	19	0.158
84	A	3	3	1.00	21	0.143
85	A	4	4	1.00	21	0.190
86	A	4	4	1.00	21	0.190
87	A	5	4	1.00	21	0.190
88	A	5	4	1.00	21	0.190
89	A	6	4	1.00	21	0.190
90	A	6	4	1.00	21	0.190
91	A	5	4	1.00	21	0.190
92	A	5	4	1.00	19	0.210
93	A	3	3	1.00	12	0.250
94	A	4	4	1.00	19	0.210
95	A	3	3	1.00	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.00	21	0.143
97	A	4	4	1.00	21	0.190
98	A	4	4	1.00	21	0.190
99	A	5	4	1.00	21	0.190
100	A	5	4	1.00	21	0.190
101	A	6	4	1.00	21	0.190
102	A	4	3	1.00	12	0.250
103	A	6	4	1.00	21	0.190
104	A	5	4	1.00	21	0.190
105	A	5	4	1.00	21	0.190
106	A	4	4	1.00	21	0.190
107	A	4	4	1.00	21	0.190
108	A	3	3	1.00	19	0.158
109	A	2	2	1.00	12	0.167
110	A	4	4	1.00	19	0.210
111	A	4	4	1.00	21	0.190
112	A	5	4	1.00	21	0.190
113	A	5	4	1.00	21	0.190
114	A	6	4	1.00	21	0.190
115	A	6	4	1.00	21	0.190
116	A	5	4	1.00	21	0.190
117	A	5	4	1.00	21	0.190
118	A	4	4	1.00	21	0.190
119	A	4	4	1.00	21	0.190
120	A	3	3	1.00	21	0.143
121	A	3	3	1.00	19	0.158
122	A	3	3	1.00	12	0.250
123	A	4	4	1.00	19	0.210
124	A	5	4	1.00	21	0.190
125	A	5	4	1.00	21	0.190
126	A	6	4	1.00	21	0.190
127	A	6	4	1.00	21	0.190
128	A	5	4	1.00	21	0.190
129	A	5	4	1.00	21	0.190
130	A	4	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	21	0.190
132	A	3	3	1.00	21	0.143
133	A	3	3	1.00	21	0.143
134	A	4	4	1.00	19	0.210
135	A	3	3	1.00	12	0.250
136	A	5	4	1.00	19	0.210
137	A	5	4	1.00	21	0.190
138	A	6	4	1.00	21	0.190
139	A	4	3	1.00	12	0.250
140	A	4	3	1.00	23	0.130
141	A	3	2	1.00	23	0.087
142	A	3	3	1.00	23	0.130
143	A	2	2	1.00	23	0.087
144	A	2	2	1.00	23	0.087
145	A	2	2	1.00	23	0.087
146	A	3	3	1.00	23	0.130
147	A	3	3	1.00	23	0.130
148	A	3	2	1.00	23	0.087
149	A	4	3	1.00	23	0.130
150	A	4	3	1.00	23	0.130
151	A	3	2	1.00	23	0.087
152	A	3	3	1.00	23	0.130
153	A	2	2	1.00	23	0.087
154	A	2	2	1.00	23	0.087
155	A	2	2	1.00	23	0.087
156	A	3	3	1.00	23	0.130
157	A	3	3	1.00	23	0.130
158	A	3	2	1.00	23	0.087
159	A	4	3	1.00	23	0.130
160	A	3	2	1.00	23	0.087
161	A	4	3	1.00	23	0.130
162	A	3	2	1.00	23	0.087
163	A	3	3	1.00	23	0.130
164	A	2	2	1.00	23	0.087
165	A	2	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	2	2	1.00	23	0.087
167	A	3	3	1.00	23	0.130
168	A	3	3	1.00	23	0.130
169	A	3	2	1.00	23	0.087
170	A	4	3	1.00	23	0.130
171	A	3	2	1.00	23	0.087
172	A	4	3	1.00	23	0.130
173	A	3	2	1.00	23	0.087
174	A	3	3	1.00	23	0.130
175	A	2	2	1.00	23	0.087
176	A	2	2	1.00	23	0.087
177	A	2	2	1.00	23	0.087
178	A	3	3	1.00	23	0.130
179	A	3	3	1.00	23	0.130
180	A	3	2	1.00	23	0.087
181	A	4	3	1.00	23	0.130
182	A	4	3	1.00	23	0.130
183	A	3	2	1.00	23	0.087
184	A	3	3	1.00	23	0.130
185	A	2	2	1.00	23	0.087
186	A	2	2	1.00	23	0.087
187	A	2	2	1.00	23	0.087
188	A	3	3	1.00	23	0.130
189	A	3	3	1.00	23	0.130
190	A	3	2	1.00	23	0.087
191	A	4	3	1.00	23	0.130
192	A	4	3	1.00	23	0.130
193	A	3	2	1.00	23	0.087
194	A	3	3	1.00	23	0.130
195	A	2	2	1.00	23	0.087
196	A	2	2	1.00	23	0.087
197	A	2	2	1.00	23	0.087
198	A	3	3	1.00	23	0.130
199	A	3	3	1.00	23	0.130
200	A	3	2	1.00	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	3	1.00	23	0.130
202	A	2	2	1.00	21	0.095
203	A	2	2	1.00	21	0.095
204	A	2	2	1.00	19	0.105
205	A	1	1	1.00	12	0.083
206	A	2	2	1.00	19	0.105
207	A	2	2	1.00	21	0.095
208	A	2	2	1.00	21	0.095
209	A	2	2	1.00	21	0.095
210	A	2	2	1.00	21	0.095
211	A	2	2	1.00	19	0.105
212	A	1	1	1.00	12	0.083
213	A	2	2	1.00	19	0.105
214	A	2	2	1.00	21	0.095
215	A	2	2	1.00	21	0.095
216	A	2	2	1.00	21	0.095
217	A	2	2	1.00	21	0.095
218	A	2	2	1.00	19	0.105
219	A	1	1	1.00	12	0.083
220	A	2	2	1.00	19	0.105
221	A	2	2	1.00	21	0.095
222	A	2	2	1.00	21	0.095
223	A	2	2	1.00	21	0.095
224	A	2	2	1.00	21	0.095
225	A	2	2	1.00	19	0.105
226	A	1	1	1.00	12	0.083
227	A	2	2	1.00	19	0.105
228	A	2	2	1.00	21	0.095
229	A	2	2	1.00	21	0.095
230	A	2	2	1.00	21	0.095
231	A	2	2	1.00	21	0.095
232	A	2	2	1.00	19	0.105
233	A	1	1	1.00	12	0.083
234	A	2	2	1.00	19	0.105
235	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	2	2	1.00	21	0.095
237	A	2	2	1.00	21	0.095
238	A	2	2	1.00	21	0.095
239	A	2	2	1.00	19	0.105
240	A	1	1	1.00	12	0.083
241	A	2	2	1.00	19	0.105
242	A	2	2	1.00	21	0.095
243	A	2	2	1.00	21	0.095
244	A	2	2	1.00	21	0.095
245	A	2	2	1.00	19	0.105
246	A	2	2	1.00	17	0.118
247	A	1	1	1.00	10	0.100
248	A	2	2	1.00	17	0.118
249	A	2	2	1.00	19	0.105
250	A	2	2	1.00	19	0.105
251	A	2	2	1.00	19	0.105
252	A	2	2	1.00	21	0.095
253	A	2	2	1.00	21	0.095
254	A	2	2	1.00	21	0.095
255	A	2	2	1.00	21	0.095
256	A	2	2	1.00	21	0.095
257	A	2	2	1.00	21	0.095
258	A	2	2	1.00	21	0.095
259	A	2	2	1.00	21	0.095
260	A	2	2	1.00	21	0.095
261	A	2	2	1.00	17	0.118
262	A	2	2	1.00	17	0.118
263	A	3	3	1.00	19	0.158
264	A	3	3	1.00	19	0.158
265	A	3	2	1.00	11	0.182
266	A	3	2	1.00	19	0.105
267	A	3	2	1.00	19	0.105
268	A	4	3	1.00	19	0.158
269	A	4	3	1.00	19	0.158
270	A	2	2	1.00	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	5	1.00	17	0.294
272	A	5	5	1.00	17	0.294
273	A	3	3	1.00	19	0.158
274	A	3	3	1.00	19	0.158
275	A	6	6	1.00	19	0.316
276	A	6	6	1.00	19	0.316
277	A	4	3	1.00	19	0.158
278	A	4	3	1.00	19	0.158
279	A	2	2	1.00	21	0.095
280	A	2	2	1.00	21	0.095
281	A	2	2	1.00	21	0.095
282	A	2	2	1.00	21	0.095
283	A	2	2	1.00	21	0.095
284	A	2	2	1.00	17	0.118
285	A	2	2	1.00	19	0.105
286	A	2	2	1.00	19	0.105
287	A	2	2	1.00	21	0.095
288	A	2	2	0.97	23	0.087
289	A	2	2	1.00	23	0.087
290	A	2	2	1.00	23	0.087
291	A	2	2	1.00	23	0.087
292	A	2	2	1.00	23	0.087
293	A	2	2	1.00	23	0.087
294	A	2	2	1.00	23	0.087

Chapter 3

Listing of integrals

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3.30 $\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx$ 188
3.31 $\int (c\cos(a+bx))^{4/3}\,dx$ 191
3.32 $\int (c\cos(a+bx))^{2/3}\,dx$ 194
3.33 $\int \sqrt[3]{c\cos(a+bx)}\,dx$ 197
3.34 $\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx$ 200
3.35 $\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx$ 203
3.36 $\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx$ 206
3.37 $\int \cos^n(a+bx)\,dx$ 209
3.38 $\int (c\cos(a+bx))^n\,dx$ 212
3.39 $\int (a\cos^2(x))^{5/2}\,dx$ 215
3.40 $\int (a\cos^2(x))^{3/2}\,dx$ 218
3.41 $\int \sqrt{a\cos^2(x)}\,dx$ 221
3.42 $\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx$ 224
3.43 $\int \frac{1}{(a\cos^2(x))^{3/2}}\,dx$ 227
3.44 $\int \frac{1}{(a\cos^2(x))^{5/2}}\,dx$ 231
3.45 $\int (a\cos^3(x))^{5/2}\,dx$ 235
3.46 $\int (a\cos^3(x))^{3/2}\,dx$ 239
3.47 $\int \sqrt{a\cos^3(x)}\,dx$ 243
3.48 $\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$ 246
3.49 $\int \frac{1}{(a\cos^3(x))^{3/2}}\,dx$ 249
3.50 $\int \frac{1}{(a\cos^3(x))^{5/2}}\,dx$ 252
3.51 $\int (a\cos^4(x))^{5/2}\,dx$ 256
3.52 $\int (a\cos^4(x))^{3/2}\,dx$ 260
3.53 $\int \sqrt{a\cos^4(x)}\,dx$ 263
3.54 $\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx$ 266
3.55 $\int \frac{1}{(a\cos^4(x))^{3/2}}\,dx$ 269
3.56 $\int \frac{1}{(a\cos^4(x))^{5/2}}\,dx$ 272
3.57 $\int (b\cos^m(c+dx))^n\,dx$ 276
3.58 $\int (c\cos^m(a+bx))^{5/2}\,dx$ 279
3.59 $\int (c\cos^m(a+bx))^{3/2}\,dx$ 282
3.60 $\int \sqrt{c\cos^m(a+bx)}\,dx$ 285

3.61 $\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx$ . . . 288
3.62 $\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx$ . . . 291
3.63 $\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx$ . . . 294
3.64 $\int (c\cos^m(a+bx))^{\frac{1}{m}}\,dx$ . . . 297
3.65 $\int (a(b\cos(c+dx))^p)^n\,dx$ . . . 300
3.66 $\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 303
3.67 $\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 307
3.68 $\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 311
3.69 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 315
3.70 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 319
3.71 $\int \sqrt{b\cos(c+dx)}\,dx$ . . . 323
3.72 $\int \sqrt{b\cos(c+dx)}\,\sec(c+dx)\,dx$ . . . 326
3.73 $\int \sqrt{b\cos(c+dx)}\,\sec^2(c+dx)\,dx$ . . . 329
3.74 $\int \sqrt{b\cos(c+dx)}\,\sec^3(c+dx)\,dx$ . . . 333
3.75 $\int \sqrt{b\cos(c+dx)}\,\sec^4(c+dx)\,dx$ . . . 337
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3.77 $\int \sqrt{b\cos(c+dx)}\,\sec^6(c+dx)\,dx$ . . . 345
3.78 $\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$ . . . 349
3.79 $\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx$ . . . 353
3.80 $\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx$ . . . 357
3.81 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx$ . . . 361
3.82 $\int (b\cos(c+dx))^{3/2}\,dx$ . . . 365
3.83 $\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$ . . . 368
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3.88 $\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx$ . . . 386
3.89 $\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx$ . . . 390
3.90 $\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$ . . . 394
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3.94 $\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$ . . . 409
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3.96 $\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$ . . . 416
3.97 $\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$ . . . 419
3.98 $\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$ . . . 423
3.99 $\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx$ . . . 427
3.100 $\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx$ . . . 431
3.101 $\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx$ . . . 435
3.102 $\int (b\cos(c+dx))^{7/2}\,dx$ . . . 439

3.103 $\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 442
3.104 $\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 446
3.105 $\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 450
3.106 $\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 454
3.107 $\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 458
3.108 $\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 462
3.109 $\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$ 466
3.110 $\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 469
3.111 $\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 473
3.112 $\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 477
3.113 $\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 481
3.114 $\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 485
3.115 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 489
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3.117 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 497
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3.119 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 505
3.120 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 509
3.121 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 512
3.122 $\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$ 515
3.123 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 519
3.124 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 523
3.125 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 527
3.126 $\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 531
3.127 $\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ 535
3.128 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ 539
3.129 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ 543
3.130 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ 547
3.131 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ 551

3.132 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 555
3.133 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 558
3.134 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 561
3.135 $\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 565
3.136 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 569
3.137 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 573
3.138 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 577
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3.140 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 585
3.141 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$ . . . 589
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3.144 $\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$ . . . 601
3.145 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 604
3.146 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 607
3.147 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 610
3.148 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 614
3.149 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ . . . 618
3.150 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$ . . . 623
3.151 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$ . . . 627
3.152 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\,dx$ . . . 632
3.153 $\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$ . . . 636
3.154 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 639
3.155 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 642
3.156 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ . . . 645
3.157 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ . . . 648
3.158 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ . . . 652
3.159 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$ . . . 656
3.160 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$ . . . 661
3.161 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$ . . . 664

3.162 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\,dx$ 668
3.163 $\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$ 673
3.164 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ 676
3.165 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ 679
3.166 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 682
3.167 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ 685
3.168 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$ 688
3.169 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$ 692
3.170 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx$ 696
3.171 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 701
3.172 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 704
3.173 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 708
3.174 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 711
3.175 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$ 714
3.176 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx$ 717
3.177 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$ 720
3.178 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 723
3.179 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 726
3.180 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 730
3.181 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$ 733
3.182 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 738
3.183 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 742
3.184 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 745
3.185 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 748
3.186 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$ 751
3.187 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx$ 754

3.188 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx$ . . . 757
3.189 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$ . . . 760
3.190 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$ . . . 764
3.191 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$ . . . 767
3.192 $\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 772
3.193 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 776
3.194 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 779
3.195 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 782
3.196 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 785
3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 788
3.198 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx$ . . . 791
3.199 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx$ . . . 794
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3.202 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$ . . . 806
3.203 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$ . . . 809
3.204 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$ . . . 812
3.205 $\int \sqrt[3]{b\cos(c+dx)}\,dx$ . . . 815
3.206 $\int \sqrt[3]{b\cos(c+dx)}\,\sec(c+dx)\,dx$ . . . 818
3.207 $\int \sqrt[3]{b\cos(c+dx)}\,\sec^2(c+dx)\,dx$ . . . 821
3.208 $\int \sqrt[3]{b\cos(c+dx)}\,\sec^3(c+dx)\,dx$ . . . 824
3.209 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx$ . . . 827
3.210 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx$ . . . 830
3.211 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx$ . . . 833
3.212 $\int (b\cos(c+dx))^{2/3}\,dx$ . . . 836
3.213 $\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx$ . . . 839
3.214 $\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx$ . . . 842
3.215 $\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx$ . . . 845
3.216 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx$ . . . 848
3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx$ . . . 851
3.218 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx$ . . . 854
3.219 $\int (b\cos(c+dx))^{4/3}\,dx$ . . . 857
3.220 $\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx$ . . . 860
3.221 $\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx$ . . . 863
3.222 $\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx$ . . . 866
3.223 $\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 869

3.224 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 872
3.225 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 875
3.226 $\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 878
3.227 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 881
3.228 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 884
3.229 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$ . . . 887
3.230 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ . . . 890
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3.232 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ . . . 896
3.233 $\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$ . . . 899
3.234 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ . . . 902
3.235 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ . . . 905
3.236 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$ . . . 908
3.237 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 911
3.238 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 914
3.239 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 917
3.240 $\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 920
3.241 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 923
3.242 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 926
3.243 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$ . . . 929
3.244 $\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx$ . . . 932
3.245 $\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx$ . . . 935
3.246 $\int \cos(c+dx)(b\cos(c+dx))^n\,dx$ . . . 938
3.247 $\int (b\cos(c+dx))^n\,dx$ . . . 941
3.248 $\int (b\cos(c+dx))^n\sec(c+dx)\,dx$ . . . 944
3.249 $\int (b\cos(c+dx))^n\sec^2(c+dx)\,dx$ . . . 947
3.250 $\int (b\cos(c+dx))^n\sec^3(c+dx)\,dx$ . . . 950
3.251 $\int (b\cos(c+dx))^n\sec^4(c+dx)\,dx$ . . . 953
3.252 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx$ . . . 956
3.253 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx$ . . . 959
3.254 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,dx$ . . . 962
3.255 $\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$ . . . 965
3.256 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$ . . . 968
3.257 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx$ . . . 971

3.258 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx$ 974
3.259 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx$ 977
3.260 $\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx$ 980
3.261 $\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$ 983
3.262 $\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 986
3.263 $\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$ 989
3.264 $\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 992
3.265 $\int \cos^3(x)\csc^{\frac{9}{2}}(x)\,dx$ 995
3.266 $\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx$ 998
3.267 $\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 1001
3.268 $\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx$ 1004
3.269 $\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 1007
3.270 $\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx$ 1011
3.271 $\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx$ 1014
3.272 $\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 1018
3.273 $\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$ 1022
3.274 $\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 1025
3.275 $\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$ 1028
3.276 $\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 1032
3.277 $\int \sqrt{\csc(a+bx)}\sec^4(a+bx)\,dx$ 1036
3.278 $\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ 1039
3.279 $\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx$ 1043
3.280 $\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx$ 1046
3.281 $\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$ 1049
3.282 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$ 1052
3.283 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$ 1055
3.284 $\int \cos^m(e+fx)\csc^n(e+fx)\,dx$ 1058
3.285 $\int (a\cos(e+fx))^m\csc^n(e+fx)\,dx$ 1061
3.286 $\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx$ 1064
3.287 $\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx$ 1067
3.288 $\int (a\cos(e+fx))^m(b\csc(e+fx))^{7/2}\,dx$ 1070
3.289 $\int (a\cos(e+fx))^m(b\csc(e+fx))^{5/2}\,dx$ 1073
3.290 $\int (a\cos(e+fx))^m(b\csc(e+fx))^{3/2}\,dx$ 1076
3.291 $\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx$ 1079

3.292 $\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx$ 1082
3.293 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx$ 1085
3.294 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx$ 1088

3.1 $\int \cos(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sin(a+bx)}{b}$$

[Out] sin(b*x+a)/b

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2717}

$$\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x],x]

[Out] Sin[a + b*x]/b

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)\,dx = \frac{\sin(a+bx)}{b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.
time = 0.06, size = 21, normalized size = 2.10

$$\frac{\cos(bx)\sin(a)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[a + b*x],x]

[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b

Maple [A]
time = 0.08, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\sin(bx+a)}{b}$	11
default	$\frac{\sin(bx+a)}{b}$	11
risch	$\frac{\sin(bx+a)}{b}$	11
norman	$\frac{2\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}$	30
meijerg	$\frac{\cos(a)\sin(bx)}{b}-\frac{\sin(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(bx)}{\sqrt{\pi}}\right)}{b}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] sin(b*x+a)/b
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="maxima")
```

```
[Out] sin(b*x + a)/b
```

Fricas [A]
time = 0.35, size = 10, normalized size = 1.00

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/b
```

Sympy [A]
time = 0.04, size = 12, normalized size = 1.20

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for}\ b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

Giac [A]
time = 0.45, size = 10, normalized size = 1.00

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="giac")
```

```
[Out] sin(b*x + a)/b
```

Mupad [B]
time = 0.25, size = 10, normalized size = 1.00

$$\frac{\sin(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x),x)
```

```
[Out] sin(a + b*x)/b
```

3.2 $\int \cos^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}$$

```
[Out] 1/2*x+1/2*cos(b*x+a)*sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2,x]
```

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\,dx &= \frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A]
time = 0.04, size = 27, normalized size = 1.08

method	result	size
risch	$\frac{x}{2}+\frac{\sin(2bx+2a)}{4b}$	19
derivativedivides	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
norman	$\frac{\frac{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+x\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\frac{x}{2}-\frac{\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{x\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.88

$$\frac{2\,bx+2\,a+\sin\left(2\,bx+2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 0.36, size = 22, normalized size = 0.88

$$\frac{bx+\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.
time = 0.07, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

Giac [**A**]
time = 0.49, size = 18, normalized size = 0.72

$$\frac{1}{2}x + \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

Mupad [**B**]
time = 0.23, size = 18, normalized size = 0.72

$$\frac{x}{2} + \frac{\sin(2a + 2bx)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2,x)
```

```
[Out] x/2 + sin(2*a + 2*b*x)/(4*b)
```

3.3 $\int \cos^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] sin(b*x+a)/b-1/3*sin(b*x+a)^3/b
```

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{2713\}$

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3,x]
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.00

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3,x]
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.85

method	result	size
derivativedivides	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
default	$\frac{(2+\cos^2(bx+a))\sin(bx+a)}{3b}$	22
risch	$\frac{3\sin(bx+a)}{4b}+\frac{\sin(3bx+3a)}{12b}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.85

$$-\frac{\sin(bx+a)^3-3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Fricas [A]
time = 0.35, size = 21, normalized size = 0.81

$$\frac{(\cos(bx+a)^2+2)\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [A]
time = 0.09, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)\cos^2(a+bx)}{b} & \text{for } b\neq 0 \\ x\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
0)), (x*cos(a)**3, True))
```

Giac [A]
time = 0.47, size = 22, normalized size = 0.85

$$-\frac{\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Mupad [B]
time = 0.07, size = 24, normalized size = 0.92

$$\frac{3\,\sin\left(a+b\,x\right)-\sin\left(a+b\,x\right)^{3}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3,x)
```

```
[Out] (3*sin(a + b*x) - sin(a + b*x)^3)/(3*b)
```

3.4 $\int \cos^4(a+bx)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{8}+\frac{3\cos(a+bx)\sin(a+bx)}{8b}+\frac{\cos^3(a+bx)\sin(a+bx)}{4b}$$

```
[Out] 3/8*x+3/8*cos(b*x+a)*sin(b*x+a)/b+1/4*cos(b*x+a)^3*sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3\sin(a+bx)\cos(a+bx)}{8b}+\frac{3x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[a + b*x]*Sin[a + b*x])/(8*b) + (Cos[a + b*x]^3*Sin[a + b*x
])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(a+bx)\,dx &= \frac{\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{3}{4}\int \cos^2(a+bx)\,dx\\
&= \frac{3\cos(a+bx)\sin(a+bx)}{8b}+\frac{\cos^3(a+bx)\sin(a+bx)}{4b}+\frac{3\int 1\,dx}{8}\\
&= \frac{3x}{8}+\frac{3\cos(a+bx)\sin(a+bx)}{8b}+\frac{\cos^3(a+bx)\sin(a+bx)}{4b}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 0.72

$$\frac{12(a+bx)+8\sin(2(a+bx))+\sin(4(a+bx))}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4,x]
```

```
[Out] (12*(a + b*x) + 8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)])/(32*b)
```

Maple [A]
time = 0.08, size = 38, normalized size = 0.83

method	result
risch	$\frac{3x}{8}+\frac{\sin(4bx+4a)}{32b}+\frac{\sin(2bx+2a)}{4b}$
derivativedivides	$\frac{\frac{\left(\cos^{3}(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{\frac{\left(\cos^{3}(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}}{b}$
norman	$\frac{\frac{3x}{8}+\frac{5\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{4b}-\frac{3\left(\tan^{3}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3\left(\tan^{5}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{5\left(\tan^{7}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{3x\left(\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2}+\frac{9x\left(\tan^{4}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4}+3x(\text{ta}}{\left(1+\tan^{2}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)
```

Maxima [A]
time = 0.28, size = 33, normalized size = 0.72

$$\frac{12\,bx+12\,a+\sin\left(4\,bx+4\,a\right)+8\,\sin\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(12*b*x + 12*a + sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 0.39, size = 36, normalized size = 0.78

$$\frac{3\,bx+\left(2\,\cos\left(bx+a\right)^{3}+3\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*b*x + (2*cos(b*x + a)^3 + 3*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(41) = 82$.
time = 0.16, size = 95, normalized size = 2.07

$$\begin{cases} \frac{3x\sin^4{(a+bx)}}{8} + \frac{3x\sin^2{(a+bx)}\cos^2{(a+bx)}}{4} + \frac{3x\cos^4{(a+bx)}}{8} + \frac{3\sin^3{(a+bx)}\cos{(a+bx)}}{8b} + \frac{5\sin{(a+bx)}\cos^3{(a+bx)}}{8b} & \text{for } b \neq 0 \\ x\cos^4{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x*sin(a + b*x)**4/8 + 3*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
3*x*cos(a + b*x)**4/8 + 3*sin(a + b*x)**3*cos(a + b*x)/(8*b) + 5*sin(a + b*
x)*cos(a + b*x)**3/(8*b), Ne(b, 0)), (x*cos(a)**4, True))
```

Giac [A]
time = 0.46, size = 32, normalized size = 0.70

$$\frac{3}{8}\,x + \frac{\sin\left(4\,bx + 4\,a\right)}{32\,b} + \frac{\sin\left(2\,bx + 2\,a\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*b*x + 4*a)/b + 1/4*sin(2*b*x + 2*a)/b
```

Mupad [B]
time = 0.19, size = 31, normalized size = 0.67

$$\frac{3\,x}{8} + \frac{\frac{\sin(2\,a+2\,b\,x)}{4} + \frac{\sin(4\,a+4\,b\,x)}{32}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4,x)
```

```
[Out] (3*x)/8 + (sin(2*a + 2*b*x)/4 + sin(4*a + 4*b*x)/32)/b
```

3.5 $\int \cos^5(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{\sin(a+bx)}{b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}$$

```
[Out] sin(b*x+a)/b-2/3*sin(b*x+a)^3/b+1/5*sin(b*x+a)^5/b
```

Rubi [A]
time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$\frac{\sin^5(a+bx)}{5b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^5,x]
```

```
[Out] Sin[a + b*x]/b - (2*Sin[a + b*x]^3)/(3*b) + Sin[a + b*x]^5/(5*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(a+bx)\,dx&=-\frac{\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(a+bx)\right)}{b}\\&=\frac{\sin(a+bx)}{b}-\frac{2\sin^3(a+bx)}{3b}+\frac{\sin^5(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 1.07

$$\frac{5\sin(a+bx)}{8b}+\frac{5\sin(3(a+bx))}{48b}+\frac{\sin(5(a+bx))}{80b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^5,x]
```

```
[Out] (5*Sin[a + b*x])/(8*b) + (5*Sin[3*(a + b*x)])/(48*b) + Sin[5*(a + b*x)]/(80
*b)
```

Maple [A]
time = 0.07, size = 32, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{5b}$	32
default	$\frac{\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{5b}$	32
risch	$\frac{5\sin(bx+a)}{8b}+\frac{\sin(5bx+5a)}{80b}+\frac{5\sin(3bx+3a)}{48b}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^5,x,method=_RETURNVERBOSE)
[Out] 1/5/b*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.83

$$\frac{3\,\sin\left(bx+a\right)^5-10\,\sin\left(bx+a\right)^3+15\,\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5,x, algorithm="maxima")
[Out] 1/15*(3*sin(b*x + a)^5 - 10*sin(b*x + a)^3 + 15*sin(b*x + a))/b
```

Fricas [A]
time = 0.35, size = 33, normalized size = 0.80

$$\frac{\left(3\,\cos\left(bx+a\right)^4+4\,\cos\left(bx+a\right)^2+8\right)\sin\left(bx+a\right)}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5,x, algorithm="fricas")
[Out] 1/15*(3*cos(b*x + a)^4 + 4*cos(b*x + a)^2 + 8)*sin(b*x + a)/b
```

Sympy [A]
time = 0.23, size = 58, normalized size = 1.41

$$\begin{cases} \frac{8\sin^5\left(a+bx\right)}{15b}+\frac{4\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)}{3b}+\frac{\sin\left(a+bx\right)\cos^4\left(a+bx\right)}{b} & \text{for}\ b\neq 0\\ x\cos^5\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**5,x)
```

```
[Out] Piecewise((8*sin(a + b*x)**5/(15*b) + 4*sin(a + b*x)**3*cos(a + b*x)**2/(3*
b) + sin(a + b*x)*cos(a + b*x)**4/b, Ne(b, 0)), (x*cos(a)**5, True))
```

Giac [A]
time = 0.46, size = 34, normalized size = 0.83

$$\frac{3 \sin{(bx+a)}^5 - 10 \sin{(bx+a)}^3 + 15 \sin{(bx+a)}}{15\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^5,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(b*x + a)^5 - 10*sin(b*x + a)^3 + 15*sin(b*x + a))/b
```

Mupad [B]
time = 0.10, size = 31, normalized size = 0.76

$$\frac{\frac{\sin(a+b\,x)^5}{5} - \frac{2\sin(a+b\,x)^3}{3} + \sin{(a+b\,x)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^5,x)
```

```
[Out] (sin(a + b*x) - (2*sin(a + b*x)^3)/3 + sin(a + b*x)^5/5)/b
```

3.6 $\int \cos^6(a+bx)\,dx$

Optimal. Leaf size=67

$$\frac{5x}{16}+\frac{5\cos(a+bx)\sin(a+bx)}{16b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}$$

```
[Out] 5/16*x+5/16*cos(b*x+a)*sin(b*x+a)/b+5/24*cos(b*x+a)^3*sin(b*x+a)/b+1/6*cos(
b*x+a)^5*sin(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{\sin(a+bx)\cos^5(a+bx)}{6b}+\frac{5\sin(a+bx)\cos^3(a+bx)}{24b}+\frac{5\sin(a+bx)\cos(a+bx)}{16b}+\frac{5x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[a + b*x]*Sin[a + b*x])/(16*b) + (5*Cos[a + b*x]^3*Sin[a +
 b*x])/(24*b) + (Cos[a + b*x]^5*Sin[a + b*x])/(6*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^6(a+bx)\,dx &= \frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{5}{6}\int \cos^4(a+bx)\,dx\\ &= \frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{5}{8}\int \cos^2(a+bx)\,dx\\ &= \frac{5\cos(a+bx)\sin(a+bx)}{16b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}+\frac{5}{16}\int\\ &= \frac{5x}{16}+\frac{5\cos(a+bx)\sin(a+bx)}{16b}+\frac{5\cos^3(a+bx)\sin(a+bx)}{24b}+\frac{\cos^5(a+bx)\sin(a+bx)}{6b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 0.64

$$\frac{60a+60bx+45\sin(2(a+bx))+9\sin(4(a+bx))+\sin(6(a+bx))}{192b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^6,x]
```

```
[Out] (60*a + 60*b*x + 45*Sin[2*(a + b*x)] + 9*Sin[4*(a + b*x)] + Sin[6*(a + b*x)
])/(192*b)
```

Maple [A]
time = 0.09, size = 48, normalized size = 0.72

method	result
risch	$\frac{5x}{16}+\frac{\sin(6bx+6a)}{192b}+\frac{3\sin(4bx+4a)}{64b}+\frac{15\sin(2bx+2a)}{64b}$
derivativedivides	$\frac{\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
default	$\frac{\frac{\left(\cos^5(bx+a)+\frac{5\left(\cos^3(bx+a)\right)}{4}+\frac{15\cos(bx+a)}{8}\right)\sin(bx+a)}{6}+\frac{5bx}{16}+\frac{5a}{16}}{b}$
norman	$\frac{5x}{16}+\frac{11\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{8b}-\frac{5\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}+\frac{15\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}-\frac{15\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{4b}+\frac{5\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{24b}-\frac{11\left(\tan^{11}\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{8b}+15$ … $\left(1+\tan^2\left(\right.\right.$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^6,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/6*(cos(b*x+a)^5+5/4*cos(b*x+a)^3+15/8*cos(b*x+a))*sin(b*x+a)+5/16*b*
x+5/16*a)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.72

$$-\frac{4\sin\left(2bx+2a\right)^3-60bx-60a-9\sin\left(4bx+4a\right)-48\sin\left(2bx+2a\right)}{192b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6,x, algorithm="maxima")
```

```
[Out] -1/192*(4*sin(2*b*x + 2*a)^3 - 60*b*x - 60*a - 9*sin(4*b*x + 4*a) - 48*sin(
2*b*x + 2*a))/b
```

Fricas [A]
time = 0.38, size = 46, normalized size = 0.69

$$\frac{15\,bx+\left(8\,\cos\left(bx+a\right)^{5}+10\,\cos\left(bx+a\right)^{3}+15\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(15*b*x + (8*cos(b*x + a)^5 + 10*cos(b*x + a)^3 + 15*cos(b*x + a))*sin
(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(61) = 122$.
time = 0.37, size = 139, normalized size = 2.07

$$\begin{cases} \frac{5x\sin^{6}\left(a+bx\right)}{16}+\frac{15x\sin^{4}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{16}+\frac{15x\sin^{2}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{16}+\frac{5x\cos^{6}\left(a+bx\right)}{16}+\frac{5\sin^{5}\left(a+bx\right)\cos\left(a+bx\right)}{16b}+\frac{5\sin^{3}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{6b}+\frac{11\sin\left(a+bx\right)\cos^{5}\left(a+bx\right)}{16b} & \text{for}\: b\neq 0 \\ x\cos^{6}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**6,x)
```

```
[Out] Piecewise((5*x*sin(a + b*x)**6/16 + 15*x*sin(a + b*x)**4*cos(a + b*x)**2/16
 + 15*x*sin(a + b*x)**2*cos(a + b*x)**4/16 + 5*x*cos(a + b*x)**6/16 + 5*sin
(a + b*x)**5*cos(a + b*x)/(16*b) + 5*sin(a + b*x)**3*cos(a + b*x)**3/(6*b)
+ 11*sin(a + b*x)*cos(a + b*x)**5/(16*b), Ne(b, 0)), (x*cos(a)**6, True))
```

Giac [A]
time = 0.45, size = 46, normalized size = 0.69

$$\frac{5}{16}\,x+\frac{\sin\left(6\,bx+6\,a\right)}{192\,b}+\frac{3\,\sin\left(4\,bx+4\,a\right)}{64\,b}+\frac{15\,\sin\left(2\,bx+2\,a\right)}{64\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*b*x + 6*a)/b + 3/64*sin(4*b*x + 4*a)/b + 15/64*sin(2*b
*x + 2*a)/b
```

Mupad [B]
time = 0.22, size = 42, normalized size = 0.63

$$\frac{5\,x}{16}+\frac{\frac{15\,\sin(2\,a+2\,b\,x)}{64}+\frac{3\,\sin(4\,a+4\,b\,x)}{64}+\frac{\sin(6\,a+6\,b\,x)}{192}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^6,x)
```

```
[Out] (5*x)/16 + ((15*sin(2*a + 2*b*x))/64 + (3*sin(4*a + 4*b*x))/64 + sin(6*a +
6*b*x)/192)/b
```

3.7 $\int \cos^7(a+bx)\,dx$

Optimal. Leaf size=54

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}$$

```
[Out] sin(b*x+a)/b-sin(b*x+a)^3/b+3/5*sin(b*x+a)^5/b-1/7*sin(b*x+a)^7/b
```

Rubi [A]
time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2713}

$$-\frac{\sin^7(a+bx)}{7b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^3(a+bx)}{b}+\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^7,x]
```

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/b + (3*Sin[a + b*x]^5)/(5*b) - Sin[a + b*x]
^7/(7*b)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^7(a+bx)\,dx &= -\frac{\operatorname{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{b}+\frac{3\sin^5(a+bx)}{5b}-\frac{\sin^7(a+bx)}{7b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 59, normalized size = 1.09

$$\frac{35\sin(a+bx)}{64b}+\frac{7\sin(3(a+bx))}{64b}+\frac{7\sin(5(a+bx))}{320b}+\frac{\sin(7(a+bx))}{448b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^7,x]
```

```
[Out] (35*Sin[a + b*x])/(64*b) + (7*Sin[3*(a + b*x)])/(64*b) + (7*Sin[5*(a + b*x)
])/(320*b) + Sin[7*(a + b*x)]/(448*b)
```

Maple [A]
time = 0.06, size = 42, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx+a)\right)}{5}\right)\sin(bx+a)}{7b}$	42
default	$\frac{\left(\frac{16}{5}+\cos^{6}(bx+a)+\frac{6\left(\cos^{4}(bx+a)\right)}{5}+\frac{8\left(\cos^{2}(bx+a)\right)}{5}\right)\sin(bx+a)}{7b}$	42
risch	$\frac{35\sin(bx+a)}{64b}+\frac{\sin(7bx+7a)}{448b}+\frac{7\sin(5bx+5a)}{320b}+\frac{7\sin(3bx+3a)}{64b}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^7,x,method=_RETURNVERBOSE)
[Out] 1/7/b*(16/5+cos(b*x+a)^6+6/5*cos(b*x+a)^4+8/5*cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A]
time = 0.27, size = 44, normalized size = 0.81

$$-\frac{5\,\sin\left(bx+a\right)^{7}-21\,\sin\left(bx+a\right)^{5}+35\,\sin\left(bx+a\right)^{3}-35\,\sin\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7,x, algorithm="maxima")
[Out] -1/35*(5*sin(b*x + a)^7 - 21*sin(b*x + a)^5 + 35*sin(b*x + a)^3 - 35*sin(b*
x + a))/b
```

Fricas [A]
time = 0.36, size = 43, normalized size = 0.80

$$\frac{\left(5\,\cos\left(bx+a\right)^{6}+6\,\cos\left(bx+a\right)^{4}+8\,\cos\left(bx+a\right)^{2}+16\right)\sin\left(bx+a\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7,x, algorithm="fricas")
[Out] 1/35*(5*cos(b*x + a)^6 + 6*cos(b*x + a)^4 + 8*cos(b*x + a)^2 + 16)*sin(b*x
+ a)/b
```

Sympy [A]
time = 0.55, size = 78, normalized size = 1.44

$$\begin{cases} \frac{16\sin^{7}\left(a+bx\right)}{35b}+\frac{8\sin^{5}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{5b}+\frac{2\sin^{3}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{b}+\frac{\sin\left(a+bx\right)\cos^{6}\left(a+bx\right)}{b} & \text{for } b\neq 0 \\ x\cos^{7}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**7,x)
```

```
[Out] Piecewise((16*sin(a + b*x)**7/(35*b) + 8*sin(a + b*x)**5*cos(a + b*x)**2/(5
*b) + 2*sin(a + b*x)**3*cos(a + b*x)**4/b + sin(a + b*x)*cos(a + b*x)**6/b,
 Ne(b, 0)), (x*cos(a)**7, True))
```

Giac [A]
time = 0.47, size = 44, normalized size = 0.81

$$-\frac{5 \sin\left(bx+a\right)^{7}-21 \sin\left(bx+a\right)^{5}+35 \sin\left(bx+a\right)^{3}-35 \sin\left(bx+a\right)}{35 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^7,x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(b*x + a)^7 - 21*sin(b*x + a)^5 + 35*sin(b*x + a)^3 - 35*sin(b*
x + a))/b
```

Mupad [B]
time = 0.09, size = 43, normalized size = 0.80

$$-\frac{\sin\left(a+b\,x\right)\,\left(5\sin\left(a+b\,x\right)^6-21\sin\left(a+b\,x\right)^4+35\sin\left(a+b\,x\right)^2-35\right)}{35\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^7,x)
```

```
[Out] -(sin(a + b*x)*(35*sin(a + b*x)^2 - 21*sin(a + b*x)^4 + 5*sin(a + b*x)^6 -
35))/(35*b)
```

3.8 $\int \cos^8(a+bx)\,dx$

Optimal. Leaf size=88

$$\frac{35x}{128}+\frac{35\cos(a+bx)\sin(a+bx)}{128b}+\frac{35\cos^3(a+bx)\sin(a+bx)}{192b}+\frac{7\cos^5(a+bx)\sin(a+bx)}{48b}+\frac{\cos^7(a+bx)\sin(a+bx)}{8b}$$

```
[Out] 35/128*x+35/128*cos(b*x+a)*sin(b*x+a)/b+35/192*cos(b*x+a)^3*sin(b*x+a)/b+7/
48*cos(b*x+a)^5*sin(b*x+a)/b+1/8*cos(b*x+a)^7*sin(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 8}

$$\frac{\sin(a+bx)\cos^7(a+bx)}{8b}+\frac{7\sin(a+bx)\cos^5(a+bx)}{48b}+\frac{35\sin(a+bx)\cos^3(a+bx)}{192b}+\frac{35\sin(a+bx)\cos(a+bx)}{128b}+\frac{35x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^8,x]
```

```
[Out] (35*x)/128 + (35*Cos[a + b*x]*Sin[a + b*x])/(128*b) + (35*Cos[a + b*x]^3*Si
n[a + b*x])/(192*b) + (7*Cos[a + b*x]^5*Sin[a + b*x])/(48*b) + (Cos[a + b*x
]^7*Sin[a + b*x])/(8*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\begin{aligned}\int \cos^8(a+bx)\,dx &= \frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \frac{7}{8}\int \cos^6(a+bx)\,dx \\ &= \frac{7\cos^5(a+bx)\sin(a+bx)}{48b} + \frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \frac{35}{48}\int \cos^4(a+bx)\,dx \\ &= \frac{35\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{7\cos^5(a+bx)\sin(a+bx)}{48b} + \frac{\cos^7(a+bx)\sin(a+bx)}{8b} + \\ &= \frac{35\cos(a+bx)\sin(a+bx)}{128b} + \frac{35\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{7\cos^5(a+bx)\sin(a+bx)}{48b} \\ &= \frac{35x}{128} + \frac{35\cos(a+bx)\sin(a+bx)}{128b} + \frac{35\cos^3(a+bx)\sin(a+bx)}{192b} + \frac{7\cos^5(a+bx)\sin(a}{48b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.62

$$\frac{840a + 840bx + 672\sin(2(a+bx)) + 168\sin(4(a+bx)) + 32\sin(6(a+bx)) + 3\sin(8(a+bx))}{3072b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^8,x]
[Out] (840*a + 840*b*x + 672*Sin[2*(a + b*x)] + 168*Sin[4*(a + b*x)] + 32*Sin[6*(
a + b*x)] + 3*Sin[8*(a + b*x)])/(3072*b)
```

Maple [A]
time = 0.11, size = 58, normalized size = 0.66

method	result
derivativedivides	$\frac{\frac{\left(\cos^7(bx+a)+\frac{7\left(\cos^5(bx+a)\right)}{6}+\frac{35\left(\cos^3(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
default	$\frac{\frac{\left(\cos^7(bx+a)+\frac{7\left(\cos^5(bx+a)\right)}{6}+\frac{35\left(\cos^3(bx+a)\right)}{24}+\frac{35\cos(bx+a)}{16}\right)\sin(bx+a)}{8}+\frac{35bx}{128}+\frac{35a}{128}}{b}$
risch	$\frac{35x}{128}+\frac{\sin(8bx+8a)}{1024b}+\frac{\sin(6bx+6a)}{96b}+\frac{7\sin(4bx+4a)}{128b}+\frac{7\sin(2bx+2a)}{32b}$
norman	$\frac{35x}{128}+\frac{93\tan\left(\frac{bx}{2}+\frac{a}{2}\right)}{64b}+\frac{91\left(\tan^3\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{1799\left(\tan^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{1085\left(\tan^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}+\frac{1085\left(\tan^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{192b}-\frac{1799\left(\tan^{11}\left(\frac{b}{2}\right.\right.}{192b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^8,x,method=_RETURNVERBOSE)
[Out] 1/b*(1/8*(cos(b*x+a)^7+7/6*cos(b*x+a)^5+35/24*cos(b*x+a)^3+35/16*cos(b*x+a)
)*sin(b*x+a)+35/128*b*x+35/128*a)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.67

$$-\frac{128 \sin\left(2\,bx+2\,a\right)^{3}-840\,bx-840\,a-3\,\sin\left(8\,bx+8\,a\right)-168\,\sin\left(4\,bx+4\,a\right)-768\,\sin\left(2\,bx+2\,a\right)}{3072\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/3072*(128*sin(2*b*x + 2*a)^3 - 840*b*x - 840*a - 3*sin(8*b*x + 8*a) - 16
8*sin(4*b*x + 4*a) - 768*sin(2*b*x + 2*a))/b
```

Fricas [A]
time = 0.36, size = 56, normalized size = 0.64

$$\frac{105\,bx+\left(48\,\cos\left(bx+a\right)^{7}+56\,\cos\left(bx+a\right)^{5}+70\,\cos\left(bx+a\right)^{3}+105\,\cos\left(bx+a\right)\right)\sin\left(bx+a\right)}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(105*b*x + (48*cos(b*x + a)^7 + 56*cos(b*x + a)^5 + 70*cos(b*x + a)^3
 + 105*cos(b*x + a))*sin(b*x + a))/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(82) = 164$.
time = 0.85, size = 184, normalized size = 2.09

$$\begin{cases} \frac{35x\sin^{8}\left(a+bx\right)}{128}+\frac{35x\sin^{6}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{32}+\frac{105x\sin^{4}\left(a+bx\right)\cos^{4}\left(a+bx\right)}{64}+\frac{35x\sin^{2}\left(a+bx\right)\cos^{6}\left(a+bx\right)}{32}+\frac{35x\cos^{8}\left(a+bx\right)}{128}+\frac{35\sin^{7}\left(a+bx\right)\cos\left(a+bx\right)}{128b}+\frac{385\sin^{5}\left(a+bx\right)\cos^{3}\left(a+bx\right)}{384b}+\frac{511\sin^{3}\left(a+bx\right)\cos^{5}\left(a+bx\right)}{384b}+\frac{93\sin\left(a+bx\right)\cos^{7}\left(a+bx\right)}{128b} & \text{for}\ b\neq 0 \\ x\cos^{8}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**8,x)
```

```
[Out] Piecewise((35*x*sin(a + b*x)**8/128 + 35*x*sin(a + b*x)**6*cos(a + b*x)**2/
32 + 105*x*sin(a + b*x)**4*cos(a + b*x)**4/64 + 35*x*sin(a + b*x)**2*cos(a
+ b*x)**6/32 + 35*x*cos(a + b*x)**8/128 + 35*sin(a + b*x)**7*cos(a + b*x)/(
128*b) + 385*sin(a + b*x)**5*cos(a + b*x)**3/(384*b) + 511*sin(a + b*x)**3*
cos(a + b*x)**5/(384*b) + 93*sin(a + b*x)*cos(a + b*x)**7/(128*b), Ne(b, 0)
), (x*cos(a)**8, True))
```

Giac [A]
time = 0.45, size = 60, normalized size = 0.68

$$\frac{35}{128}\,x+\frac{\sin\left(8\,bx+8\,a\right)}{1024\,b}+\frac{\sin\left(6\,bx+6\,a\right)}{96\,b}+\frac{7\,\sin\left(4\,bx+4\,a\right)}{128\,b}+\frac{7\,\sin\left(2\,bx+2\,a\right)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*b*x + 8*a)/b + 1/96*sin(6*b*x + 6*a)/b + 7/128*sin(
4*b*x + 4*a)/b + 7/32*sin(2*b*x + 2*a)/b
```

Mupad [B]
time = 0.28, size = 53, normalized size = 0.60

$$\frac{35\,x}{128}+\frac{\frac{7\,\sin\left(2\,a+2\,b\,x\right)}{32}+\frac{7\,\sin\left(4\,a+4\,b\,x\right)}{128}+\frac{\sin\left(6\,a+6\,b\,x\right)}{96}+\frac{\sin\left(8\,a+8\,b\,x\right)}{1024}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^8,x)
```

```
[Out] (35*x)/128 + ((7*sin(2*a + 2*b*x))/32 + (7*sin(4*a + 4*b*x))/128 + sin(6*a
+ 6*b*x)/96 + sin(8*a + 8*b*x)/1024)/b
```

3.9 $\int \cos^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=65

$$\frac{10\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{21b}+\frac{10\sqrt{\cos(a+bx)}\,\sin(a+bx)}{21b}+\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}$$

```
[Out] 10/21*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1
/2*b*x),2^(1/2))/b+2/7*cos(b*x+a)^(5/2)*sin(b*x+a)/b+10/21*sin(b*x+a)*cos(b
*x+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2715, 2720}

$$\frac{10F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}+\frac{10\sin(a+bx)\sqrt{\cos(a+bx)}}{21b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(7/2),x]
```

```
[Out] (10*EllipticF[(a + b*x)/2, 2])/(21*b) + (10*Sqrt[Cos[a + b*x]]*Sin[a + b*x]
)/(21*b) + (2*Cos[a + b*x]^(5/2)*Sin[a + b*x])/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{7}{2}}(a+bx)\,dx&=\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}+\frac{5}{7}\int \cos^{\frac{3}{2}}(a+bx)\,dx\\&=\frac{10\sqrt{\cos(a+bx)}\,\sin(a+bx)}{21b}+\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}+\frac{5}{21}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\&=\frac{10F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b}+\frac{10\sqrt{\cos(a+bx)}\,\sin(a+bx)}{21b}+\frac{2\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{7b}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 51, normalized size = 0.78

$$\frac{20\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}\,(23\sin(a+bx)+3\sin(3(a+bx)))}{42b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(7/2),x]
```

```
[Out] (20*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(23*Sin[a + b*x] + 3*Sin
[3*(a + b*x)]))/(42*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(81) = 162$.
time = 0.09, size = 199, normalized size = 3.06

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(48\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-72\left(\right.\right.}{21\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(48*cos(1/2*b
*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(1/2*b*
x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)
*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-2*sin(1/2*b
*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1
/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 80, normalized size = 1.23

$$\frac{2\left(3\cos(bx+a)^2+5\right)\sqrt{\cos(bx+a)}\sin(bx+a)-5i\sqrt{2}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+5i\sqrt{2}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*(3*cos(b*x + a)^2 + 5)*sqrt(cos(b*x + a))*sin(b*x + a) - 5*I*sqrt(2
)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + 5*I*sqrt(2)*w
eierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3060 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(7/2), x)
```

Mupad [**B**]
time = 0.29, size = 42, normalized size = 0.65

$$-\frac{2\cos\left(a+b\,x\right)^{9/2}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(a+b\,x\right)^2\right)}{9\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(7/2),x)
```

```
[Out] -(2*cos(a + b*x)^(9/2)*sin(a + b*x)*hypergeom([1/2, 9/4], 13/4, cos(a + b*x
)^2))/(9*b*(sin(a + b*x)^2)^(1/2))
```

3.10 $\int \cos^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{5b}$$

```
[Out] 6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2/5*cos(b*x+a)^(3/2)*sin(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2715, 2719}

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(
5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{5}{2}}(a+bx)\,dx &= \frac{2\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\cos(a+bx)}\,dx\\ &= \frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{5b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 40, normalized size = 0.95

$$\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,\sin(2(a+bx))}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(5/2),x]
```

```
[Out] (6*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[2*(a + b*x)])/(5*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(62) = 124$.
time = 0.03, size = 202, normalized size = 4.81

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\left(-8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+8\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2(\mathrm{si}\right.}{5\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(-8*cos(1/2*b*
x+1/2*a)*sin(1/2*b*x+1/2*a)^6+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*s
in(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*si
n(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(
1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*
b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 74, normalized size = 1.76

$$\frac{2\cos(bx+a)^{\frac{3}{2}}\sin(bx+a)+3i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-3i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*cos(b*x + a)^(3/2)*sin(b*x + a) + 3*I*sqrt(2)*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) - 3*I*sqrt(2)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x +
 a))))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(5/2), x)
```

Mupad [**B**]
time = 0.17, size = 42, normalized size = 1.00

$$-\frac{2\cos\left(a+b\,x\right)^{7/2}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(a+b\,x\right)^2\right)}{7\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(5/2),x)
```

```
[Out] -(2*cos(a + b*x)^(7/2)*sin(a + b*x)*hypergeom([1/2, 7/4], 11/4, cos(a + b*x
)^2))/(7*b*(sin(a + b*x)^2)^(1/2))
```

3.11 $\int \cos^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=42

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}$$

```
[Out] 2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2/3*sin(b*x+a)*cos(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2715, 2720}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(
3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(a+bx)\,dx &= \frac{2\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\ &= \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sqrt{\cos(a+bx)}\,\sin(a+bx)}{3b}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 36, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\sin(a+bx)\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*Sin[a + b*x]))/(3*b)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(62) = 124$.
time = 0.03, size = 179, normalized size = 4.26

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\quad}\right.}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\quad\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(4*sin(1/2*b*x
+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)+(sin
(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(cos(1/2
*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/
sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 68, normalized size = 1.62

$$\frac{2\sqrt{\cos(bx+a)}\sin(bx+a)-i\sqrt{2}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(cos(b*x + a))*sin(b*x + a) - I*sqrt(2)*weierstrassPInverse(-4,
0, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(2)*weierstrassPInverse(-4, 0, co
s(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{\frac{3}{2}}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(3/2), x)
```

Mupad [B]
time = 0.08, size = 35, normalized size = 0.83

$$\frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{3\,b}+\frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(3/2),x)
```

```
[Out] (2*ellipticF(a/2 + (b*x)/2, 2))/(3*b) + (2*cos(a + b*x)^(1/2)*sin(a + b*x))
/(3*b)
```

3.12 $\int \sqrt{\cos(a+bx)}\,dx$

Optimal. Leaf size=16

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2719}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[a + b*x]],x]
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{\cos(a+bx)}\,dx = \frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[a + b*x]],x]
[Out] (2*EllipticE[(a + b*x)/2, 2])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(42) = 84$.
time = 0.10, size = 133, normalized size = 8.31

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\operatorname{EllipticE}\left(\cos\left(\right.\right.}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,b}$
risch	$-\frac{i\sqrt{2}\,\sqrt{\left(e^{2i(bx+a)}+1\right)e^{-i(bx+a)}}}{b}-\frac{i\left(-\frac{2\left(e^{2i(bx+a)}+1\right)}{\sqrt{\left(e^{2i(bx+a)}+1\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(\right.}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*((2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*(sin(1/2*b*x+1/2*
a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)/sin(1/2*b*x+1
/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 57, normalized size = 3.56

$$\frac{i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) +
 I*sin(b*x + a))) - I*sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(cos(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(b*x + a)), x)
```

Mupad [**B**]
time = 0.12, size = 15, normalized size = 0.94

$$\frac{2\,\mathrm{E}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(1/2),x)
```

```
[Out] (2*ellipticE(a/2 + (b*x)/2, 2))/b
```

3.13 $\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=16

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b
*x),2^(1/2))/b
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2720}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx = \frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/b
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.03, size = 18, normalized size = 1.12

method	result	size
default	$\frac{2\,\mathrm{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2}\middle\vert\sqrt{2}\right)}{b}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] 2/b*InverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 51, normalized size = 3.19

$$\frac{-i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a)) + I*s
qrt(2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(1/2),x)
[Out] Integral(1/sqrt(cos(a + b*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(b*x + a)), x)
```

Mupad [**B**]
time = 0.10, size = 15, normalized size = 0.94

$$\frac{2\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(1/2),x)
```

```
[Out] (2*ellipticF(a/2 + (b*x)/2, 2))/b
```

3.14 $\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] -2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))/b+2*sin(b*x+a)/b/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2719}

$$\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}\,dx\\ &= -\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 1.00

$$-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b}+\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-3/2),x]
```

```
[Out] (-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(62) = 124.
time = 0.04, size = 182, normalized size = 4.79

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\right.\right.}}{\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1
/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)
^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(
cos(1/2*b*x+1/2*a),2^(1/2)))/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/sin(1/2*b*x+1/2*a)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 93, normalized size = 2.45

$$\frac{-i\sqrt{2}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+i\sqrt{2}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+2\sqrt{\cos(bx+a)}\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*cos(b*x + a)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*sqrt(co
s(b*x + a))*sin(b*x + a))/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(cos(a + b*x)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-3/2), x)
```

Mupad [B]
time = 0.25, size = 42, normalized size = 1.11

$$\frac{2\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{\cos\left(a+b\,x\right)}\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(3/2),x)
```

```
[Out] (2*sin(a + b*x)*hypergeom([-1/4, 1/2], 3/4, cos(a + b*x)^2))/(b*cos(a + b*x
)^(1/2)*(sin(a + b*x)^2)^(1/2))
```

3.15 $\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=42

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))/b+2/3*sin(b*x+a)/b/cos(b*x+a)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2720}

$$\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-5/2),x]
```

```
[Out] (2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sin[a + b*x])/(3*b*Cos[a + b*x]^(3
/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx&=\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx\\&=\frac{2F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [**A**]
time = 0.04, size = 36, normalized size = 0.86

$$\frac{2\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\frac{\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-5/2),x]
```

```
[Out] (2*(EllipticF[(a + b*x)/2, 2] + Sin[a + b*x]/Cos[a + b*x]^(3/2)))/(3*b)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(62) = 124$.
time = 0.04, size = 213, normalized size = 5.07

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right.}{3\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-2*sin(1/2*b*x+1/2*a)
^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-
1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))*((2*cos(1/2*b*x+1/2*a)^2-1)
*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+1/2*a)^2)
^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(3/2)/sin(1/2*b*x+1/2*a)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 92, normalized size = 2.19

$$\frac{-i\sqrt{2}\cos(bx+a)^2\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\cos(bx+a)^2\,\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))+2\sqrt{\cos(bx+a)}\sin(bx+a)}{3b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*x + a) + I*
sin(b*x + a)) + I*sqrt(2)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*x
 + a) - I*sin(b*x + a)) + 2*sqrt(cos(b*x + a))*sin(b*x + a))/(b*cos(b*x + a
)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(5/2),x)
```

```
[Out] Integral(cos(a + b*x)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-5/2), x)
```

Mupad [B]
time = 0.27, size = 42, normalized size = 1.00

$$\frac{2\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(a+b\,x\right)}^2\right)}{3\,b\,{\cos\left(a+b\,x\right)}^{3/2}\,\sqrt{{\sin\left(a+b\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(5/2),x)
```

```
[Out] (2*sin(a + b*x)*hypergeom([-3/4, 1/2], 1/4, cos(a + b*x)^2))/(3*b*cos(a + b
*x)^(3/2)*(sin(a + b*x)^2)^(1/2))
```

3.16 $\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}$$

```
[Out] -6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/
2*b*x),2^(1/2))/b+2/5*sin(b*x+a)/b/cos(b*x+a)^(5/2)+6/5*sin(b*x+a)/b/cos(b*
x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2716, 2719}

$$-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-7/2),x]
```

```
[Out] (-6*EllipticE[(a + b*x)/2, 2])/(5*b) + (2*Sin[a + b*x])/(5*b*Cos[a + b*x]^(
5/2)) + (6*Sin[a + b*x])/(5*b*Sqrt[Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx\\ &= \frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}-\frac{3}{5}\int \sqrt{\cos(a+bx)}\,dx\\ &= -\frac{6E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{5b}+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{6\sin(a+bx)}{5b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 59, normalized size = 0.91

$$\frac{-6\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|\,2\right)+3\sin(2(a+bx))+2\tan(a+bx)}{5b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-7/2),x]
```

```
[Out] (-6*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] + 3*Sin[2*(a + b*x)] + 2*T
an[a + b*x])/(5*b*Cos[a + b*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(81) = 162.
time = 0.05, size = 358, normalized size = 5.51

method	result
default	$-\frac{2\sqrt{-\left(-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(24\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(-(-2*cos(1/2*b*x+1/2*a)^2+1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(8*sin(1/2*b
*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1
/2*a)^3*(24*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6-12*(2*sin(1/2*b*x+1/2*a
)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1
/2))*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+12*(2*
sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/
2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+8*sin(1/2*b*x+1/2*a)^2*cos(1/2*b
*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*E
```

```
llipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4+sin(1/2*b*x+
1/2*a)^2)^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 110, normalized size = 1.69

$$\frac{-3i\sqrt{2}\cos(bx+a)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+3i\sqrt{2}\cos(bx+a)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+2\left(3\cos(bx+a)^2+1\right)\sqrt{\cos(bx+a)}\sin(bx+a)}{5b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*cos(b*x + a)^3*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a)))
 + 2*(3*cos(b*x + a)^2 + 1)*sqrt(cos(b*x + a))*sin(b*x + a))/(b*cos(b*x + a
)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-7/2), x)
```

Mupad [B]
time = 0.31, size = 42, normalized size = 0.65

$$\frac{2\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ {\cos\left(a+b\,x\right)}^2\right)}{5\,b\,{\cos\left(a+b\,x\right)}^{5/2}\,\sqrt{{\sin\left(a+b\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(7/2),x)
```

```
[Out] (2*sin(a + b*x)*hypergeom([-5/4, 1/2], -1/4, cos(a + b*x)^2))/(5*b*cos(a +
b*x)^(5/2)*(sin(a + b*x)^2)^(1/2))
```

3.17 $\int (c\cos(a+bx))^{7/2}\,dx$

Optimal. Leaf size=98

$$\frac{10c^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{21b\sqrt{c\cos(a+bx)}}+\frac{10c^3\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{21b}+\frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b}$$

```
[Out] 2/7*c*(c*cos(b*x+a))^(5/2)*sin(b*x+a)/b+10/21*c^4*(cos(1/2*a+1/2*b*x)^2)^(1
/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/
2)/b/(c*cos(b*x+a))^(1/2)+10/21*c^3*sin(b*x+a)*(c*cos(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 2721, 2720}

$$\frac{10c^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{21b\sqrt{c\cos(a+bx)}}+\frac{10c^3\sin(a+bx)\sqrt{c\cos(a+bx)}}{21b}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(7/2),x]
```

```
[Out] (10*c^4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(21*b*Sqrt[c*Cos[a +
b*x]]) + (10*c^3*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(21*b) + (2*c*(c*Cos[a
+ b*x])^(5/2)*Sin[a + b*x])/(7*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{7/2}\,dx = \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b} + \frac{1}{7}(5c^2)\int (c\cos(a+bx))^{3/2}\,dx$$
$$= \frac{10c^3\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{21b} + \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b} + \frac{1}{21}(5c^4)\int -$$
$$= \frac{10c^3\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{21b} + \frac{2c(c\cos(a+bx))^{5/2}\sin(a+bx)}{7b} + \frac{\left(5c^4\sqrt{\cos(a\right.}}{2}$$
$$= \frac{10c^4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{21b\sqrt{c\cos(a+bx)}} + \frac{10c^3\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{21b} + \frac{2c(c\cos(}{}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.78

$$\frac{c^3\sqrt{c\cos(a+bx)}\left(20F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,(23\sin(a+bx)+3\sin(3(a+bx)))\right)}{42b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(7/2),x]
[Out] (c^3*Sqrt[c*Cos[a + b*x]]*(20*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]
]*(23*Sin[a + b*x] + 3*Sin[3*(a + b*x)])))/(42*b*Sqrt[Cos[a + b*x]])
```

Maple [A]
time = 0.05, size = 210, normalized size = 2.14

method	result
default	$-\frac{2\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,c^4\left(48\left(\cos^9\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-120\left(\cos^7\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+128\left(\cos^5\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-72\right.}{21\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/21*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^4*(48*cos
(1/2*b*x+1/2*a)^9-120*cos(1/2*b*x+1/2*a)^7+128*cos(1/2*b*x+1/2*a)^5-72*cos(
1/2*b*x+1/2*a)^3+5*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)
^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2))+16*cos(1/2*b*x+1/2*a))/(-c*(2*
sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(2*
cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(7/2),x, algorithm="maxima")
[Out] integrate((c*cos(b*x + a))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 95, normalized size = 0.97

$$\frac{-5i\,\sqrt{2}\,c^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a))+5i\,\sqrt{2}\,c^{\frac{7}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\,\sin(bx+a))+2\left(3\,c^{3}\cos(bx+a)^{2}+5\,c^{3}\right)\sqrt{c\cos(bx+a)}\,\sin(bx+a)}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(7/2),x, algorithm="fricas")
[Out] 1/21*(-5*I*sqrt(2)*c^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(
b*x + a)) + 5*I*sqrt(2)*c^(7/2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I
*sin(b*x + a)) + 2*(3*c^3*cos(b*x + a)^2 + 5*c^3)*sqrt(c*cos(b*x + a))*sin(
b*x + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(7/2),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\cos(a+b\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(7/2),x)
[Out] int((c*cos(a + b*x))^(7/2), x)
```

3.18 $\int (c\cos(a+bx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{6c^2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}}+\frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b}$$

```
[Out] 2/5*c*(c*cos(b*x+a))^(3/2)*sin(b*x+a)/b+6/5*c^2*(cos(1/2*a+1/2*b*x)^2)^(1/2
)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(c*cos(b*x+a))^(
1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2719}

$$\frac{6c^2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{5b\sqrt{\cos(a+bx)}}+\frac{2c\sin(a+bx)(c\cos(a+bx))^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(5/2),x]
```

```
[Out] (6*c^2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*Sqrt[Cos[a + b*
x]]) + (2*c*(c*Cos[a + b*x])^(3/2)*Sin[a + b*x])/(5*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{5/2}\,dx = \frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b} + \frac{1}{5}(3c^2)\int \sqrt{c\cos(a+bx)}\,dx$$
$$= \frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b} + \frac{\left(3c^2\sqrt{c\cos(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{5\sqrt{\cos(a+bx)}}$$
$$= \frac{6c^2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5b\sqrt{\cos(a+bx)}} + \frac{2c(c\cos(a+bx))^{3/2}\sin(a+bx)}{5b}$$

Mathematica [A]
time = 0.05, size = 62, normalized size = 0.89

$$\frac{(c\cos(a+bx))^{5/2}\left(6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\sin(2(a+bx))\right)}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((c*Cos[a + b*x])^(5/2)*(6*EllipticE[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*S
in[2*(a + b*x)]))/(5*b*Cos[a + b*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(86) = 172$.
time = 0.04, size = 213, normalized size = 3.04

method	result
default	$-\frac{2\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,c^3\left(-8\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+8\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{5\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^3*(-8*cos(
1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6+8*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*
a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)
*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-
c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(
c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(5/2),x, algorithm="maxima")
[Out] integrate((c*cos(b*x + a))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 91, normalized size = 1.30

$$\frac{2\sqrt{c\cos(bx+a)}\,c^2\cos(bx+a)\sin(bx+a)+3i\sqrt{2}\,c^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-3i\sqrt{2}\,c^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(5/2),x, algorithm="fricas")
[Out] 1/5*(2*sqrt(c*cos(b*x + a))*c^2*cos(b*x + a)*sin(b*x + a) + 3*I*sqrt(2)*c^(
5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin
(b*x + a))) - 3*I*sqrt(2)*c^(5/2)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(5/2),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\cos(a+b\,x))^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(5/2),x)
[Out] int((c*cos(a + b*x))^(5/2), x)
```

3.19 $\int (c\cos(a+bx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2c^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{3b}$$

```
[Out] 2/3*c^2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a
+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(c*cos(b*x+a))^(1/2)+2/3*c*sin(b*x+a)
*(c*cos(b*x+a))^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 2721, 2720}

$$\frac{2c^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{c\cos(a+bx)}}+\frac{2c\sin(a+bx)\sqrt{c\cos(a+bx)}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(3/2),x]
```

```
[Out] (2*c^2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*Sqrt[c*Cos[a + b*
x]]) + (2*c*Sqrt[c*Cos[a + b*x]]*Sin[a + b*x])/(3*b)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{3/2}\,dx = \frac{2c\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{3b} + \frac{1}{3}c^2\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx$$

$$= \frac{2c\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{3b} + \frac{\left(c^2\sqrt{\cos(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3\sqrt{c\cos(a+bx)}}$$

$$= \frac{2c^2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b\sqrt{c\cos(a+bx)}} + \frac{2c\sqrt{c\cos(a+bx)}\,\sin(a+bx)}{3b}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 0.83

$$\frac{2(c\cos(a+bx))^{3/2}\left(F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,\sin(a+bx)\right)}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(3/2),x]
[Out] (2*(c*Cos[a + b*x])^(3/2)*(EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*S
in[a + b*x]))/(3*b*Cos[a + b*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(86) = 172$.
time = 0.04, size = 190, normalized size = 2.71

method	result
default	$-\frac{2\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\,c^2\left(4\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\cos\left(\frac{bx}{2}+\frac{a}{2}\right)+\sqrt{\cdots}\right.}{3\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c^2*(4*sin(1
/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)-2*sin(1/2*b*x+1/2*a)^2*cos(1/2*b*x+1/2*a
)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticF(c
os(1/2*b*x+1/2*a),2^(1/2)))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^
2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*cos(b*x + a))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 77, normalized size = 1.10

$$\frac{-i\sqrt{2}\,c^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)+i\,\sin(bx+a))+i\sqrt{2}\,c^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(bx+a)-i\,\sin(bx+a))+2\,\sqrt{c\cos(bx+a)}\,c\sin(bx+a)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(3/2),x, algorithm="fricas")
[Out] 1/3*(-I*sqrt(2)*c^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x
 + a)) + I*sqrt(2)*c^(3/2)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(
b*x + a)) + 2*sqrt(c*cos(b*x + a))*c*sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\cos(a+bx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(3/2),x)
[Out] Integral((c*cos(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\cos(a+b\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(3/2),x)
[Out] int((c*cos(a + b*x))^(3/2), x)
```

3.20 $\int \sqrt{c\cos(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b
*x),2^(1/2))*(c*cos(b*x+a))^(1/2)/b/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2719}

$$\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\cos(a+bx)}\,dx &= \frac{\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{\sqrt{\cos(a+bx)}}\\ &= \frac{2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Cos[a + b*x]],x]
[Out] (2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*Sqrt[Cos[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(60) = 120$.
time = 0.07, size = 142, normalized size = 3.74

method	result
default	$\frac{2\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\, c\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+1}\,\mathrm{EllipticE}(}{\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}}$
risch	$-\frac{i\sqrt{2}\,\sqrt{c\left(e^{2i(bx+a)}+1\right)e^{-i(bx+a)}}}{b}-\frac{i\left(-\frac{2\left(c\,e^{2i(bx+a)}+c\right)}{c\sqrt{e^{i(bx+a)}\left(c\,e^{2i(bx+a)}+c\right)}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)*c*(sin(1/2*b*x+
1/2*a)^2)^(1/2)*(-2*cos(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cos(1/2*b*x+1/2
*a),2^(1/2))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+1/2*a)^2))^(1/2)/sin(1
/2*b*x+1/2*a)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*cos(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 63, normalized size = 1.66

$$\frac{i\sqrt{2}\sqrt{c}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))-i\sqrt{2}\sqrt{c}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] (I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*
x + a) + I*sin(b*x + a))) - I*sqrt(2)*sqrt(c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x + a))))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\cos{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(sqrt(c*cos(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{c\,\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(1/2),x)
```

```
[Out] int((c*cos(a + b*x))^(1/2), x)
```

3.21 $\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b\sqrt{c\cos(a+bx)}}$$

```
[Out] 2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b
*x),2^(1/2))*cos(b*x+a)^(1/2)/b/(c*cos(b*x+a))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2720}

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx &= \frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{\sqrt{c\cos(a+bx)}}\\ &= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{b\sqrt{c\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Cos[a + b*x]],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(b*Sqrt[c*Cos[a + b*x]])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.04, size = 54, normalized size = 1.42

method	result	size
default	$\frac{2\sqrt{2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{am}^{-1}\left(\frac{bx}{2}+\frac{a}{2}\middle|\sqrt{2}\right)}{b\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)*(2*cos(1/2*b*x+1/2*a)^2-1)^(1/2)*I
nverseJacobiAM(1/2*b*x+1/2*a,2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 60, normalized size = 1.58

$$\frac{-i\sqrt{2}\sqrt{c}\ \mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{2}\sqrt{c}\ \mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a))}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a
)) + I*sqrt(2)*sqrt(c)*weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x
+ a)))/(b*c)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cos{(a+bx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(1/2),x)
[Out] Integral(1/sqrt(c*cos(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(c*cos(b*x + a)), x)
```

Mupad [B]
time = 0.15, size = 33, normalized size = 0.87

$$\frac{2\,\sqrt{\cos\left(a+b\,x\right)}\,\mathrm{F}\left(\frac{a}{2}+\frac{b\,x}{2}\middle|2\right)}{b\,\sqrt{c\,\cos\left(a+b\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(1/2),x)
[Out] (2*cos(a + b*x)^(1/2)*ellipticF(a/2 + (b*x)/2, 2))/(b*(c*cos(a + b*x))^(1/2
))
```

3.22 $\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{bc^2\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}$$

```
[Out] 2*sin(b*x+a)/b/c/(c*cos(b*x+a))^(1/2)-2*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/
2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*(c*cos(b*x+a))^(1/2)/b/c
^2/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2719}

$$\frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{bc^2\sqrt{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(b*c^2*Sqrt[Cos[a + b*x
]]) + (2*Sin[a + b*x])/(b*c*Sqrt[c*Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx &= \frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}} - \frac{\int \sqrt{c\cos(a+bx)}\,dx}{c^2}\\ &= \frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}} - \frac{\sqrt{c\cos(a+bx)}\int \sqrt{\cos(a+bx)}\,dx}{c^2\sqrt{\cos(a+bx)}}\\ &= -\frac{2\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{bc^2\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{bc\sqrt{c\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.74

$$\frac{2\left(-\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\sin(a+bx)\right)}{bc\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-3/2),x]
```

```
[Out] (2*(-(Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]) + Sin[a + b*x]))/(b*c*S
qrt[c*Cos[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.
time = 0.04, size = 198, normalized size = 2.91

method	result
default	$-\frac{2\left(-2\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)c+c\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\sqrt{2\left(\sin\right.}\right.}{c\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right)}\,\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c*(-2*cos(1/2*b*x+1/2*a)*(-2*sin(1/2*b*x+1/2*a)^4*c+c*sin(1/2*b*x+1/2*a)
^2)^(1/2)*sin(1/2*b*x+1/2*a)^2+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+
1/2*a)^2-1)^(1/2)*(-2*sin(1/2*b*x+1/2*a)^4*c+c*sin(1/2*b*x+1/2*a)^2)^(1/2)*
EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*
b*x+1/2*a)^2))^(1/2)/sin(1/2*b*x+1/2*a)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2
)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate((c*cos(b*x + a))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 104, normalized size = 1.53

$$\frac{-i\sqrt{2}\sqrt{c}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+i\sqrt{2}\sqrt{c}\cos(bx+a)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+2\sqrt{c\cos(bx+a)}\sin(bx+a)}{bc^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*sqrt(c)*cos(b*x + a)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + I*sqrt(2)*sqrt(c)*cos(b*x + a)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) - I*sin(b*x +
a))) + 2*sqrt(c*cos(b*x + a))*sin(b*x + a))/(b*c^2*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c\cos(a+bx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(3/2),x)
[Out] Integral((c*cos(a + b*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(3/2),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\,\cos(a+b\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(3/2),x)
[Out] int(1/(c*cos(a + b*x))^(3/2), x)
```

3.23 $\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

```
[Out] 2/3*sin(b*x+a)/b/c/(c*cos(b*x+a))^(3/2)+2/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/co
s(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)/b/c
^2/(c*cos(b*x+a))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2721, 2720}

$$\frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{3bc^2\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])/(3*b*c^2*Sqrt[c*Cos[a + b*
x]]) + (2*Sin[a + b*x])/(3*b*c*(c*Cos[a + b*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{5/2}}\,dx = \frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}} + \frac{\int \frac{1}{\sqrt{c\cos(a+bx)}}\,dx}{3c^2}$$
$$= \frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}} + \frac{\sqrt{\cos(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{3c^2\sqrt{c\cos(a+bx)}}$$
$$= \frac{2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)}{3bc^2\sqrt{c\cos(a+bx)}} + \frac{2\sin(a+bx)}{3bc(c\cos(a+bx))^{3/2}}$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.71

$$\frac{2\left(\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|\,2\right)+\tan(a+bx)\right)}{3bc^2\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-5/2),x]
```

```
[Out] (2*(Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + Tan[a + b*x]))/(3*b*c^2*
Sqrt[c*Cos[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.
time = 0.04, size = 241, normalized size = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(bx+a)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)\right.}{3c^2\sqrt{-c\left(2\left(\sin^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-\left(\sin^2\left(\frac{bx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*Elli
pticF(cos(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2-2*sin(1/2*b*x+1/2*a)
^2*cos(1/2*b*x+1/2*a)+(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-
1)^(1/2)*EllipticF(cos(1/2*b*x+1/2*a),2^(1/2)))/c^2*(c*(2*cos(1/2*b*x+1/2*a
)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/(-c*(2*sin(1/2*b*x+1/2*a)^4-sin(1/2*b*x+
1/2*a)^2))^(1/2)/(2*cos(1/2*b*x+1/2*a)^2-1)/sin(1/2*b*x+1/2*a)/(c*(2*cos(1/
2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 103, normalized size = 1.43

$$\frac{-i\sqrt{2}\sqrt{c}\cos\left(bx+a\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right)\right)+i\sqrt{2}\sqrt{c}\cos\left(bx+a\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right)\right)+2\sqrt{c\cos\left(bx+a\right)}\sin\left(bx+a\right)}{3\,bc^{3}\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(c)*cos(b*x + a)^2*weierstrassPInverse(-4, 0, cos(b*x +
 a) + I*sin(b*x + a)) + I*sqrt(2)*sqrt(c)*cos(b*x + a)^2*weierstrassPInvers
e(-4, 0, cos(b*x + a) - I*sin(b*x + a)) + 2*sqrt(c*cos(b*x + a))*sin(b*x +
a))/(b*c^3*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\cos\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(5/2),x)
```

```
[Out] Integral((c*cos(a + b*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,\cos\left(a+b\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(5/2),x)
[Out] int(1/(c*cos(a + b*x))^(5/2), x)
```

3.24 $\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{6\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^4\sqrt{\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}+\frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}$$

```
[Out] 2/5*sin(b*x+a)/b/c/(c*cos(b*x+a))^(5/2)+6/5*sin(b*x+a)/b/c^3/(c*cos(b*x+a))
^(1/2)-6/5*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/
2*a+1/2*b*x),2^(1/2))*(c*cos(b*x+a))^(1/2)/b/c^4/cos(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2721, 2719}

$$-\frac{6E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{c\cos(a+bx)}}{5bc^4\sqrt{\cos(a+bx)}}+\frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}+\frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[c*Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2])/(5*b*c^4*Sqrt[Cos[a + b
*x]]) + (2*Sin[a + b*x])/(5*b*c*(c*Cos[a + b*x])^(5/2)) + (6*Sin[a + b*x])/
(5*b*c^3*Sqrt[c*Cos[a + b*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{7/2}}\,dx = \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{3\int \frac{1}{(c\cos(a+bx))^{3/2}}\,dx}{5c^2}$$
$$= \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}} - \frac{3\int \sqrt{c\cos(a+bx)}\,dx}{5c^4}$$
$$= \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}} - \frac{\left(3\sqrt{c\cos(a+bx)}\right)\int \sqrt{\cos(a+}}{5c^4\sqrt{\cos(a+bx)}}$$
$$= -\frac{6\sqrt{c\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{5bc^4\sqrt{\cos(a+bx)}} + \frac{2\sin(a+bx)}{5bc(c\cos(a+bx))^{5/2}} + \frac{6\sin(a+bx)}{5bc^3\sqrt{c\cos(a+b}}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.68

$$\frac{-6\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right) + 6\sin(a+bx) + 2\sec(a+bx)\tan(a+bx)}{5bc^3\sqrt{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-7/2),x]
[Out] (-6*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2] + 6*Sin[a + b*x] + 2*Sec[a
 + b*x]*Tan[a + b*x])/(5*b*c^3*Sqrt[c*Cos[a + b*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(112) = 224$.
time = 0.06, size = 366, normalized size = 3.66

method	result
default	$-\frac{2\sqrt{c\left(2\left(\cos^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}\left(24\cos\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\sin^6\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-12\sqrt{2\left(\sin^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(7/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(c*(2*cos(1/2*b*x+1/2*a)^2-1)*sin(1/2*b*x+1/2*a)^2)^(1/2)/c^4/sin(1/2*
b*x+1/2*a)^3/(8*sin(1/2*b*x+1/2*a)^6-12*sin(1/2*b*x+1/2*a)^4+6*sin(1/2*b*x+
1/2*a)^2-1)*(24*cos(1/2*b*x+1/2*a)*sin(1/2*b*x+1/2*a)^6-12*(2*sin(1/2*b*x+1
/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(cos(1/2*b*x+1/2*a),
2^(1/2))*sin(1/2*b*x+1/2*a)^4-24*sin(1/2*b*x+1/2*a)^4*cos(1/2*b*x+1/2*a)+12
```

```
*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/2)*(sin(1/2*b*x+1/2*a)^2)^(1/2)*EllipticE(co
s(1/2*b*x+1/2*a),2^(1/2))*sin(1/2*b*x+1/2*a)^2+8*sin(1/2*b*x+1/2*a)^2*cos(1
/2*b*x+1/2*a)-3*(sin(1/2*b*x+1/2*a)^2)^(1/2)*(2*sin(1/2*b*x+1/2*a)^2-1)^(1/
2)*EllipticE(cos(1/2*b*x+1/2*a),2^(1/2)))*(-2*sin(1/2*b*x+1/2*a)^4*c+c*sin(
1/2*b*x+1/2*a)^2)^(1/2)/(c*(2*cos(1/2*b*x+1/2*a)^2-1))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 121, normalized size = 1.21

$$\frac{-3i\sqrt{2}\sqrt{c}\cos(bx+a)^3\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)+i\sin(bx+a)))+3i\sqrt{2}\sqrt{c}\cos(bx+a)^3\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(bx+a)-i\sin(bx+a)))+2\sqrt{c\cos(bx+a)}\,(3\cos(bx+a)^2+1)\sin(bx+a)}{5\,bc^4\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(c)*cos(b*x + a)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*I*sqrt(2)*sqrt(c)*cos(b
*x + a)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(b*x + a) -
I*sin(b*x + a))) + 2*sqrt(c*cos(b*x + a))*(3*cos(b*x + a)^2 + 1)*sin(b*x +
a))/(b*c^4*cos(b*x + a)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(c\cos(a+b\,x))^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(7/2),x)
```

```
[Out] int(1/(c*cos(a + b*x))^(7/2), x)
```

3.25 $\int \cos^{\frac{4}{3}}(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{3\cos^{\frac{7}{3}}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sin(a+bx)}{7b\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/7*cos(b*x+a)^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x+a)/
b/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$-\frac{3\sin(a+bx)\cos^{\frac{7}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(7*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{4}{3}}(a+bx)\,dx = -\frac{3\cos^{\frac{7}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.04, size = 53, normalized size = 1.00

$$-\frac{3\cos^{\frac{7}{3}}(a+bx)\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(4/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(7/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(7*b)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \cos^{\frac{4}{3}}\left(bx+a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(4/3),x)
```

```
[Out] int(cos(b*x+a)^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(4/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(4/3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(4/3), x)
```

Mupad [B]
time = 0.19, size = 42, normalized size = 0.79

$$-\frac{3\cos\left(a+b\,x\right)^{7/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{6};\ \frac{13}{6};\ \cos\left(a+b\,x\right)^2\right)}{7\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(4/3),x)
```

```
[Out] -(3*cos(a + b*x)^(7/3)*sin(a + b*x)*hypergeom([1/2, 7/6], 13/6, cos(a + b*x
)^2))/(7*b*(sin(a + b*x)^2)^(1/2))
```

3.26 $\int \cos^{\frac{2}{3}}(a+bx)\,dx$

Optimal. Leaf size=53

$$-\frac{3\cos^{\frac{5}{3}}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sin(a+bx)}{5b\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/5*cos(b*x+a)^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x+a)/
b/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2722\}$

$$-\frac{3\sin(a+bx)\cos^{\frac{5}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2]*Si
n[a + b*x])/(5*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{2}{3}}(a+bx)\,dx = -\frac{3\cos^{\frac{5}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.00

$$-\frac{3\cos^{\frac{5}{3}}(a+bx)\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(5/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a
 + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(5*b)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \cos^{\frac{2}{3}}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(2/3),x)
```

```
[Out] int(cos(b*x+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{\frac{2}{3}}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(2/3), x)
```

Mupad [B]
time = 0.19, size = 42, normalized size = 0.79

$$-\frac{3\cos\left(a+b\,x\right)^{5/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{5}{6};\ \frac{11}{6};\ \cos\left(a+b\,x\right)^2\right)}{5\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(2/3),x)
```

```
[Out] -(3*cos(a + b*x)^(5/3)*sin(a + b*x)*hypergeom([1/2, 5/6], 11/6, cos(a + b*x
)^2))/(5*b*(sin(a + b*x)^2)^(1/2))
```

3.27 $\int \sqrt[3]{\cos(a+bx)}\,dx$

Optimal. Leaf size=53

$$-\frac{3\cos^{\frac{4}{3}}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sin(a+bx)}{4b\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/4*cos(b*x+a)^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+a)/b
/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$-\frac{3\sin(a+bx)\cos^{\frac{4}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]*Sin
[a + b*x])/(4*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{\cos(a+bx)}\,dx = -\frac{3\cos^{\frac{4}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.00

$$-\frac{3\cos^{\frac{4}{3}}(a+bx)\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(4/3)*Csc[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/(4*b)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \cos^{\frac{1}{3}}\left(bx+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^(1/3),x)
```

```
[Out] int(cos(b*x+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{\cos\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(1/3), x)
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.79

$$-\frac{3\cos\left(a+b\,x\right)^{4/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{2}{3};\ \frac{5}{3};\ \cos\left(a+b\,x\right)^2\right)}{4\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^(1/3),x)
```

```
[Out] -(3*cos(a + b*x)^(4/3)*sin(a + b*x)*hypergeom([1/2, 2/3], 5/3, cos(a + b*x)
^2))/(4*b*(sin(a + b*x)^2)^(1/2))
```

3.28 $\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx$

Optimal. Leaf size=53

$$-\frac{3\cos^{\frac{2}{3}}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sin(a+bx)}{2b\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/2*cos(b*x+a)^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+a)/b
/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$-\frac{3\sin(a+bx)\cos^{\frac{2}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sin
[a + b*x])/(2*b*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{\cos(a+bx)}}\,dx = -\frac{3\cos^{\frac{2}{3}}(a+bx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 53, normalized size = 1.00

$$-\frac{3\cos^{\frac{2}{3}}(a+bx)\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-1/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(2/3)*Csc[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/(2*b)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos \left(bx+a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(1/3),x)
```

```
[Out] int(1/cos(b*x+a)^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{\cos \left(a+bx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(1/3),x)
```

```
[Out] Integral(cos(a + b*x)**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(1/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(-1/3), x)
```

Mupad [B]
time = 0.21, size = 42, normalized size = 0.79

$$-\frac{3\cos\left(a+b\,x\right)^{2/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{2};\ \frac{4}{3};\ \cos\left(a+b\,x\right)^2\right)}{2\,b\,\sqrt{\sin\left(a+b\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(1/3),x)
[Out] -(3*cos(a + b*x)^(2/3)*sin(a + b*x)*hypergeom([1/3, 1/2], 4/3, cos(a + b*x)
^2))/(2*b*(sin(a + b*x)^2)^(1/2))
```

3.29 $\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx$

Optimal. Leaf size=51

$$-\frac{3\sqrt[3]{\cos(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3*cos(b*x+a)^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)/b/(
sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2722\}$

$$-\frac{3\sin(a+bx)\sqrt[3]{\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sin
[a + b*x])/(b*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{2}{3}}(a+bx)}\,dx = -\frac{3\sqrt[3]{\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 1.00

$$-\frac{3\sqrt[3]{\cos(a+bx)}\,\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-2/3),x]
```

```
[Out] (-3*Cos[a + b*x]^(1/3)*Csc[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/b
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos \left(b x +a \right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(2/3),x)
```

```
[Out] int(1/cos(b*x+a)^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{2}{3}}{\left(a + b x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(2/3),x)
```

```
[Out] Integral(cos(a + b*x)**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(2/3),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^(-2/3), x)
```

Mupad [B]
time = 0.20, size = 42, normalized size = 0.82

$$-\frac{3\cos\left(a+b\,x\right)^{1/3}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{6},\frac{1}{2};\ \frac{7}{6};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{{\sin\left(a+b\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(2/3),x)
[Out] -(3*cos(a + b*x)^(1/3)*sin(a + b*x)*hypergeom([1/6, 1/2], 7/6, cos(a + b*x)
^2))/(b*(sin(a + b*x)^2)^(1/2))
```

3.30 $\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx$

Optimal. Leaf size=51

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

```
[Out] 3*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*sin(b*x+a)/b/cos(b*x+a)^(1/3)/(
sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$\frac{3\sin(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{b\sqrt{\sin^2(a+bx)}\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*Cos[a
 + b*x]^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{4}{3}}(a+bx)}\,dx = \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sin(a+bx)}{b\sqrt[3]{\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 1.00

$$\frac{3\csc(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b\sqrt[3]{\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^(-4/3),x]
```

```
[Out] (3*Csc[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(b*Cos[a + b*x]^(1/3))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos \left(b x +a \right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^(4/3),x)
```

```
[Out] int(1/cos(b*x+a)^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^(-4/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\cos^{\frac{4}{3}}{\left(a + b x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**(4/3),x)
```

```
[Out] Integral(cos(a + b*x)**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^(-4/3), x)
```

Mupad [B]
time = 0.23, size = 42, normalized size = 0.82

$$\frac{3\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{6},\frac{1}{2};\ \frac{5}{6};\ {\cos\left(a+b\,x\right)}^2\right)}{b\cos\left(a+b\,x\right)^{1/3}\,\sqrt{{\sin\left(a+b\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(a + b*x)^(4/3),x)
```

```
[Out] (3*sin(a + b*x)*hypergeom([-1/6, 1/2], 5/6, cos(a + b*x)^2))/(b*cos(a + b*x
)^(1/3)*(sin(a + b*x)^2)^(1/2))
```

3.31 $\int (c\cos(a+bx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(c\cos(a+bx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sin(a+bx)}{7bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/7*(c*cos(b*x+a))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(b*x+a)^2)*sin(b*x
+a)/b/c/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)}{7bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(4/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(7*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{4/3}\,dx = -\frac{3(c\cos(a+bx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(a+bx)\right)\sin(a+bx)}{7bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.95

$$-\frac{3(c\cos(a+bx))^{4/3}\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(4/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(4/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(7*b)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (c\cos (bx+a))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(4/3),x)
```

```
[Out] int((c*cos(b*x+a))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(4/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(1/3)*c*cos(b*x + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(4/3),x)
```

```
[Out] Timed out
```

<u>Giac</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(4/3), x)
```

<u>Mupad</u> **[F]**
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,\cos\left(a+b\,x\right))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(4/3),x)
```

```
[Out] int((c*cos(a + b*x))^(4/3), x)
```

3.32 $\int (c\cos(a+bx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(c\cos(a+bx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sin(a+bx)}{5bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/5*(c*cos(b*x+a))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(b*x+a)^2)*sin(b*x
+a)/b/c/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)}{5bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(2/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[a + b*x]^2
]*Sin[a + b*x])/(5*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^{2/3}\,dx = -\frac{3(c\cos(a+bx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(a+bx)\right)\sin(a+bx)}{5bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.95

$$-\frac{3(c\cos(a+bx))^{2/3}\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(2/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(2/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(5*b)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c \cos \left(b x +a \right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(2/3),x)
```

```
[Out] int((c*cos(b*x+a))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \cos \left(a + b x\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\,\cos\left(a+b\,x\right))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(2/3),x)
```

```
[Out] int((c*cos(a + b*x))^(2/3), x)
```

3.33 $\int \sqrt[3]{c\cos(a+bx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3(c\cos(a+bx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sin(a+bx)}{4bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/4*(c*cos(b*x+a))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(b*x+a)^2)*sin(b*x+
a)/b/c/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)}{4bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(1/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[a + b*x]^2]
*Sin[a + b*x])/(4*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{c\cos(a+bx)}\,dx = -\frac{3(c\cos(a+bx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(a+bx)\right)\sin(a+bx)}{4bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.95

$$-\frac{3\sqrt[3]{c\cos(a+bx)}\,\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(1/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(1/3)*Cot[a + b*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Co
s[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(4*b)
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (c\cos (bx+a))^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^(1/3),x)
```

```
[Out] int((c*cos(b*x+a))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(1/3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{c\cos (a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (c\cos(a+b\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^(1/3),x)
```

```
[Out] int((c*cos(a + b*x))^(1/3), x)
```

3.34 $\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(c\cos(a+bx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sin(a+bx)}{2bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3/2*(c*cos(b*x+a))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(b*x+a)^2)*sin(b*x+
a)/b/c/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(a+bx)(c\cos(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)}{2bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-1/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]
*Sin[a + b*x])/(2*b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{c\cos(a+bx)}}\,dx=-\frac{3(c\cos(a+bx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(a+bx)\right)\sin(a+bx)}{2bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.95

$$-\frac{3\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{2b\sqrt[3]{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-1/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(2*b*(c*Cos[a + b*x])^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \cos \left(b x+a\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(1/3),x)
```

```
[Out] int(1/(c*cos(b*x+a))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(2/3)/(c*cos(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{c \cos \left(a+b x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(1/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(1/3),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,\cos\left(a+b\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(1/3),x)
[Out] int(1/(c*cos(a + b*x))^(1/3), x)
```

3.35 $\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt[3]{c\cos(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt{\sin^2(a+bx)}}$$

```
[Out] -3*(c*cos(b*x+a))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(b*x+a)^2)*sin(b*x+a)
/b/c/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(a+bx)\sqrt[3]{c\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-2/3),x]
```

```
[Out] (-3*(c*Cos[a + b*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]
*Sin[a + b*x])/(b*c*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{2/3}}\,dx = -\frac{3\sqrt[3]{c\cos(a+bx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.95

$$-\frac{3\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(c\cos(a+bx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-2/3),x]
```

```
[Out] (-3*Cot[a + b*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(b*(c*Cos[a + b*x])^(2/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \cos \left(b x + a\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(2/3),x)
```

```
[Out] int(1/(c*cos(b*x+a))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(1/3)/(c*cos(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \cos \left(a + b x\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(2/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(2/3),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,\cos\left(a+b\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(2/3),x)
[Out] int(1/(c*cos(a + b*x))^(2/3), x)
```

3.36 $\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt[3]{c\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

```
[Out] 3*hypergeom([-1/6, 1/2],[5/6],cos(b*x+a)^2)*sin(b*x+a)/b/c/(c*cos(b*x+a))^(
1/3)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$\frac{3\sin(a+bx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)}{bc\sqrt{\sin^2(a+bx)}\sqrt[3]{c\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(c*
Cos[a + b*x])^(1/3)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(c\cos(a+bx))^{4/3}}\,dx = \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(a+bx)\right)\sin(a+bx)}{bc\sqrt[3]{c\cos(a+bx)}\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 53, normalized size = 0.95

$$\frac{3\cot(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(c\cos(a+bx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^(-4/3),x]
```

```
[Out] (3*Cot[a + b*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[a + b*x]^2]*Sqrt[Sin[
a + b*x]^2])/(b*(c*Cos[a + b*x])^(4/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \cos \left(b x+a\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a))^(4/3),x)
```

```
[Out] int(1/(c*cos(b*x+a))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^(2/3)/(c^2*cos(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \cos \left(a+b x\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))**(4/3),x)
```

```
[Out] Integral((c*cos(a + b*x))**(-4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a))^(4/3),x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^(-4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(c\,\cos\left(a+b\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x))^(4/3),x)
[Out] int(1/(c*cos(a + b*x))^(4/3), x)
```

3.37 $\int \cos^n(a+bx)\,dx$

Optimal. Leaf size=64

$$-\frac{\cos^{1+n}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{b(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -cos(b*x+a)^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*sin(
b*x+a)/b/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2722}

$$-\frac{\sin(a+bx)\cos^{n+1}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{b(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^n,x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[a
+ b*x]^2]*Sin[a + b*x])/(b*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^n(a+bx)\,dx = -\frac{\cos^{1+n}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{b(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 1.00

$$-\frac{\cos^{1+n}(a+bx)\csc(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^n,x]
```

```
[Out] -((Cos[a + b*x]^(1 + n)*Csc[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \cos^{n}\left(b x + a\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^n,x)
```

```
[Out] int(cos(b*x+a)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^{n}\left(a + b x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**n,x)
```

```
[Out] Integral(cos(a + b*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^n, x)
```

Mupad [B]
time = 0.53, size = 57, normalized size = 0.89

$$-\frac{\cos\left(a+b\,x\right)^{n+1}\,\sin\left(a+b\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{n}{2}+\frac{1}{2};\ \frac{n}{2}+\frac{3}{2};\ \cos\left(a+b\,x\right)^2\right)}{b\,\sqrt{\sin\left(a+b\,x\right)^2}\,\left(n+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^n,x)
```

```
[Out] -(cos(a + b*x)^(n + 1)*sin(a + b*x)*hypergeom([1/2, n/2 + 1/2], n/2 + 3/2,
cos(a + b*x)^2))/(b*(sin(a + b*x)^2)^(1/2)*(n + 1))
```

3.38 $\int (c\cos(a+bx))^n \, dx$

Optimal. Leaf size=69

$$-\frac{(c\cos(a+bx))^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sin(a+bx)}{bc(1+n)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -(c*cos(b*x+a))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(b*x+a)^2)*
sin(b*x+a)/b/c/(1+n)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2722}

$$-\frac{\sin(a+bx)(c\cos(a+bx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(a+bx)\right)}{bc(n+1)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x])^n,x]
```

```
[Out] -(((c*Cos[a + b*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[a + b*x]^2]*Sin[a + b*x])/(b*c*(1 + n)*Sqrt[Sin[a + b*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (c\cos(a+bx))^n \, dx = -\frac{(c\cos(a+bx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(a+bx)\right)\sin(a+bx)}{bc(1+n)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.93

$$-\frac{(c\cos(a+bx))^n\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x])^n,x]
```

```
[Out] -(((c*Cos[a + b*x])^n*Cot[a + b*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(1 + n)))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(c \cos \left(b x +a \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a))^n,x)
```

```
[Out] int((c*cos(b*x+a))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^n,x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^n,x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \cos \left(a + b x\right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))**n,x)
```

```
[Out] Integral((c*cos(a + b*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a))^n,x, algorithm="giac")
[Out] integrate((c*cos(b*x + a))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\,\cos\left(a+b\,x\right))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x))^n,x)
[Out] int((c*cos(a + b*x))^n, x)
```

3.39 $\int (a\cos^2(x))^{5/2}\, dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\sqrt{a\cos^2(x)}\,\tan(x)+\frac{4}{15}a\big(a\cos^2(x)\big)^{3/2}\tan(x)+\frac{1}{5}\big(a\cos^2(x)\big)^{5/2}\tan(x)$$

```
[Out] 4/15*a*(a*cos(x)^2)^(3/2)*tan(x)+1/5*(a*cos(x)^2)^(5/2)*tan(x)+8/15*a^2*(a*
cos(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3282, 3286, 2717}

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cos^2(x)\right)^{5/2} dx = \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x) + \frac{1}{5}(4a)\int \left(a\cos^2(x)\right)^{3/2} dx$$
$$= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x) + \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x) + \frac{1}{15}\left(8a^2\right)\int \sqrt{a\cos^2(x)}\, dx$$
$$= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x) + \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x) + \frac{1}{15}\left(8a^2\sqrt{a\cos^2(x)}\sec(x)\right)\int$$
$$= \frac{8}{15}a^2\sqrt{a\cos^2(x)}\tan(x) + \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x) + \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)$$

Mathematica [A]
time = 0.01, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2\sqrt{a\cos^2(x)}\sec(x)(150\sin(x) + 25\sin(3x) + 3\sin(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(5/2),x]
[Out] (a^2*Sqrt[a*Cos[x]^2]*Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/240
```

Maple [A]
time = 0.05, size = 32, normalized size = 0.60

method	result
default	$\frac{a^3\cos(x)\sin(x)\left(3\left(\cos^4(x)\right)+4\left(\cos^2(x)\right)+8\right)}{15\sqrt{a\left(\cos^2(x)\right)}}$
risch	$-\frac{ia^2 e^{6ix}\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{160(e^{2ix}+1)} - \frac{5ia^2 e^{2ix}\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{16(e^{2ix}+1)} + \frac{5ia^2\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{16(e^{2ix}+1)} + \frac{5ia^2 e^{-2ix}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/15*a^3*cos(x)*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.52, size = 31, normalized size = 0.58

$$\frac{1}{240}\left(3\,a^2\sin(5\,x) + 25\,a^2\sin(3\,x) + 150\,a^2\sin(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

Fricas [A]
time = 0.37, size = 40, normalized size = 0.75

$$\frac{\left(3\,a^{2}\cos\left(x\right)^{4}+4\,a^{2}\cos\left(x\right)^{2}+8\,a^{2}\right)\sqrt{a\cos\left(x\right)^{2}}\,\sin\left(x\right)}{15\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos(
x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**2)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [A]
time = 0.43, size = 34, normalized size = 0.64

$$\frac{1}{15}\left(3\,a^{2}\sin\left(x\right)^{5}-10\,a^{2}\sin\left(x\right)^{3}+15\,a^{2}\sin\left(x\right)\right)\sqrt{a}\,\mathrm{sgn}(\cos\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(5/2),x, algorithm="giac")
[Out] 1/15*(3*a^2*sin(x)^5 - 10*a^2*sin(x)^3 + 15*a^2*sin(x))*sqrt(a)*sgn(cos(x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int {\left(a\cos\left(x\right)^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(5/2),x)
[Out] int((a*cos(x)^2)^(5/2), x)
```

3.40 $\int \left(a\cos^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{2}{3}a\sqrt{a\cos^2(x)}\,\tan(x)+\frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)$$

```
[Out] 1/3*(a*cos(x)^2)^(3/2)*tan(x)+2/3*a*(a*cos(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3282, 3286, 2717}

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3282

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Dist[b*((2*p - 1)/(2*p)), Int[(b*Si
n[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && Gt
Q[p, 1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cos^2(x)\right)^{3/2} dx = \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x) + \frac{1}{3}(2a)\int \sqrt{a\cos^2(x)}\, dx$$
$$= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x) + \frac{1}{3}\left(2a\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\, dx$$
$$= \frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x) + \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 0.76

$$\frac{1}{12}a\sqrt{a\cos^2(x)}\sec(x)(9\sin(x)+\sin(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(3/2),x]
[Out] (a*Sqrt[a*Cos[x]^2]*Sec[x]*(9*Sin[x] + Sin[3*x]))/12
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.71

method	result
default	$\frac{a^2\cos(x)\sin(x)(\cos^2(x)+2)}{3\sqrt{a\left(\cos^2(x)\right)}}$
risch	$-\frac{ia\,e^{4ix}\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{24(e^{2ix}+1)} - \frac{3ia\,e^{2ix}\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{8(e^{2ix}+1)} + \frac{3ia\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{8(e^{2ix}+1)} + \frac{ia\,e^{-2ix}\sqrt{a\,(}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/3*a^2*cos(x)*sin(x)*(cos(x)^2+2)/(a*cos(x)^2)^(1/2)
```

Maxima [A]
time = 0.51, size = 17, normalized size = 0.50

$$\frac{1}{12}\left(a\sin\left(3x\right)+9a\sin\left(x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(3/2),x, algorithm="maxima")
[Out] 1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

Fricas [**A**]
time = 0.37, size = 26, normalized size = 0.76

$$\frac{\left(a\cos\left(x\right)^{2}+2\,a\right)\sqrt{a\cos\left(x\right)^{2}}\,\sin\left(x\right)}{3\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**2)**(3/2),x)
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 17, normalized size = 0.50

$$-\frac{1}{3}\left(\sin\left(x\right)^{3}-3\,\sin\left(x\right)\right)a^{\frac{3}{2}}\mathrm{sgn}(\cos\left(x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/3*(sin(x)^3 - 3*sin(x))*a^(3/2)*sgn(cos(x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int\left(a\cos\left(x\right)^{2}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(3/2),x)
[Out] int((a*cos(x)^2)^(3/2), x)
```

3.41 $\int \sqrt{a\cos^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{a\cos^2(x)}\,\tan(x)$$

```
[Out] (a*cos(x)^2)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 2717}

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \sqrt{a\cos^2(x)}\,dx &= \left(\sqrt{a\cos^2(x)}\,\sec(x)\right)\int \cos(x)\,dx \\ &= \sqrt{a\cos^2(x)}\,\tan(x) \end{aligned}$$

Mathematica [A]
time = 0.00, size = 13, normalized size = 1.00

$$\sqrt{a\cos^2(x)}\,\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Maple [A]
time = 0.03, size = 15, normalized size = 1.15

method	result	size
default	$\frac{a\cos(x)\sin(x)}{\sqrt{a\left(\cos^2(x)\right)}}$	15
risch	$-\frac{i\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}\, e^{2ix}}{2(e^{2ix}+1)}+\frac{i\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}{2\,e^{2ix}+2}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*cos(x)^2)^(1/2)*a*cos(x)*sin(x)
```

Maxima [A]
time = 0.51, size = 6, normalized size = 0.46

$$\sqrt{a}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*sin(x)
```

Fricas [A]
time = 0.35, size = 15, normalized size = 1.15

$$\frac{\sqrt{a\cos(x)^2}\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

Sympy [A]
time = 0.18, size = 15, normalized size = 1.15

$$\frac{\sqrt{a\cos^2(x)}\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**2)**(1/2),x)
```

```
[Out] sqrt(a*cos(x)**2)*sin(x)/cos(x)
```

Giac [A]
time = 0.53, size = 9, normalized size = 0.69

$$\sqrt{a}\,\mathrm{sgn}(\cos(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(a)*sgn(cos(x))*sin(x)
```

Mupad [B]
time = 0.21, size = 46, normalized size = 3.54

$$\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{\cos(2\,x)+1}\,\left(\cos(2\,x)-1+\sin(2\,x)\,1\mathrm{i}\right)}{2\,\left(\cos(2\,x)\,1\mathrm{i}-\sin(2\,x)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*a^(1/2)*(cos(2*x) + 1)^(1/2)*(cos(2*x) + sin(2*x)*1i - 1))/(2*(cos
(2*x)*1i - sin(2*x) + 1i))
```

3.42 $\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}$$

```
[Out] arctanh(sin(x))*cos(x)/(a*cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3855}

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[x]^2],x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx &= \frac{\cos(x)\int \sec(x)\,dx}{\sqrt{a\cos^2(x)}}\\ &= \frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(16) = 32.
time = 0.02, size = 46, normalized size = 2.88

$$\frac{\cos(x)\left(-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[x]^2],x]
```

```
[Out] (Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]))/Sqrt[a*Cos[
x]^2]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. 2(14) = 28.
time = 0.05, size = 48, normalized size = 3.00

method	result	size
default	$\frac{\cos(x)\sqrt{a\left(\sin^2(x)\right)}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)}{\sqrt{a}\,\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$	48
risch	$-\frac{2\ln\left(\mathrm{e}^{ix}-i\right)\cos(x)}{\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}+\frac{2\ln\left(\mathrm{e}^{ix}+i\right)\cos(x)}{\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^2\mathrm{e}^{-2ix}}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] cos(x)*(a*sin(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*sin(x)^2)^(1/2)+a)/cos(x
))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(14) = 28.
time = 0.53, size = 38, normalized size = 2.38

$$\frac{\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2+2\,\sin\left(x\right)+1\right)-\log\left(\cos\left(x\right)^2+\sin\left(x\right)^2-2\,\sin\left(x\right)+1\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1))/sqrt(a)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.
time = 0.37, size = 65, normalized size = 4.06

$$\left[-\frac{\sqrt{a\cos\left(x\right)^{2}}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos\left(x\right)}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos\left(x\right)^{2}}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-a
)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos^{2}\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**2)**(1/2),x)
[Out] Integral(1/sqrt(a*cos(x)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*cos(x)^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\cos\left(x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(1/2),x)
[Out] int(1/(a*cos(x)^2)^(1/2), x)
```

3.43 $\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}$$

```
[Out] 1/2*arctanh(sin(x))*cos(x)/a/(a*cos(x)^2)^(1/2)+1/2*tan(x)/a/(a*cos(x)^2)^(
1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3283, 3286, 3855}

$$\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\tanh^{-1}(\sin(x))}{2a\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(-3/2),x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx &= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{2a}\\ &= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\int \sec(x)\,dx}{2a\sqrt{a\cos^2(x)}}\\ &= \frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 91 vs. $2(42) = 84$.
time = 0.04, size = 91, normalized size = 2.17

$$-\frac{\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\cos(2x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)-2\sin(x)\right)}{4\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(-3/2),x]
```

```
[Out] -1/4*(Cos[x]*(Log[Cos[x/2] - Sin[x/2]] + Cos[2*x]*(Log[Cos[x/2] - Sin[x/2]]
 - Log[Cos[x/2] + Sin[x/2]]) - Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]))/(a*Cos
[x]^2)^(3/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.
time = 0.05, size = 70, normalized size = 1.67

method	result	size
default	$\frac{\sqrt{a\left(\sin^2(x)\right)}\left(\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)a\left(\cos^2(x)\right)+\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}\right)}{2a^{\frac{5}{2}}\cos(x)\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$	70
risch	$-\frac{i\left(e^{2ix}-1\right)}{a\left(e^{2ix}+1\right)\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}-\frac{\ln\left(e^{ix}-i\right)\cos(x)}{a\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}+\frac{\ln\left(e^{ix}+i\right)\cos(x)}{a\sqrt{a\left(e^{2ix}+1\right)^2 e^{-2ix}}}$	109

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^(5/2)/cos(x)*(a*sin(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*sin(x)^2)^(1/2)+a)/
cos(x))*a*cos(x)^2+a^(1/2)*(a*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(34) = 68$.
time = 0.54, size = 304, normalized size = 7.24

$$\frac{4 \left(\sin \left(3 x\right)-\sin \left(x\right)\right) \cos \left(4 x\right)+\left(2 \left(2 \cos \left(2 x\right)+1\right) \cos \left(4 x\right)+\cos \left(4 x\right)^{2}+4 \cos \left(2 x\right)^{2}+\sin \left(4 x\right)^{2}+4 \sin \left(4 x\right) \sin \left(2 x\right)+4 \sin \left(2 x\right)^{2}+4 \cos \left(2 x\right)+1\right) \log \left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}+2 \sin \left(x\right)+1\right)-\left(2 \left(2 \cos \left(2 x\right)+1\right) \cos \left(4 x\right)+\cos \left(4 x\right)^{2}+4 \cos \left(2 x\right)^{2}+\sin \left(4 x\right)^{2}+4 \sin \left(4 x\right) \sin \left(2 x\right)+4 \sin \left(2 x\right)^{2}+4 \cos \left(2 x\right)+1\right) \log \left(\cos \left(x\right)^{2}+\sin \left(x\right)^{2}-2 \sin \left(x\right)+1\right)-4 \left(\cos \left(3 x\right)-\cos \left(x\right)\right) \sin \left(4 x\right)+4 \left(2 \cos \left(2 x\right)+1\right) \sin \left(3 x\right)-8 \cos \left(3 x\right) \sin \left(2 x\right)+8 \cos \left(x\right) \sin \left(2 x\right)-8 \cos \left(2 x\right) \sin \left(x\right)-4 \sin \left(x\right)}{4 \left(a \cos \left(4 x\right)^{2}+4 a \cos \left(2 x\right)^{2}+a \sin \left(4 x\right)^{2}+4 a \sin \left(4 x\right) \sin \left(2 x\right)+4 a \sin \left(2 x\right)^{2}+2 \left(2 a \cos \left(2 x\right)+a\right) \cos \left(4 x\right)+4 a \cos \left(2 x\right)+a\right) \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x)^2 + 4
*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(
2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

Fricas [A]
time = 0.37, size = 40, normalized size = 0.95

$$-\frac{\sqrt{a \cos \left(x\right)^{2}} \left(\cos \left(x\right)^{2} \log \left(-\frac{\sin \left(x\right)-1}{\sin \left(x\right)+1}\right)-2 \sin \left(x\right)\right)}{4 a^{2} \cos \left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x))
/(a^2*cos(x)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a \cos^{2}\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**2)**(3/2),x)
```

```
[Out] Integral((a*cos(x)**2)**(-3/2), x)
```

Giac [A]
time = 0.56, size = 57, normalized size = 1.36

$$\frac{\frac{\log\left(\sin\left(x\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(x\right)\right)}-\frac{\log\left(-\sin\left(x\right)+1\right)}{\sqrt{a}\,\mathrm{sgn}\left(\cos\left(x\right)\right)}-\frac{2 \sin\left(x\right)}{\left(\sin\left(x\right)^{2}-1\right)\sqrt{a}\,\mathrm{sgn}\left(\cos\left(x\right)\right)}}{4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(log(sin(x) + 1)/(sqrt(a)*sgn(cos(x))) - log(-sin(x) + 1)/(sqrt(a)*sgn(
cos(x))) - 2*sin(x)/((sin(x)^2 - 1)*sqrt(a)*sgn(cos(x))))/a
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\cos\left(x\right)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(3/2),x)
```

```
[Out] int(1/(a*cos(x)^2)^(3/2), x)
```

3.44 $\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}+\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

```
[Out] 3/8*arctanh(sin(x))*cos(x)/a^2/(a*cos(x)^2)^(1/2)+1/4*tan(x)/a/(a*cos(x)^2)
^(3/2)+3/8*tan(x)/a^2/(a*cos(x)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3283, 3286, 3855}

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rule 3283

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Dist[2*((p + 1)/(b*(2*p
+ 1))), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx = \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx}{4a}$$
$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{8a^2}$$
$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{(3\cos(x))\int \sec(x)\,dx}{8a^2\sqrt{a\cos^2(x)}}$$
$$= \frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 1.18

$$\frac{\cos^5(x)\left(-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\sec^4(x)(11\sin(x)+3\sin(3x))\right)}{16\left(a\cos^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^2)^(-5/2),x]
```

```
[Out] (Cos[x]^5*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[
x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/(16*(a*Cos[x]^2)^(5/2))
```

Maple [A]
time = 0.05, size = 89, normalized size = 1.46

method	result
default	$\frac{\sqrt{a\left(\sin^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)a\left(\cos^4(x)\right)+3\sqrt{a\left(\sin^2(x)\right)}\left(\cos^2(x)\right)\sqrt{a}+2\sqrt{a}\,\sqrt{a\left(\sin^2\right.}\right.}{8a^{\frac{7}{2}}\cos(x)^3\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$
risch	$-\frac{i\left(3\,e^{6ix}+11\,e^{4ix}-11\,e^{2ix}-3\right)}{4a^2\left(e^{2ix}+1\right)^3\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}-\frac{3\ln\left(e^{ix}-i\right)\cos(x)}{4a^2\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}+\frac{3\ln\left(e^{ix}+i\right)\cos(x)}{4a^2\sqrt{a\left(e^{2ix}+1\right)^2e^{-2ix}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^(7/2)/cos(x)^3*(a*sin(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*sin(x)^2)^(1/2)
+a)/cos(x))*a*cos(x)^4+3*(a*sin(x)^2)^(1/2)*cos(x)^2*a^(1/2)+2*a^(1/2)*(a*s
in(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(49) = 98$.
time = 0.80, size = 933, normalized size = 15.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*(
2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin(3
*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(11*s
in(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) +
 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*c
os(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 +
4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(
4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*s
in(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x
) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x
)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(
2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^
2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(3*cos(7*x)
 + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*cos(6*x) + 6*cos(
4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos(3*x) - 3*cos(x))
*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 24*(11*cos(3*x) + 3
*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*cos(3*x)*sin(2*x) +
48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))/((a^2*cos(8*x)^2 + 16*
a^2*cos(6*x)^2 + 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 1
6*a^2*sin(6*x)^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*si
n(2*x)^2 + 8*a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^
2*cos(2*x) + a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(
6*x) + 12*(4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4
*x) + 2*a^2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6
*x))*sqrt(a))
```

Fricas [A]
time = 0.39, size = 49, normalized size = 0.80

$$-\frac{\left(3\cos\left(x\right)^{4}\log\left(-\frac{\sin\left(x\right)-1}{\sin\left(x\right)+1}\right)-2\left(3\cos\left(x\right)^{2}+2\right)\sin\left(x\right)\right)\sqrt{a\cos\left(x\right)^{2}}}{16\,a^{3}\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin(
x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 35, normalized size = 0.57

$$-\frac{3\sqrt{a}\,\sin(x)^3-5\sqrt{a}\,\sin(x)}{8\left(\sin(x)^2-1\right)^2a^3\mathrm{sgn}\left(\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(a)*sin(x)^3 - 5*sqrt(a)*sin(x))/((sin(x)^2 - 1)^2*a^3*sgn(cos(
x)))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\left(a\cos(x)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*cos(x)^2)^(5/2), x)
```

3.45 $\int \left(a\cos^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=117

$$\frac{26a^2\sqrt{a\cos^3(x)}\,F\left(\frac{x}{2}\middle|2\right)}{77\cos^{\frac{3}{2}}(x)}+\frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\,\sin(x)+\frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\,\sin(x)+\frac{2}{15}a^2\cos^5(x)$$

```
[Out] 26/77*a^2*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*(a*
cos(x)^3)^(1/2)/cos(x)^(3/2)+78/385*a^2*cos(x)*sin(x)*(a*cos(x)^3)^(1/2)+26
/165*a^2*cos(x)^3*sin(x)*(a*cos(x)^3)^(1/2)+2/15*a^2*cos(x)^5*sin(x)*(a*cos
(x)^3)^(1/2)+26/77*a^2*(a*cos(x)^3)^(1/2)*tan(x)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2720}

$$\frac{26}{165}a^2\sin(x)\cos^3(x)\sqrt{a\cos^3(x)}+\frac{78}{385}a^2\sin(x)\cos(x)\sqrt{a\cos^3(x)}+\frac{26}{77}a^2\tan(x)\sqrt{a\cos^3(x)}+\frac{26a^2F\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{77\cos^{\frac{3}{2}}(x)}+\frac{2}{15}a^2\sin(x)\cos^5(x)\sqrt{a\cos^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(5/2),x]
```

```
[Out] (26*a^2*Sqrt[a*Cos[x]^3]*EllipticF[x/2, 2])/(77*Cos[x]^(3/2)) + (78*a^2*Cos
[x]*Sqrt[a*Cos[x]^3]*Sin[x])/385 + (26*a^2*Cos[x]^3*Sqrt[a*Cos[x]^3]*Sin[x]
)/165 + (2*a^2*Cos[x]^5*Sqrt[a*Cos[x]^3]*Sin[x])/15 + (26*a^2*Sqrt[a*Cos[x]
^3]*Tan[x])/77
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cos^3(x)\right)^{5/2}\,dx = \frac{\left(a^2\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{15}{2}}(x)\,dx}{\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{\left(13a^2\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{11}{2}}(x)\,dx}{15\cos^{\frac{3}{2}}(x)}$$
$$= \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{2}{15}a^2\cos^5(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{\left(39a^2\sqrt{a\cos^3\right.}{55}$$
$$= \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{2}{15}a^2\cos^5(x)$$
$$= \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{2}{15}a^2\cos^5(x)$$
$$= \frac{26a^2\sqrt{a\cos^3(x)}\,F\left(\frac{x}{2}\middle|2\right)}{77\cos^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cos(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{26}{165}a^2\cos^3(x)\sqrt{a\cos}$$

Mathematica [A]
time = 0.07, size = 61, normalized size = 0.52

$$\frac{a\left(a\cos^3(x)\right)^{3/2}\left(12480F\left(\frac{x}{2}\middle|2\right) + \sqrt{\cos(x)}\,(15465\sin(x) + 3657\sin(3x) + 749\sin(5x) + 77\sin(7x))\right)}{36960\cos^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(5/2),x]
[Out] (a*(a*Cos[x]^3)^(3/2)*(12480*EllipticF[x/2, 2] + Sqrt[Cos[x]]*(15465*Sin[x]
 + 3657*Sin[3*x] + 749*Sin[5*x] + 77*Sin[7*x])))/(36960*Cos[x]^(9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.27, size = 114, normalized size = 0.97

method	result
default	$\frac{2(-1+\cos(x))\left(77(\cos^8(x))-77(\cos^7(x))+91(\cos^6(x))-91(\cos^5(x))-195i\sqrt{\frac{1}{\cos(x)+1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\ \text{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)\right.}{1155\cos(x)^8\sin(x)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/1155*(-1+cos(x))*(77*cos(x)^8-77*cos(x)^7+91*cos(x)^6-91*cos(x)^5-195*I*(
1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x
),I)*sin(x)+117*cos(x)^4-117*cos(x)^3+195*cos(x)^2-195*cos(x))*(cos(x)+1)^2
*(a*cos(x)^3)^(5/2)/cos(x)^8/sin(x)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 92, normalized size = 0.79

$$\frac{195i\,\sqrt{2}\,a^{\frac{5}{2}}\cos\left(x\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(x\right)+i\,\sin\left(x\right))-195i\,\sqrt{2}\,a^{\frac{5}{2}}\cos\left(x\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(x\right)-i\,\sin\left(x\right))+2\left(77\,a^{2}\cos\left(x\right)^{6}+91\,a^{2}\cos\left(x\right)^{4}+117\,a^{2}\cos\left(x\right)^{2}+195\,a^{2}\right)\sqrt{a\cos\left(x\right)^{3}}\,\sin\left(x\right)}{1155\,\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1155*(195*I*sqrt(2)*a^(5/2)*cos(x)*weierstrassPInverse(-4, 0, cos(x) + I*
sin(x)) - 195*I*sqrt(2)*a^(5/2)*cos(x)*weierstrassPInverse(-4, 0, cos(x) -
I*sin(x)) + 2*(77*a^2*cos(x)^6 + 91*a^2*cos(x)^4 + 117*a^2*cos(x)^2 + 195*a
^2)*sqrt(a*cos(x)^3)*sin(x))/cos(x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cos\left(x\right)^3\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(5/2),x)
```

```
[Out] int((a*cos(x)^3)^(5/2), x)
```

3.46 $\int \left(a\cos^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{14a\sqrt{a\cos^3(x)}\,E\left(\frac{x}{2}\middle|2\right)}{15\cos^{\frac{3}{2}}(x)}+\frac{14}{45}a\sqrt{a\cos^3(x)}\,\sin(x)+\frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\,\sin(x)$$

```
[Out] 14/15*a*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2))*(a*co
s(x)^3)^(1/2)/cos(x)^(3/2)+14/45*a*sin(x)*(a*cos(x)^3)^(1/2)+2/9*a*cos(x)^2
*sin(x)*(a*cos(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.300$, Rules used = {3286, 2715, 2719}

$$\frac{14}{45}a\sin(x)\sqrt{a\cos^3(x)}+\frac{14aE\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{15\cos^{\frac{3}{2}}(x)}+\frac{2}{9}a\sin(x)\cos^2(x)\sqrt{a\cos^3(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(3/2),x]
```

```
[Out] (14*a*Sqrt[a*Cos[x]^3]*EllipticE[x/2, 2])/(15*Cos[x]^(3/2)) + (14*a*Sqrt[a*
Cos[x]^3]*Sin[x])/45 + (2*a*Cos[x]^2*Sqrt[a*Cos[x]^3]*Sin[x])/9
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cos^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{9}{2}}(x)\,dx}{\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{\left(7a\sqrt{a\cos^3(x)}\right)\int \cos^{\frac{5}{2}}(x)\,dx}{9\cos^{\frac{3}{2}}(x)}$$
$$= \frac{14}{45}a\sqrt{a\cos^3(x)}\,\sin(x) + \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\,\sin(x) + \frac{\left(7a\sqrt{a\cos^3(x)}\right)\int \sqrt{\cos}}{15\cos^{\frac{3}{2}}(x)}$$
$$= \frac{14a\sqrt{a\cos^3(x)}\,E\left(\frac{x}{2}\middle|2\right)}{15\cos^{\frac{3}{2}}(x)} + \frac{14}{45}a\sqrt{a\cos^3(x)}\,\sin(x) + \frac{2}{9}a\cos^2(x)\sqrt{a\cos^3(x)}\,\sin(x)$$

Mathematica [A]
time = 0.05, size = 50, normalized size = 0.75

$$\frac{\left(a\cos^3(x)\right)^{3/2}\left(168E\left(\frac{x}{2}\middle|2\right) + \sqrt{\cos(x)}\,(38\sin(2x) + 5\sin(4x))\right)}{180\cos^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(3/2),x]
```

```
[Out] ((a*Cos[x]^3)^(3/2)*(168*EllipticE[x/2, 2] + Sqrt[Cos[x]]*(38*Sin[2*x] + 5*
Sin[4*x])))/(180*Cos[x]^(9/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.08, size = 198, normalized size = 2.96

method	result
default	$-\frac{2\left(5\left(\cos^6(x)\right)+21i\cos(x)\sin(x)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,-21i\cos(x)\sin(x)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(5*cos(x)^6+21*I*cos(x)*sin(x)*EllipticE(I*(-1+cos(x))/sin(x),I)*(1/(
cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)-21*I*cos(x)*sin(x)*EllipticF(I*(
-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)+21*I*si
n(x)*EllipticE(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)
+1))^(1/2)-21*I*sin(x)*EllipticF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/
```

```
2)*(cos(x)/(cos(x)+1))^(1/2)+2*cos(x)^4+14*cos(x)^2-21*cos(x))*(a*cos(x)^3)
^(3/2)/cos(x)^5/sin(x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 66, normalized size = 0.99

$$-\frac{7}{15} i \sqrt{2} a^{\frac{3}{2}} \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(x) + i \sin(x))) + \frac{7}{15} i \sqrt{2} a^{\frac{3}{2}} \mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(x) - i \sin(x))) + \frac{2}{45} \sqrt{a \cos(x)^{3}} (5 a \cos(x)^{2} + 7 a) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] -7/15*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(x) + I*sin(x))) + 7/15*I*sqrt(2)*a^(3/2)*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(x) - I*sin(x))) + 2/45*sqrt(a*cos(x)^3)*(5*a*cos(x)^
2 + 7*a)*sin(x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**3)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\cos\left(x\right)^3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(3/2),x)
```

```
[Out] int((a*cos(x)^3)^(3/2), x)
```

3.47 $\int \sqrt{a\cos^3(x)}\,dx$

Optimal. Leaf size=44

$$\frac{2\sqrt{a\cos^3(x)}\,F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)}+\frac{2}{3}\sqrt{a\cos^3(x)}\,\tan(x)$$

```
[Out] 2/3*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(1/2))*(a*cos(x)
^3)^(1/2)/cos(x)^(3/2)+2/3*(a*cos(x)^3)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 2720}

$$\frac{2}{3}\tan(x)\sqrt{a\cos^3(x)}+\frac{2F\left(\frac{x}{2}\middle|2\right)\sqrt{a\cos^3(x)}}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[x]^3],x]
```

```
[Out] (2*Sqrt[a*Cos[x]^3]*EllipticF[x/2, 2])/(3*Cos[x]^(3/2)) + (2*Sqrt[a*Cos[x]^
3]*Tan[x])/3
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\cos^3(x)}\,dx = \frac{\sqrt{a\cos^3(x)}\int \cos^{\frac{3}{2}}(x)\,dx}{\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2}{3}\sqrt{a\cos^3(x)}\tan(x) + \frac{\sqrt{a\cos^3(x)}\int \frac{1}{\sqrt{\cos(x)}}\,dx}{3\cos^{\frac{3}{2}}(x)}$$
$$= \frac{2\sqrt{a\cos^3(x)}\,F\left(\frac{x}{2}\middle|2\right)}{3\cos^{\frac{3}{2}}(x)} + \frac{2}{3}\sqrt{a\cos^3(x)}\tan(x)$$

Mathematica [A]
time = 0.02, size = 37, normalized size = 0.84

$$\frac{2\sqrt{a\cos^3(x)}\left(F\left(\frac{x}{2}\middle|2\right)+\sqrt{\cos(x)}\sin(x)\right)}{3\cos^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[x]^3],x]
[Out] (2*Sqrt[a*Cos[x]^3]*(EllipticF[x/2, 2] + Sqrt[Cos[x]]*Sin[x]))/(3*Cos[x]^(3
/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.09, size = 76, normalized size = 1.73

method	result
default	$-\frac{2(-1+\cos(x))\left(i\sqrt{\frac{1}{\cos(x)+1}}\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)\sin(x)-\left(\cos^2(x)\right)+\cos(x)\right)(\cos(x)+1)^2\sqrt{a\left(\cos^3\left(x\right.\right.}}{3\sin(x)^3\cos(x)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(-1+cos(x))*(I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*Elliptic
F(I*(-1+cos(x))/sin(x),I)*sin(x)-cos(x)^2+cos(x))*(cos(x)+1)^2*(a*cos(x)^3)
^(1/2)/sin(x)^3/cos(x)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cos(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 59, normalized size = 1.34

$$\frac{i\sqrt{2}\sqrt{a}\cos(x)\,\text{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x))-i\sqrt{2}\sqrt{a}\cos(x)\,\text{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x))+2\sqrt{a\cos(x)^3}\sin(x)}{3\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(I*sqrt(2)*sqrt(a)*cos(x)*weierstrassPInverse(-4, 0, cos(x) + I*sin(x))
 - I*sqrt(2)*sqrt(a)*cos(x)*weierstrassPInverse(-4, 0, cos(x) - I*sin(x)) +
 2*sqrt(a*cos(x)^3)*sin(x))/cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*cos(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cos(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^3)^(1/2),x)
```

```
[Out] int((a*cos(x)^3)^(1/2), x)
```

3.48 $\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$

Optimal. Leaf size=42

$$-\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}}+\frac{2\cos(x)\sin(x)}{\sqrt{a\cos^3(x)}}$$

```
[Out] -2*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),2^(1/2
))/(a*cos(x)^3)^(1/2)+2*cos(x)*sin(x)/(a*cos(x)^3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2719}

$$\frac{2\sin(x)\cos(x)}{\sqrt{a\cos^3(x)}}-\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[x]^3],x]
```

```
[Out] (-2*Cos[x]^(3/2)*EllipticE[x/2, 2])/Sqrt[a*Cos[x]^3] + (2*Cos[x]*Sin[x])/Sq
rt[a*Cos[x]^3]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx &= \frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\cos^3(x)}}\\ &= \frac{2\cos(x)\sin(x)}{\sqrt{a\cos^3(x)}} - \frac{\cos^{\frac{3}{2}}(x)\int \sqrt{\cos(x)}\,dx}{\sqrt{a\cos^3(x)}}\\ &= -\frac{2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{\sqrt{a\cos^3(x)}} + \frac{2\cos(x)\sin(x)}{\sqrt{a\cos^3(x)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 0.74

$$\frac{-2\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)+\sin(2x)}{\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[x]^3],x]
```

```
[Out] (-2*Cos[x]^(3/2)*EllipticE[x/2, 2] + Sin[2*x])/Sqrt[a*Cos[x]^3]
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 191, normalized size = 4.55

method	result
default	$\frac{2(\cos(x)+1)^2(-1+\cos(x))^2\left(i\cos(x)\sin(x)\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,-i\cos(x)\sin(x)\,\mathrm{EllipticF}\left(\frac{i(-1+\cos}{\sin(x}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(cos(x)+1)^2*(-1+cos(x))^2*(I*cos(x)*sin(x)*EllipticE(I*(-1+cos(x))/sin(x
),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)-I*cos(x)*sin(x)*Ellipti
cF(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)+I
*sin(x)*EllipticE(I*(-1+cos(x))/sin(x),I)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos
(x)+1))^(1/2)-I*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*
(-1+cos(x))/sin(x),I)*sin(x)-cos(x)+1)*cos(x)/(a*cos(x)^3)^(1/2)/sin(x)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*cos(x)^3), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 71, normalized size = 1.69

$$\frac{i\sqrt{2}\sqrt{a}\cos(x)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x))) - i\sqrt{2}\sqrt{a}\cos(x)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x))) + 2\sqrt{a\cos(x)^3}\sin(x)}{a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(a)*cos(x)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(x) + I*sin(x))) - I*sqrt(2)*sqrt(a)*cos(x)^2*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(x) - I*sin(x))) + 2*sqrt(a*cos(x)^3)*sin(x))
/(a*cos(x)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\cos^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*cos(x)**3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*cos(x)^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a\cos(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(1/2),x)
[Out] int(1/(a*cos(x)^3)^(1/2), x)
```

3.49 $\int \frac{1}{\left(a\cos^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=71

$$\frac{10\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)}{21a\sqrt{a\cos^3(x)}}+\frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}}$$

```
[Out] 10/21*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticF(sin(1/2*x),2^(
1/2))/a/(a*cos(x)^3)^(1/2)+10/21*sin(x)/a/(a*cos(x)^3)^(1/2)+2/7*sec(x)*tan
(x)/a/(a*cos(x)^3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2720}

$$\frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{10\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)}{21a\sqrt{a\cos^3(x)}}+\frac{2\tan(x)\sec(x)}{7a\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(-3/2),x]
```

```
[Out] (10*Cos[x]^(3/2)*EllipticF[x/2, 2])/(21*a*Sqrt[a*Cos[x]^3]) + (10*Sin[x])/(
21*a*Sqrt[a*Cos[x]^3]) + (2*Sec[x]*Tan[x])/(7*a*Sqrt[a*Cos[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a\cos^3(x)\right)^{3/2}}\,dx &= \frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\cos^3(x)}}\\
&= \frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}}+\frac{\left(5\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\cos^3(x)}}\\
&= \frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}}+\frac{\left(5\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\cos(x)}}\,dx}{21a\sqrt{a\cos^3(x)}}\\
&= \frac{10\cos^{\frac{3}{2}}(x)F\left(\frac{x}{2}\middle|2\right)}{21a\sqrt{a\cos^3(x)}}+\frac{10\sin(x)}{21a\sqrt{a\cos^3(x)}}+\frac{2\sec(x)\tan(x)}{7a\sqrt{a\cos^3(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 44, normalized size = 0.62

$$
\frac{2\cos^2(x)\left(5\cos^{\frac{5}{2}}(x)F\left(\frac{x}{2}\middle|2\right)+5\cos(x)\sin(x)+3\tan(x)\right)}{21\left(a\cos^3(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(-3/2),x]
```

```
[Out] (2*Cos[x]^2*(5*Cos[x]^(5/2)*EllipticF[x/2, 2] + 5*Cos[x]*Sin[x] + 3*Tan[x])
)/(21*(a*Cos[x]^3)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.23, size = 87, normalized size = 1.23

method	result
default	$-\frac{2(\cos(x)+1)^2(-1+\cos(x))\left(5i(\cos^3(x))\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\ \text{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)-5(\cos^3(x))+5(\cos^2(x))-\right.}{21\sin(x)^3(a(\cos^3(x)))^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(cos(x)+1)^2*(-1+cos(x))*(5*I*cos(x)^3*sin(x)*(1/(cos(x)+1))^(1/2)*(c
os(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-5*cos(x)^3+5*cos(
x)^2-3*cos(x)+3)*cos(x)/sin(x)^3/(a*cos(x)^3)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="maxima")
[Out] integrate((a*cos(x)^3)^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 74, normalized size = 1.04

$$\frac{5i\,\sqrt{2}\,\sqrt{a}\,\cos\left(x\right)^{5}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(x\right)+i\,\sin\left(x\right))-5i\,\sqrt{2}\,\sqrt{a}\,\cos\left(x\right)^{5}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(x\right)-i\,\sin\left(x\right))+2\,\sqrt{a\cos\left(x\right)^{3}}\,\left(5\,\cos\left(x\right)^{2}+3\right)\sin\left(x\right)}{21\,a^{2}\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="fricas")
[Out] 1/21*(5*I*sqrt(2)*sqrt(a)*cos(x)^5*weierstrassPInverse(-4, 0, cos(x) + I*si
n(x)) - 5*I*sqrt(2)*sqrt(a)*cos(x)^5*weierstrassPInverse(-4, 0, cos(x) - I*
sin(x)) + 2*sqrt(a*cos(x)^3)*(5*cos(x)^2 + 3)*sin(x))/(a^2*cos(x)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**3)**(3/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*cos(x)^3)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a\cos\left(x\right)^{3}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(3/2),x)
[Out] int(1/(a*cos(x)^3)^(3/2), x)
```

3.50 $\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}}+\frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}}+\frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}}$$

```
[Out] -154/195*cos(x)^(3/2)*(cos(1/2*x)^2)^(1/2)/cos(1/2*x)*EllipticE(sin(1/2*x),
2^(1/2))/a^2/(a*cos(x)^3)^(1/2)+154/195*cos(x)*sin(x)/a^2/(a*cos(x)^3)^(1/2
)+154/585*tan(x)/a^2/(a*cos(x)^3)^(1/2)+22/117*sec(x)^2*tan(x)/a^2/(a*cos(x
)^3)^(1/2)+2/13*sec(x)^4*tan(x)/a^2/(a*cos(x)^3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2716, 2719}

$$\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\cos^3(x)}}+\frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}}-\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}}+\frac{2\tan(x)\sec^4(x)}{13a^2\sqrt{a\cos^3(x)}}+\frac{22\tan(x)\sec^2(x)}{117a^2\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^3)^(-5/2),x]
```

```
[Out] (-154*Cos[x]^(3/2)*EllipticE[x/2, 2])/(195*a^2*Sqrt[a*Cos[x]^3]) + (154*Cos
[x]*Sin[x])/(195*a^2*Sqrt[a*Cos[x]^3]) + (154*Tan[x])/(585*a^2*Sqrt[a*Cos[x
]^3]) + (22*Sec[x]^2*Tan[x])/(117*a^2*Sqrt[a*Cos[x]^3]) + (2*Sec[x]^4*Tan[x
])/(13*a^2*Sqrt[a*Cos[x]^3])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
```

```
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\cos^3(x)\right)^{5/2}}\,dx = \frac{\cos^{\frac{3}{2}}(x)\int \frac{1}{\cos^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} + \frac{\left(11\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} + \frac{\left(77\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}} + \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} + \frac{\left(77\cos^{\frac{3}{2}}(x)\right)\int \frac{1}{\cos^{\frac{3}{2}}(x)}}{195a^2\sqrt{a\cos^3(x)}}$$

$$= \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\cos^3(x)}} + \frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}} + \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}} + \frac{2\sec^4(x)\tan(x)}{13a^2\sqrt{a\cos^3(x)}} -$$

$$= -\frac{154\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\cos^3(x)}} + \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\cos^3(x)}} + \frac{154\tan(x)}{585a^2\sqrt{a\cos^3(x)}} + \frac{22\sec^2(x)\tan(x)}{117a^2\sqrt{a\cos^3(x)}}$$

Mathematica [A]
time = 0.07, size = 57, normalized size = 0.49

$$\frac{-462\cos^{\frac{3}{2}}(x)E\left(\frac{x}{2}\middle|2\right) + 462\cos(x)\sin(x) + 2(77 + 55\sec^2(x) + 45\sec^4(x))\tan(x)}{585a^2\sqrt{a\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^3)^(-5/2),x]
[Out] (-462*Cos[x]^(3/2)*EllipticE[x/2, 2] + 462*Cos[x]*Sin[x] + 2*(77 + 55*Sec[x
]^2 + 45*Sec[x]^4)*Tan[x])/(585*a^2*Sqrt[a*Cos[x]^3])
```

Maple [C] Result contains complex when optimal does not.
time = 0.34, size = 223, normalized size = 1.91

method	result

default	$-\frac{2(\cos(x)+1)^2(-1+\cos(x))^2\left(231i(\cos^7(x))\sin(x)\sqrt{\frac{1}{\cos(x)+1}}\,\sqrt{\frac{\cos(x)}{\cos(x)+1}}\,\operatorname{EllipticF}\left(\frac{i(-1+\cos(x))}{\sin(x)},i\right)-231i(\cos^7(x))\sin(x)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/585*(cos(x)+1)^2*(-1+cos(x))^2*(231*I*cos(x)^7*sin(x)*(1/(cos(x)+1))^(1/
2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)
^7*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+co
s(x))/sin(x),I)+231*I*cos(x)^6*sin(x)*(1/(cos(x)+1))^(1/2)*(cos(x)/(cos(x)+
1))^(1/2)*EllipticF(I*(-1+cos(x))/sin(x),I)-231*I*cos(x)^6*sin(x)*(1/(cos(x
)+1))^(1/2)*(cos(x)/(cos(x)+1))^(1/2)*EllipticE(I*(-1+cos(x))/sin(x),I)+231
*cos(x)^7-154*cos(x)^6-22*cos(x)^4-10*cos(x)^2-45)*cos(x)/sin(x)^5/(a*cos(x
)^3)^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(x)^3)^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 92, normalized size = 0.79

$$\frac{231i\sqrt{2}\sqrt{a}\cos(x)^8\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)+i\sin(x)))-231i\sqrt{2}\sqrt{a}\cos(x)^8\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(x)-i\sin(x)))+2\left(231\cos(x)^6+77\cos(x)^4+55\cos(x)^2+45\right)\sqrt{a\cos(x)^3}\sin(x)}{585a^3\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/585*(231*I*sqrt(2)*sqrt(a)*cos(x)^8*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(x) + I*sin(x))) - 231*I*sqrt(2)*sqrt(a)*cos(x)^8*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(x) - I*sin(x))) + 2*(231*cos(x
)^6 + 77*cos(x)^4 + 55*cos(x)^2 + 45)*sqrt(a*cos(x)^3)*sin(x))/(a^3*cos(x)^
8)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**3)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(x)^3)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\cos\left(x\right)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^3)^(5/2),x)
```

```
[Out] int(1/(a*cos(x)^3)^(5/2), x)
```

3.51 $\int \left(a\cos^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$\frac{63}{256}a^2x\sqrt{a\cos^4(x)}\,\sec^2(x)+\frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\,\sin(x)+\frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x)+\frac{9}{80}a^2\cos^5($$

```
[Out] 63/256*a^2*x*sec(x)^2*(a*cos(x)^4)^(1/2)+21/128*a^2*cos(x)*sin(x)*(a*cos(x)
^4)^(1/2)+21/160*a^2*cos(x)^3*sin(x)*(a*cos(x)^4)^(1/2)+9/80*a^2*cos(x)^5*s
in(x)*(a*cos(x)^4)^(1/2)+1/10*a^2*cos(x)^7*sin(x)*(a*cos(x)^4)^(1/2)+63/256
*a^2*(a*cos(x)^4)^(1/2)*tan(x)
```

Rubi [A]
time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{21}{128}a^2\sin(x)\cos(x)\sqrt{a\cos^4(x)}+\frac{63}{256}a^2\tan(x)\sqrt{a\cos^4(x)}+\frac{63}{256}a^2x\sec^2(x)\sqrt{a\cos^4(x)}+\frac{1}{10}a^2\sin(x)\cos^7(x)\sqrt{a\cos^4(x)}+\frac{9}{80}a^2\sin(x)\cos^5(x)\sqrt{a\cos^4(x)}+\frac{21}{160}a^2\sin(x)\cos^3(x)\sqrt{a\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(5/2),x]
```

```
[Out] (63*a^2*x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/256 + (21*a^2*Cos[x]*Sqrt[a*Cos[x]^4]*
Sin[x])/128 + (21*a^2*Cos[x]^3*Sqrt[a*Cos[x]^4]*Sin[x])/160 + (9*a^2*Cos[x]
^5*Sqrt[a*Cos[x]^4]*Sin[x])/80 + (a^2*Cos[x]^7*Sqrt[a*Cos[x]^4]*Sin[x])/10
+ (63*a^2*Sqrt[a*Cos[x]^4]*Tan[x])/256
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cos^4(x)\right)^{5/2}\,dx = \left(a^2\sqrt{a\cos^4(x)}\,\sec^2(x)\right)\int \cos^{10}(x)\,dx$$
$$= \frac{1}{10}a^2\cos^7(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{10}\left(9a^2\sqrt{a\cos^4(x)}\,\sec^2(x)\right)\int \cos^8(x)\,dx$$
$$= \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{10}a^2\cos^7(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{80}\Big(63a^2\sqrt{a\cos^4}$$
$$= \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{10}a^2\cos^7(x)\sqrt{}$$
$$= \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{}$$
$$= \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{9}{80}a^2\cos^5(x)\sqrt{}$$
$$= \frac{63}{256}a^2x\sqrt{a\cos^4(x)}\,\sec^2(x) + \frac{21}{128}a^2\cos(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{21}{160}a^2\cos^3(x)\sqrt{a\cos}$$

Mathematica [A]
time = 0.08, size = 53, normalized size = 0.40

$$\frac{a\left(a\cos^4(x)\right)^{3/2}\sec^6(x)(2520x + 2100\sin(2x) + 600\sin(4x) + 150\sin(6x) + 25\sin(8x) + 2\sin(10x))}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(5/2),x]
```

```
[Out] (a*(a*Cos[x]^4)^(3/2)*Sec[x]^6*(2520*x + 2100*Sin[2*x] + 600*Sin[4*x] + 150
*Sin[6*x] + 25*Sin[8*x] + 2*Sin[10*x]))/10240
```

Maple [A]
time = 0.41, size = 57, normalized size = 0.43

method	result
default	$\frac{\left(a\left(\cos^4(x)\right)\right)^{\frac{5}{2}}\left(128\left(\cos^9(x)\right)\sin(x)+144\left(\cos^7(x)\right)\sin(x)+168\left(\cos^5(x)\right)\sin(x)+210\left(\cos^3(x)\right)\sin(x)+315\cos(x)\sin(x)+315x\right)}{1280\cos(x)^{10}}$
risch	$\frac{63a^2e^{2ix}\sqrt{a\left(e^{2ix}+1\right)^4e^{-4ix}}\,x}{256\left(e^{2ix}+1\right)^2} - \frac{ia^2e^{12ix}\sqrt{a\left(e^{2ix}+1\right)^4e^{-4ix}}}{10240\left(e^{2ix}+1\right)^2} - \frac{5ia^2e^{10ix}\sqrt{a\left(e^{2ix}+1\right)^4e^{-4ix}}}{4096\left(e^{2ix}+1\right)^2} - \frac{105ia^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1280*(a*cos(x)^4)^(5/2)*(128*cos(x)^9*sin(x)+144*cos(x)^7*sin(x)+168*cos(
x)^5*sin(x)+210*cos(x)^3*sin(x)+315*cos(x)*sin(x)+315*x)/cos(x)^10
```

Maxima [**A**]
time = 0.50, size = 85, normalized size = 0.64

$$\frac{63}{256}a^{\frac{5}{2}}x+\frac{315\,a^{\frac{5}{2}}\tan\left(x\right)^{9}+1470\,a^{\frac{5}{2}}\tan\left(x\right)^{7}+2688\,a^{\frac{5}{2}}\tan\left(x\right)^{5}+2370\,a^{\frac{5}{2}}\tan\left(x\right)^{3}+965\,a^{\frac{5}{2}}\tan\left(x\right)}{1280\,\left(\tan\left(x\right)^{10}+5\,\tan\left(x\right)^{8}+10\,\tan\left(x\right)^{6}+10\,\tan\left(x\right)^{4}+5\,\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x + 1/1280*(315*a^(5/2)*tan(x)^9 + 1470*a^(5/2)*tan(x)^7 + 2
688*a^(5/2)*tan(x)^5 + 2370*a^(5/2)*tan(x)^3 + 965*a^(5/2)*tan(x))/(tan(x)^
10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

Fricas [**A**]
time = 0.37, size = 68, normalized size = 0.52

$$\frac{\sqrt{a\cos\left(x\right)^{4}}\,\left(315\,a^{2}x+\left(128\,a^{2}\cos\left(x\right)^{9}+144\,a^{2}\cos\left(x\right)^{7}+168\,a^{2}\cos\left(x\right)^{5}+210\,a^{2}\cos\left(x\right)^{3}+315\,a^{2}\cos\left(x\right)\right)\sin\left(x\right)\right)}{1280\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1280*sqrt(a*cos(x)^4)*(315*a^2*x + (128*a^2*cos(x)^9 + 144*a^2*cos(x)^7 +
 168*a^2*cos(x)^5 + 210*a^2*cos(x)^3 + 315*a^2*cos(x))*sin(x))/cos(x)^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**4)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 57, normalized size = 0.43

$$\frac{1}{10240}\left(2520\,a^{2}x+2\,a^{2}\sin\left(10\,x\right)+25\,a^{2}\sin\left(8\,x\right)+150\,a^{2}\sin\left(6\,x\right)+600\,a^{2}\sin\left(4\,x\right)+2100\,a^{2}\sin\left(2\,x\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10240*(2520*a^2*x + 2*a^2*sin(10*x) + 25*a^2*sin(8*x) + 150*a^2*sin(6*x)
+ 600*a^2*sin(4*x) + 2100*a^2*sin(2*x))*sqrt(a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a \cos \left(x\right)^4\right)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(5/2),x)
```

```
[Out] int((a*cos(x)^4)^(5/2), x)
```

3.52 $\int \left(a \cos^4(x)\right)^{3/2} \, dx$

Optimal. Leaf size=78

$$\frac{5}{16} a x \sqrt{a \cos^4(x)} \sec^2(x)+\frac{5}{24} a \cos(x) \sqrt{a \cos^4(x)} \sin(x)+\frac{1}{6} a \cos^3(x) \sqrt{a \cos^4(x)} \sin(x)+\frac{5}{16} a \sqrt{a \cos^4(x)} \text{ t}$$

```
[Out] 5/16*a*x*sec(x)^2*(a*cos(x)^4)^(1/2)+5/24*a*cos(x)*sin(x)*(a*cos(x)^4)^(1/2
)+1/6*a*cos(x)^3*sin(x)*(a*cos(x)^4)^(1/2)+5/16*a*(a*cos(x)^4)^(1/2)*tan(x)
```

Rubi [A]

time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{5}{24} a \sin(x) \cos(x) \sqrt{a \cos^4(x)} + \frac{5}{16} a \tan(x) \sqrt{a \cos^4(x)} + \frac{5}{16} a x \sec^2(x) \sqrt{a \cos^4(x)} + \frac{1}{6} a \sin(x) \cos^3(x) \sqrt{a \cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(3/2),x]
```

```
[Out] (5*a*x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/16 + (5*a*Cos[x]*Sqrt[a*Cos[x]^4]*Sin[x])
/24 + (a*Cos[x]^3*Sqrt[a*Cos[x]^4]*Sin[x])/6 + (5*a*Sqrt[a*Cos[x]^4]*Tan[x]
)/16
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\cos^4(x)\right)^{3/2} dx = \left(a\sqrt{a\cos^4(x)}\,\sec^2(x)\right)\int \cos^6(x)\,dx$$
$$= \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{6}\left(5a\sqrt{a\cos^4(x)}\,\sec^2(x)\right)\int \cos^4(x)\,dx$$
$$= \frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{8}\left(5a\sqrt{a\cos^4(x)}\,\mathrm{sec}\right.$$
$$= \frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{5}{16}a\sqrt{a\cos^4(x)}\,\tan($$
$$= \frac{5}{16}ax\sqrt{a\cos^4(x)}\,\sec^2(x) + \frac{5}{24}a\cos(x)\sqrt{a\cos^4(x)}\,\sin(x) + \frac{1}{6}a\cos^3(x)\sqrt{a\cos^4(x)}\,\mathrm{si}$$

Mathematica [A]
time = 0.04, size = 38, normalized size = 0.49

$$\frac{1}{192}\left(a\cos^4(x)\right)^{3/2}\sec^6(x)(60x + 45\sin(2x) + 9\sin(4x) + \sin(6x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(3/2),x]
[Out] ((a*Cos[x]^4)^(3/2)*Sec[x]^6*(60*x + 45*Sin[2*x] + 9*Sin[4*x] + Sin[6*x]))/
192
```

Maple [A]
time = 0.15, size = 41, normalized size = 0.53

method	result
default	$\frac{\left(a\left(\cos^4(x)\right)\right)^{\frac{3}{2}}\left(8\left(\cos^5(x)\right)\sin(x)+10\left(\cos^3(x)\right)\sin(x)+15\cos(x)\sin(x)+15x\right)}{48\cos(x)^6}$
risch	$\frac{5a\,e^{2ix}\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}\;x}{16\left(e^{2ix}+1\right)^2} - \frac{ia\,e^{8ix}\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}}{384\left(e^{2ix}+1\right)^2} - \frac{3ia\,e^{6ix}\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}}{128\left(e^{2ix}+1\right)^2} - \frac{15ia\,e^{4ix}\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/48*(a*cos(x)^4)^(3/2)*(8*cos(x)^5*sin(x)+10*cos(x)^3*sin(x)+15*cos(x)*sin
(x)+15*x)/cos(x)^6
```

Maxima [A]
time = 0.49, size = 55, normalized size = 0.71

$$\frac{5}{16}\,a^{\frac{3}{2}}x + \frac{15\,a^{\frac{3}{2}}\tan(x)^5 + 40\,a^{\frac{3}{2}}\tan(x)^3 + 33\,a^{\frac{3}{2}}\tan(x)}{48\left(\tan(x)^6 + 3\,\tan(x)^4 + 3\,\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(x)^4)^(3/2),x, algorithm="maxima")

[Out] 5/16*a^(3/2)*x + 1/48*(15*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 33*a^(3/2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)

Fricas [A]
time = 0.37, size = 42, normalized size = 0.54

$$\frac{\sqrt{a\cos\left(x\right)^{4}}\,\left(15\,ax+\left(8\,a\cos\left(x\right)^{5}+10\,a\cos\left(x\right)^{3}+15\,a\cos\left(x\right)\right)\sin\left(x\right)\right)}{48\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(x)^4)^(3/2),x, algorithm="fricas")

[Out] 1/48*sqrt(a*cos(x)^4)*(15*a*x + (8*a*cos(x)^5 + 10*a*cos(x)^3 + 15*a*cos(x))*sin(x))/cos(x)^2

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(x)**4)**(3/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep

Giac [A]
time = 0.46, size = 25, normalized size = 0.32

$$\frac{1}{192}\,a^{\frac{3}{2}}\left(60\,x+\sin\left(6\,x\right)+9\,\sin\left(4\,x\right)+45\,\sin\left(2\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(x)^4)^(3/2),x, algorithm="giac")

[Out] 1/192*a^(3/2)*(60*x + sin(6*x) + 9*sin(4*x) + 45*sin(2*x))

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a\cos\left(x\right)^{4}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(x)^4)^(3/2),x)

[Out] int((a*cos(x)^4)^(3/2), x)

3.53 $\int \sqrt{a\cos^4(x)}\, dx$

Optimal. Leaf size=36

$$\frac{1}{2}x\sqrt{a\cos^4(x)}\,\sec^2(x)+\frac{1}{2}\sqrt{a\cos^4(x)}\,\tan(x)$$

```
[Out] 1/2*x*sec(x)^2*(a*cos(x)^4)^(1/2)+1/2*(a*cos(x)^4)^(1/2)*tan(x)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 2715, 8}

$$\frac{1}{2}\tan(x)\sqrt{a\cos^4(x)}+\frac{1}{2}x\sec^2(x)\sqrt{a\cos^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Cos[x]^4],x]
```

```
[Out] (x*Sqrt[a*Cos[x]^4]*Sec[x]^2)/2 + (Sqrt[a*Cos[x]^4]*Tan[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a\cos^4(x)}\,dx = \left(\sqrt{a\cos^4(x)}\,\sec^2(x)\right)\int \cos^2(x)\,dx$$
$$= \frac{1}{2}\sqrt{a\cos^4(x)}\,\tan(x) + \frac{1}{2}\left(\sqrt{a\cos^4(x)}\,\sec^2(x)\right)\int 1\,dx$$
$$= \frac{1}{2}x\sqrt{a\cos^4(x)}\,\sec^2(x) + \frac{1}{2}\sqrt{a\cos^4(x)}\,\tan(x)$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.69

$$\frac{1}{2}\sqrt{a\cos^4(x)}\,\sec^2(x)(x+\cos(x)\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Cos[x]^4],x]
[Out] (Sqrt[a*Cos[x]^4]*Sec[x]^2*(x + Cos[x]*Sin[x]))/2
```

Maple [A]
time = 0.08, size = 22, normalized size = 0.61

method	result	size
default	$\frac{\sqrt{a\left(\cos^4(x)\right)}\,(\cos(x)\sin(x)+x)}{2\cos(x)^2}$	22
risch	$\frac{\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}\,e^{2ix}x}{2(e^{2ix}+1)^2} - \frac{i\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}\,e^{4ix}}{8(e^{2ix}+1)^2} + \frac{i\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}}{8(e^{2ix}+1)^2}$	102

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(a*cos(x)^4)^(1/2)*(cos(x)*sin(x)+x)/cos(x)^2
```

Maxima [A]
time = 0.51, size = 22, normalized size = 0.61

$$\frac{1}{2}\sqrt{a}\,x + \frac{\sqrt{a}\,\tan(x)}{2\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(a)*x + 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

Fricas [**A**]
time = 0.39, size = 21, normalized size = 0.58

$$\frac{\sqrt{a\cos\left(x\right)^{4}}\left(\cos\left(x\right)\sin\left(x\right)+x\right)}{2\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a*cos(x)^4)*(cos(x)*sin(x) + x)/cos(x)^2
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 13, normalized size = 0.36

$$\frac{1}{4}\,\sqrt{a}\left(2\,x+\sin\left(2\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a)*(2*x + sin(2*x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\cos\left(x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x)^4)^(1/2),x)
```

```
[Out] int((a*cos(x)^4)^(1/2), x)
```

3.54 $\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)\sin(x)}{\sqrt{a\cos^4(x)}}$$

```
[Out] cos(x)*sin(x)/(a*cos(x)^4)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3286, 3852, 8}

$$\frac{\sin(x)\cos(x)}{\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Cos[x]^4],x]
```

```
[Out] (Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a\cos^4(x)}}\,dx = \frac{\cos^2(x)\int \sec^2(x)\,dx}{\sqrt{a\cos^4(x)}}$$
$$= -\frac{\cos^2(x)\text{Subst}(\int 1\,dx, x, -\tan(x))}{\sqrt{a\cos^4(x)}}$$
$$= \frac{\cos(x)\sin(x)}{\sqrt{a\cos^4(x)}}$$

Mathematica [A]
time = 0.00, size = 15, normalized size = 1.00

$$\frac{\cos(x)\sin(x)}{\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Cos[x]^4],x]
[Out] (Cos[x]*Sin[x])/Sqrt[a*Cos[x]^4]
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.93

method	result	size
default	$\frac{\cos(x)\sin(x)}{\sqrt{a\left(\cos^4(x)\right)}}$	14
risch	$\frac{2i\left(1+e^{-2ix}\right)}{\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(1/2),x,method=_RETURNVERBOSE)
[Out] cos(x)*sin(x)/(a*cos(x)^4)^(1/2)
```

Maxima [A]
time = 0.50, size = 6, normalized size = 0.40

$$\frac{\tan(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] tan(x)/sqrt(a)
```

Fricas [A]
time = 0.37, size = 18, normalized size = 1.20

$$\frac{\sqrt{a\cos\left(x\right)^{4}}\,\sin\left(x\right)}{a\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*cos(x)^4)*sin(x)/(a*cos(x)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**4)**(1/2),x)
[Out] Timed out
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.40

$$\frac{\tan\left(x\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(1/2),x, algorithm="giac")
[Out] tan(x)/sqrt(a)
```

Mupad [B]
time = 0.23, size = 6, normalized size = 0.40

$$\frac{\tan(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(1/2),x)
[Out] tan(x)/a^(1/2)
```

3.55 $\int \frac{1}{\left(a\cos^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{\cos(x)\sin(x)}{a\sqrt{a\cos^4(x)}}+\frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}$$

```
[Out] cos(x)*sin(x)/a/(a*cos(x)^4)^(1/2)+2/3*sin(x)^2*tan(x)/a/(a*cos(x)^4)^(1/2)
+1/5*sin(x)^2*tan(x)^3/a/(a*cos(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3286, 3852}

$$\frac{\sin(x)\cos(x)}{a\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}+\frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(-3/2),x]
```

```
[Out] (Cos[x]*Sin[x])/(a*Sqrt[a*Cos[x]^4]) + (2*Sin[x]^2*Tan[x])/(3*a*Sqrt[a*Cos[
x]^4]) + (Sin[x]^2*Tan[x]^3)/(5*a*Sqrt[a*Cos[x]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos^4(x))^{3/2}}\,dx = \frac{\cos^2(x)\int \sec^6(x)\,dx}{a\sqrt{a\cos^4(x)}}$$
$$= -\frac{\cos^2(x)\text{Subst}\left(\int (1+2x^2+x^4)\ dx, x, -\tan(x)\right)}{a\sqrt{a\cos^4(x)}}$$
$$= \frac{\cos(x)\sin(x)}{a\sqrt{a\cos^4(x)}} + \frac{2\sin^2(x)\tan(x)}{3a\sqrt{a\cos^4(x)}} + \frac{\sin^2(x)\tan^3(x)}{5a\sqrt{a\cos^4(x)}}$$

Mathematica [A]
time = 0.02, size = 30, normalized size = 0.45

$$\frac{\cos(x)(8+6\cos(2x)+\cos(4x))\sin(x)}{15\,(a\cos^4(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(-3/2),x]
[Out] (Cos[x]*(8 + 6*Cos[2*x] + Cos[4*x])*Sin[x])/(15*(a*Cos[x]^4)^(3/2))
```

Maple [A]
time = 0.09, size = 29, normalized size = 0.43

method	result	size
default	$\frac{\sin(x)\left(8\left(\cos^4(x)\right)+4\left(\cos^2(x)\right)+3\right)\cos(x)}{15\left(a\left(\cos^4(x)\right)\right)^{\frac{3}{2}}}$	29
risch	$\frac{16i(5+11\cos(2x)+9i\sin(2x))}{15a(e^{2ix}+1)^3\sqrt{a\left(e^{2ix}+1\right)^4 e^{-4ix}}}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/15*sin(x)*(8*cos(x)^4+4*cos(x)^2+3)*cos(x)/(a*cos(x)^4)^(3/2)
```

Maxima [A]
time = 0.50, size = 22, normalized size = 0.33

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^(3/2)
```

Fricas [A]
time = 0.37, size = 33, normalized size = 0.49

$$\frac{\sqrt{a\cos(x)^4}\,\left(8\,\cos(x)^4+4\,\cos(x)^2+3\right)\sin(x)}{15\,a^2\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="fricas")
[Out] 1/15*sqrt(a*cos(x)^4)*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**4)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [A]
time = 0.45, size = 22, normalized size = 0.33

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(3/2),x, algorithm="giac")
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^(3/2)
```

Mupad [B]
time = 0.54, size = 36, normalized size = 0.54

$$\frac{4\,\sin(x)}{5\,a^{3/2}\cos(x)^3}+\frac{\sin(x)}{5\,a^{3/2}\cos(x)^5}-\frac{8\sin(x)^3}{15\,a^{3/2}\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(3/2),x)
[Out] (4*sin(x))/(5*a^(3/2)*cos(x)^3) + sin(x)/(5*a^(3/2)*cos(x)^5) - (8*sin(x)^3
)/(15*a^(3/2)*cos(x)^3)
```

3.56 $\int \frac{1}{\left(a\cos^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=117

$$\frac{\cos(x)\sin(x)}{a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}}+\frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^7(x)}{9a^2\sqrt{a\cos^4(x)}}$$

```
[Out] cos(x)*sin(x)/a^2/(a*cos(x)^4)^(1/2)+4/3*sin(x)^2*tan(x)/a^2/(a*cos(x)^4)^(
1/2)+6/5*sin(x)^2*tan(x)^3/a^2/(a*cos(x)^4)^(1/2)+4/7*sin(x)^2*tan(x)^5/a^2
/(a*cos(x)^4)^(1/2)+1/9*sin(x)^2*tan(x)^7/a^2/(a*cos(x)^4)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3286, 3852}

$$\frac{\sin(x)\cos(x)}{a^2\sqrt{a\cos^4(x)}}+\frac{\sin^2(x)\tan^7(x)}{9a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}}+\frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}}+\frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x]^4)^(-5/2),x]
```

```
[Out] (Cos[x]*Sin[x])/(a^2*Sqrt[a*Cos[x]^4]) + (4*Sin[x]^2*Tan[x])/(3*a^2*Sqrt[a*
Cos[x]^4]) + (6*Sin[x]^2*Tan[x]^3)/(5*a^2*Sqrt[a*Cos[x]^4]) + (4*Sin[x]^2*T
an[x]^5)/(7*a^2*Sqrt[a*Cos[x]^4]) + (Sin[x]^2*Tan[x]^7)/(9*a^2*Sqrt[a*Cos[x
]^4])
```

Rule 3286

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p])), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos^4(x))^{5/2}}\,dx = \frac{\cos^2(x)\int \sec^{10}(x)\,dx}{a^2\sqrt{a\cos^4(x)}}$$
$$= -\frac{\cos^2(x)\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx, x, -\tan(x)\right)}{a^2\sqrt{a\cos^4(x)}}$$
$$= \frac{\cos(x)\sin(x)}{a^2\sqrt{a\cos^4(x)}} + \frac{4\sin^2(x)\tan(x)}{3a^2\sqrt{a\cos^4(x)}} + \frac{6\sin^2(x)\tan^3(x)}{5a^2\sqrt{a\cos^4(x)}} + \frac{4\sin^2(x)\tan^5(x)}{7a^2\sqrt{a\cos^4(x)}} + \frac{\sin^2(x)\,\mathrm{t}}{9a^2\sqrt{a\,\mathrm{c}}}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.40

$$\frac{(128+130\cos(2x)+46\cos(4x)+10\cos(6x)+\cos(8x))\sec^6(x)\tan(x)}{315a^2\sqrt{a\cos^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x]^4)^(-5/2),x]
[Out] ((128 + 130*Cos[2*x] + 46*Cos[4*x] + 10*Cos[6*x] + Cos[8*x])*Sec[x]^6*Tan[x
])/(315*a^2*Sqrt[a*Cos[x]^4])
```

Maple [A]
time = 0.17, size = 41, normalized size = 0.35

method	result	size
default	$\frac{\sin(x)\left(128\left(\cos^8(x)\right)+64\left(\cos^6(x)\right)+48\left(\cos^4(x)\right)+40\left(\cos^2(x)\right)+35\right)\cos(x)}{315\left(a\left(\cos^4(x)\right)\right)^{\frac{5}{2}}}$	41
risch	$\frac{256i\left(126\,\mathrm{e}^{6ix}+84\,\mathrm{e}^{4ix}+9+37\cos(2x)+35i\sin(2x)\right)}{315a^2\left(\mathrm{e}^{2ix}+1\right)^7\sqrt{a\left(\mathrm{e}^{2ix}+1\right)^4\mathrm{e}^{-4ix}}}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/315*sin(x)*(128*cos(x)^8+64*cos(x)^6+48*cos(x)^4+40*cos(x)^2+35)*cos(x)/(
a*cos(x)^4)^(5/2)
```

Maxima [A]
time = 0.49, size = 34, normalized size = 0.29

$$\frac{35\tan(x)^9+180\tan(x)^7+378\tan(x)^5+420\tan(x)^3+315\tan(x)}{315\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^(5/2)
```

Fricas [A]
time = 0.39, size = 45, normalized size = 0.38

$$\frac{\left(128 \cos \left(x\right)^{8}+64 \cos \left(x\right)^{6}+48 \cos \left(x\right)^{4}+40 \cos \left(x\right)^{2}+35\right) \sqrt{a \cos \left(x\right)^{4}}\, \sin \left(x\right)}{315\, a^{3} \cos \left(x\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sqrt(a*
cos(x)^4)*sin(x)/(a^3*cos(x)^11)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)**4)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [A]
time = 0.46, size = 34, normalized size = 0.29

$$\frac{35 \tan \left(x\right)^{9}+180 \tan \left(x\right)^{7}+378 \tan \left(x\right)^{5}+420 \tan \left(x\right)^{3}+315 \tan \left(x\right)}{315\, a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^(5/2)
```

Mupad [B]
time = 3.74, size = 306, normalized size = 2.62

$$\frac{{\mathrm{e}}^{x\,4{}\mathrm{i}}\,\sqrt{a\,{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}\,2048{}\mathrm{i}}{5\,a^3\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^5\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}-\frac{{\mathrm{e}}^{x\,4{}\mathrm{i}}\,\sqrt{a\,{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}\,4096{}\mathrm{i}}{3\,a^3\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^6\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}+\frac{{\mathrm{e}}^{x\,4{}\mathrm{i}}\,\sqrt{a\,{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}\,12288{}\mathrm{i}}{7\,a^3\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^7\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}-\frac{{\mathrm{e}}^{x\,4{}\mathrm{i}}\,\sqrt{a\,{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}\,1024{}\mathrm{i}}{a^3\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^8\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}+\frac{{\mathrm{e}}^{x\,4{}\mathrm{i}}\,\sqrt{a\,{\left(\frac{{\mathrm{e}}^{-x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{x\,1{}\mathrm{i}}}{2}\right)}^4}\,2048{}\mathrm{i}}{9\,a^3\,{\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+1\right)}^9\,\left({\mathrm{e}}^{x\,2{}\mathrm{i}}+2\,{\mathrm{e}}^{x\,4{}\mathrm{i}}+{\mathrm{e}}^{x\,6{}\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)^4)^(5/2),x)
```

```
[Out] (exp(x*4i)*(a*(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*2048i)/(5*a^3*(exp(x*2i
) + 1)^5*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) - (exp(x*4i)*(a*(exp(-x*1i)
/2 + exp(x*1i)/2)^4)^(1/2)*4096i)/(3*a^3*(exp(x*2i) + 1)^6*(exp(x*2i) + 2*e
xp(x*4i) + exp(x*6i))) + (exp(x*4i)*(a*(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2
)*12288i)/(7*a^3*(exp(x*2i) + 1)^7*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) -
 (exp(x*4i)*(a*(exp(-x*1i)/2 + exp(x*1i)/2)^4)^(1/2)*1024i)/(a^3*(exp(x*2i)
 + 1)^8*(exp(x*2i) + 2*exp(x*4i) + exp(x*6i))) + (exp(x*4i)*(a*(exp(-x*1i)/
2 + exp(x*1i)/2)^4)^(1/2)*2048i)/(9*a^3*(exp(x*2i) + 1)^9*(exp(x*2i) + 2*ex
p(x*4i) + exp(x*6i)))
```

3.57 $\int (b\cos^m(c+dx))^n\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(c+dx)\,(b\cos^m(c+dx))^n\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+mn)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -cos(d*x+c)*(b*cos(d*x+c)^m)^n*hypergeom([1/2, 1/2*m*n+1/2],[1/2*m*n+3/2],c
os(d*x+c)^2)*sin(d*x+c)/d/(m*n+1)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3287, 2722}

$$-\frac{\sin(c+dx)\cos(c+dx)\,(b\cos^m(c+dx))^n\ {}_2F_1\left(\frac{1}{2},\frac{1}{2}(mn+1);\frac{1}{2}(mn+3);\cos^2(c+dx)\right)}{d(mn+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x]^m)^n,x]
```

```
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x]^m)^n*Hypergeometric2F1[1/2, (1 + m*n)/2, (3
 + m*n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + m*n)*Sqrt[Sin[c + d*x]^2])
)
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (b\cos^m(c+dx))^n \, dx = \left(\cos^{-mn}(c+dx)\,(b\cos^m(c+dx))^n\right)\int \cos^{mn}(c+dx)\,dx$$
$$= -\frac{\cos(c+dx)\,(b\cos^m(c+dx))^n \;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+mn);\frac{1}{2}(3+mn);\cos^2(c+dx)\right)\sin(c}{d(1+mn)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 0.92

$$-\frac{(b\cos^m(c+dx))^n\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+mn),\frac{1}{2}(3+mn),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(1+mn)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x]^m)^n,x]
```

```
[Out] -(((b*Cos[c + d*x]^m)^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + m*n)/2, (3
 + m*n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + m*n)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (b(\cos^m(dx+c)))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c)^m)^n,x)
```

```
[Out] int((b*cos(d*x+c)^m)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)^m)^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c)^m)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)^m)^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^m)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos^{m}\left(c+dx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)**m)**n,x)
```

```
[Out] Integral((b*cos(c + d*x)**m)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c)^m)^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c)^m)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\cos\left(c+d\,x\right)}^m\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x)^m)^n,x)
```

```
[Out] int((b*cos(c + d*x)^m)^n, x)
```

3.58 $\int (c\cos^m(a+bx))^{5/2}\,dx$

Optimal. Leaf size=89

$$-\frac{2c^2\cos^{1+2m}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\cos^2(a+bx)\right)\sin(a+bx)}{b(2+5m)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -2*c^2*cos(b*x+a)^(1+2*m)*hypergeom([1/2, 1/2+5/4*m],[3/2+5/4*m],cos(b*x+a)
^2)*sin(b*x+a)*(c*cos(b*x+a)^m)^(1/2)/b/(2+5*m)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{2c^2\sin(a+bx)\cos^{2m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\cos^2(a+bx)\right)}{b(5m+2)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(5/2),x]
```

```
[Out] (-2*c^2*Cos[a + b*x]^(1 + 2*m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2
, (2 + 5*m)/4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 5*m)*Sqrt
[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (c\cos^m(a+bx))^{5/2}\,dx = \left(c^2\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\right)\int \cos^{\frac{5m}{2}}(a+bx)\,dx$$
$$= -\frac{2c^2\cos^{1+2m}(a+bx)\sqrt{c\cos^m(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\cos^2(a+\right.}{b(2+5m)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.11, size = 74, normalized size = 0.83

$$-\frac{2(c\cos^m(a+bx))^{5/2}\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+5m),\frac{1}{4}(6+5m),\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(2+5m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(5/2),x]
```

```
[Out] (-2*(c*Cos[a + b*x]^m)^(5/2)*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/
4, (6 + 5*m)/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + 5*m))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (c(\cos^m(bx+a)))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(5/2),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (c\cos(a+b\,x)^m)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x)^m)^(5/2),x)
```

```
[Out] int((c*cos(a + b*x)^m)^(5/2), x)
```

3.59 $\int (c\cos^m(a+bx))^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{2c\cos^{1+m}(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\cos^2(a+bx)\right)\sin(a+bx)}{b(2+3m)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -2*c*cos(b*x+a)^(1+m)*hypergeom([1/2, 1/2+3/4*m],[3/2+3/4*m],cos(b*x+a)^2)*
sin(b*x+a)*(c*cos(b*x+a)^m)^(1/2)/b/(2+3*m)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{2c\sin(a+bx)\cos^{m+1}(a+bx)\sqrt{c\cos^m(a+bx)}\;{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\cos^2(a+bx)\right)}{b(3m+2)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(3/2),x]
```

```
[Out] (-2*c*Cos[a + b*x]^(1 + m)*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (2
 + 3*m)/4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + 3*m)*Sqrt[S
in[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (c\cos^m(a+bx))^{3/2}\,dx = \left(c\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\right)\int \cos^{\frac{3m}{2}}(a+bx)\,dx$$
$$= -\frac{2c\cos^{1+m}(a+bx)\sqrt{c\cos^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\cos^2(a+bx)\right)\text{si}}{b(2+3m)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.08, size = 72, normalized size = 0.87

$$-\frac{2(c\cos^m(a+bx))^{3/2}\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2+3m),\frac{3(2+m)}{4},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(2+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(3/2),x]
```

```
[Out] (-2*(c*Cos[a + b*x]^m)^(3/2)*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/
4, (3*(2 + m))/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + 3*m))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (c(\cos^m(bx+a)))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(3/2),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(3/2),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: BINDING-STACK overflow at size
10240. Stack can probably be resized.Proceed with caution.
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\cos^m{(a+bx)})^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*cos(a + b*x)**m)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\cos\left(a+b\,x\right)}^m\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x)^m)^(3/2),x)
```

```
[Out] int((c*cos(a + b*x)^m)^(3/2), x)
```

3.60 $\int \sqrt{c \cos^m(a+bx)}\,dx$

Optimal. Leaf size=74

$$-\frac{2\cos(a+bx)\sqrt{c\cos^m(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\cos^2(a+bx)\right)\sin(a+bx)}{b(2+m)\sqrt{\sin^2(a+bx)}}$$

```
[Out] -2*cos(b*x+a)*hypergeom([1/2, 1/2+1/4*m],[3/2+1/4*m],cos(b*x+a)^2)*sin(b*x+
a)*(c*cos(b*x+a)^m)^(1/2)/b/(2+m)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\cos^m(a+bx)}\;{}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\cos^2(a+bx)\right)}{b(m+2)\sqrt{\sin^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (-2*Cos[a + b*x]*Sqrt[c*Cos[a + b*x]^m]*Hypergeometric2F1[1/2, (2 + m)/4, (
6 + m)/4, Cos[a + b*x]^2]*Sin[a + b*x])/(b*(2 + m)*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{c\cos^m(a+bx)}\,dx = \left(\cos^{-\frac{m}{2}}(a+bx)\sqrt{c\cos^m(a+bx)}\right)\int \cos^{\frac{m}{2}}(a+bx)\,dx$$
$$= -\frac{2\cos(a+bx)\sqrt{c\cos^m(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\cos^2(a+bx)\right)\sin(a+bx)}{b(2+m)\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.04, size = 68, normalized size = 0.92

$$-\frac{2\sqrt{c\cos^m(a+bx)}\,\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{4},\frac{6+m}{4},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (-2*Sqrt[c*Cos[a + b*x]^m]*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (
6 + m)/4, Cos[a + b*x]^2]*Sqrt[Sin[a + b*x]^2])/(b*(2 + m))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(\cos^m(bx+a)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(1/2),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)^m), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c\cos^m{(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(c*cos(a + b*x)**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*cos(b*x + a)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c\cos\left(a+b\,x\right)^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x)^m)^(1/2),x)
```

```
[Out] int((c*cos(a + b*x)^m)^(1/2), x)
```

3.61 $\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\cos(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\cos^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -2*cos(b*x+a)*hypergeom([1/2, 1/2-1/4*m],[3/2-1/4*m],cos(b*x+a)^2)*sin(b*x+
a)/b/(2-m)/(c*cos(b*x+a)^m)^(1/2)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{2\sin(a+bx)\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\cos^2(a+bx)\right)}{b(2-m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (-2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]^
2]*Sin[a + b*x])/(b*(2 - m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx = \frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{m}{2}}(a+bx)\,dx}{\sqrt{c\cos^m(a+bx)}}$$
$$= -\frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\cos^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{c\cos^m(a+bx)}\,\sqrt{\sin^2(a+bx)}}$$

Mathematica [**A**]
time = 0.05, size = 72, normalized size = 0.90

$$\frac{2\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2-m}{4},\frac{6-m}{4},\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{b(-2+m)\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Cos[a + b*x]^m],x]
```

```
[Out] (2*Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Cos[a + b*x]^2
]*Sqrt[Sin[a + b*x]^2])/(b*(-2 + m)*Sqrt[c*Cos[a + b*x]^m])
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\left(\cos^m(bx+a)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a)^m)^(1/2),x)
```

```
[Out] int(1/(c*cos(b*x+a)^m)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)^m), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c\cos^m(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*cos(a + b*x)**m), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*cos(b*x + a)^m), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\cos\left(a+b\,x\right)^m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x)^m)^(1/2),x)
```

```
[Out] int(1/(c*cos(a + b*x)^m)^(1/2), x)
```

3.62 $\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\cos^{1-m}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),\frac{3(2-m)}{4},\cos^2(a+bx)\right)\sin(a+bx)}{bc(2-3m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -2*cos(b*x+a)^(1-m)*hypergeom([1/2, 1/2-3/4*m],[3/2-3/4*m],cos(b*x+a)^2)*si
n(b*x+a)/b/c/(2-3*m)/(c*cos(b*x+a)^m)^(1/2)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{2\sin(a+bx)\cos^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\cos^2(a+bx)\right)}{bc(2-3m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(-3/2),x]
```

```
[Out] (-2*Cos[a + b*x]^(1 - m)*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4,
 Cos[a + b*x]^2]*Sin[a + b*x])/(b*c*(2 - 3*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt[S
in[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(c\cos^m(a+bx))^{3/2}}\,dx = \frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{3m}{2}}(a+bx)\,dx}{c\sqrt{c\cos^m(a+bx)}}$$
$$= -\frac{2\cos^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\cos^2(a+bx)\right)\sin(a+bx)}{bc(2-3m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.07, size = 72, normalized size = 0.81

$$-\frac{\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-3m),-\frac{3}{4}(-2+m),\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{\left(b-\frac{3bm}{2}\right)(c\cos^m(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(-3/2),x]
[Out] -((Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4, Cos[a
+ b*x]^2]*Sqrt[Sin[a + b*x]^2])/((b - (3*b*m)/2)*(c*Cos[a + b*x]^m)^(3/2)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\left(\cos^m(bx+a)\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a)^m)^(3/2),x)
[Out] int(1/(c*cos(b*x+a)^m)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="maxima")
[Out] integrate((c*cos(b*x + a)^m)^(-3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\cos^{m}\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*cos(a + b*x)**m)**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,{\cos\left(a+b\,x\right)}^m\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x)^m)^(3/2),x)
```

```
[Out] int(1/(c*cos(a + b*x)^m)^(3/2), x)
```

3.63 $\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=89

$$-\frac{2\cos^{1-2m}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\cos^2(a+bx)\right)\sin(a+bx)}{bc^2(2-5m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

```
[Out] -2*cos(b*x+a)^(1-2*m)*hypergeom([1/2, 1/2-5/4*m],[3/2-5/4*m],cos(b*x+a)^2)*
sin(b*x+a)/b/c^2/(2-5*m)/(c*cos(b*x+a)^m)^(1/2)/(sin(b*x+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{2\sin(a+bx)\cos^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\cos^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\sin^2(a+bx)}\sqrt{c\cos^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^(-5/2),x]
```

```
[Out] (-2*Cos[a + b*x]^(1 - 2*m)*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4,
 Cos[a + b*x]^2]*Sin[a + b*x])/(b*c^2*(2 - 5*m)*Sqrt[c*Cos[a + b*x]^m]*Sqrt
[Sin[a + b*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{(c\cos^m(a+bx))^{5/2}}\,dx = \frac{\cos^{\frac{m}{2}}(a+bx)\int \cos^{-\frac{5m}{2}}(a+bx)\,dx}{c^2\sqrt{c\cos^m(a+bx)}}$$
$$= -\frac{2\cos^{1-2m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-5m);\frac{1}{4}(6-5m);\cos^2(a+bx)\right)\sin(a+bx)}{bc^2(2-5m)\sqrt{c\cos^m(a+bx)}\sqrt{\sin^2(a+bx)}}$$

Mathematica [A]
time = 0.07, size = 74, normalized size = 0.83

$$-\frac{\cot(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(2-5m),\frac{1}{4}(6-5m),\cos^2(a+bx)\right)\sqrt{\sin^2(a+bx)}}{\left(b-\frac{5bm}{2}\right)(c\cos^m(a+bx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^(-5/2),x]
[Out] -((Cot[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Cos[a + b*
x]^2]*Sqrt[Sin[a + b*x]^2])/((b - (5*b*m)/2)*(c*Cos[a + b*x]^m)^(5/2)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{1}{(c(\cos^m(bx+a)))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(b*x+a)^m)^(5/2),x)
[Out] int(1/(c*cos(b*x+a)^m)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="maxima")
[Out] integrate((c*cos(b*x + a)^m)^(-5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c \cos^m (a + bx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)**m)**(5/2),x)
```

```
[Out] Integral((c*cos(a + b*x)**m)**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*cos(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,{\cos\left(a+b\,x\right)}^m\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*cos(a + b*x)^m)^(5/2),x)
```

```
[Out] int(1/(c*cos(a + b*x)^m)^(5/2), x)
```

3.64 $\int (c \cos^m(a+bx))^{\frac{1}{m}} \, dx$

Optimal. Leaf size=24

$$\frac{(c \cos^m(a+bx))^{\frac{1}{m}} \tan(a+bx)}{b}$$

```
[Out] (c*cos(b*x+a)^m)^(1/m)*tan(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2717}

$$\frac{\tan(a+bx) (c \cos^m(a+bx))^{\frac{1}{m}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[a + b*x]^m)^m^(-1),x]
```

```
[Out] ((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (c \cos^m(a+bx))^{\frac{1}{m}} \, dx &= \left((c \cos^m(a+bx))^{\frac{1}{m}} \sec(a+bx)\right) \int \cos(a+bx) \, dx \\ &= \frac{(c \cos^m(a+bx))^{\frac{1}{m}} \tan(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{(c\cos^m(a+bx))^{\frac{1}{m}}\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[a + b*x]^m)^m^(-1),x]
```

```
[Out] ((c*Cos[a + b*x]^m)^m^(-1)*Tan[a + b*x])/b
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (c(\cos^m(bx+a)))^{\frac{1}{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(b*x+a)^m)^(1/m),x)
```

```
[Out] int((c*cos(b*x+a)^m)^(1/m),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="maxima")
```

```
[Out] integrate((c*cos(b*x + a)^m)^(1/m), x)
```

Fricas [A]
time = 0.37, size = 15, normalized size = 0.62

$$\frac{c^{\left(\frac{1}{m}\right)}\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="fricas")
```

```
[Out] c^(1/m)*sin(b*x + a)/b
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

time = 0.44, size = 61, normalized size = 2.54

$$\begin{cases} x(c\cos^m(a))^{\frac{1}{m}} & \text{for } b = 0 \\ x(0^m c)^{\frac{1}{m}} & \text{for } a = -bx + \frac{\pi}{2} \vee a = -bx + \frac{3\pi}{2} \\ \frac{(c\cos^m(a+bx))^{\frac{1}{m}}\sin(a+bx)}{b\cos(a+bx)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)**m)**(1/m),x)
```

```
[Out] Piecewise((x*(c*cos(a)**m)**(1/m), Eq(b, 0)), (x*(0**m*c)**(1/m), Eq(a, -b*
x + pi/2) | Eq(a, -b*x + 3*pi/2)), ((c*cos(a + b*x)**m)**(1/m)*sin(a + b*x)
/(b*cos(a + b*x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(24) = 48.
time = 1.20, size = 300, normalized size = 12.50

$$\frac{2\left(|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^3-|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)+4|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2-|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^3+|c|^{\left(\frac{1}{m}\right)}\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)\right)}{b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+2b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+b\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^4+b\tan\left(\frac{1}{2}bx+\frac{1}{2}a+\frac{\pi\mathrm{sgn}(c)}{4m}-\frac{\pi}{4m}\right)^2+2b\tan\left(\frac{1}{2}bx+\frac{1}{2}a\right)^2+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(b*x+a)^m)^(1/m),x, algorithm="giac")
```

```
[Out] 2*(abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/2
*b*x + 1/2*a)^3 - abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*
pi/m)^2*tan(1/2*b*x + 1/2*a) + 4*abs(c)^(1/m)*tan(1/2*b*x + 1/2*a + 1/4*pi*
sgn(c)/m - 1/4*pi/m)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/m)*tan(1/2*b*x + 1/
2*a)^3 + abs(c)^(1/m)*tan(1/2*b*x + 1/2*a))/(b*tan(1/2*b*x + 1/2*a + 1/4*pi
*sgn(c)/m - 1/4*pi/m)^2*tan(1/2*b*x + 1/2*a)^4 + 2*b*tan(1/2*b*x + 1/2*a +
1/4*pi*sgn(c)/m - 1/4*pi/m)^2*tan(1/2*b*x + 1/2*a)^2 + b*tan(1/2*b*x + 1/2*
a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/m - 1/4*pi/m)^2 + 2*b*tan(1/2*
b*x + 1/2*a)^2 + b)
```

Mupad [B]
time = 0.40, size = 40, normalized size = 1.67

$$\frac{\sin(2\,a+2\,b\,x)\,\left(c\cos(a+b\,x)^m\right)^{1/m}}{b\,\left(\cos(2\,a+2\,b\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(a + b*x)^m)^(1/m),x)
```

```
[Out] (sin(2*a + 2*b*x)*(c*cos(a + b*x)^m)^(1/m))/(b*(cos(2*a + 2*b*x) + 1))
```

3.65 $\int (a(b\cos(c+dx))^p)^n \, dx$

Optimal. Leaf size=80

$$-\frac{\cos(c+dx)\left(a(b\cos(c+dx))^p\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+np),\frac{1}{2}(3+np),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+np)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -cos(d*x+c)*(a*(b*cos(d*x+c))^p)^n*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/
2],cos(d*x+c)^2)*sin(d*x+c)/d/(n*p+1)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3287, 2722}

$$-\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\cos^2(c+dx)\right)\left(a(b\cos(c+dx))^p\right)^n}{d(np+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Cos[c + d*x])^p)^n,x]
```

```
[Out] -((Cos[c + d*x]*(a*(b*Cos[c + d*x])^p)^n*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + n*p)*Sqrt[Sin[c + d*x]
^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3287

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[b^IntPart[p]*((b*(c*Sin[e + f*x])^n)^FracPart[p]/(c*Sin[e + f*x])^(n*Fr
acPart[p])), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int (a(b\cos(c+dx))^p)^n \, dx = \left((b\cos(c+dx))^{-np}(a(b\cos(c+dx))^p)^n\right) \int (b\cos(c+dx))^{np} \, dx$$
$$= -\frac{\cos(c+dx)\,(a(b\cos(c+dx))^p)^n \, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(1+np); \frac{1}{2}(3+np); \cos^2(c+dx)\right) \sin}{d(1+np)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.92

$$-\frac{(a(b\cos(c+dx))^p)^n \cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2}(1+np), \frac{1}{2}(3+np), \cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(1+np)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Cos[c + d*x])^p)^n,x]
```

```
[Out] -(((a*(b*Cos[c + d*x])^p)^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2
, (3 + n*p)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + n*p)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (a(b\cos(dx+c))^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*cos(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*cos(d*x+c))^p)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*cos(d*x + c))^p*a)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*cos(d*x + c))^p*a)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(b\cos(c+dx))^p)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*cos(c + d*x))**p)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*cos(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*cos(d*x + c))^p*a)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,(b\,\cos(c+d\,x))^p)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*cos(c + d*x))^p)^n,x)
```

```
[Out] int((a*(b*cos(c + d*x))^p)^n, x)
```

3.66 $\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=123

$$\frac{30b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d}+\frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d}+\frac{2(b\cos$$

```
[Out] 18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d
*x+c)/b^4/d+30/77*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+30/7
7*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^4d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^2d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}+\frac{30b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (30*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c + d*
x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(77*b^2*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(
11*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)\sqrt{b\cos(c+dx)}\,dx &= \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^5}\\
&= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^4d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^3}\\
&= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^4d} + 4\\
&= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + 2\\
&= \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^2d} + 2\\
&= \frac{30b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\sin(c+dx)}{77d}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 83, normalized size = 0.67

$$\frac{\sqrt{b\cos(c+dx)}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(5(c+dx)))\right)}{616d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(
290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Sqrt[
Cos[c + d*x]])
```

Maple [A]
time = 0.14, size = 234, normalized size = 1.90

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{77\sqrt{-b\left(2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(448*cos(
1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040
*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 98, normalized size = 0.80

$$\frac{2\left(7\cos\left(dx+c\right)^{4}+9\cos\left(dx+c\right)^{2}+15\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+15i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)}{77\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*sin
(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^5\,\sqrt{b\,\cos\left(c+d\,x\right)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2), x)
```

3.67 $\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=97

$$\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d}$$

```
[Out] 14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*x+
c)/b^3/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^3d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45bd}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]
]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*(b*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(9*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

`[-1, n, 1] && IntegerQ[2*n]`

Rubi steps

$$\int \cos^4(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^4}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^2}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{7}{1}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{(}{}$$
$$= \frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd}$$

Mathematica [A]
time = 0.10, size = 75, normalized size = 0.77

$$\frac{\sqrt{b\cos(c+dx)}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(38\sin(2(c+dx))+5\sin(4(c+dx)))\right)}{180d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(
38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(109) = 218$.
time = 0.09, size = 221, normalized size = 2.28

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(160*cos(
1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos
(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*
cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 101, normalized size = 1.04

$$\frac{2\left(5\cos\left(dx+c\right)^{3}+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{45d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^4\,\sqrt{b\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2), x)
```

3.68 $\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d}$$

```
[Out] 2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^2/d+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$,
Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^2d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*
x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{1}{21}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{(5}{}$$
$$= \frac{10b\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.77

$$\frac{\sqrt{b\cos(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}(23\sin(c+dx)+3\sin(3(c+dx)))\right)}{42d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(2
3*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 208, normalized size = 2.19

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*cos(1
/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/
2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 88, normalized size = 0.93

$$\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,\left(3\,\cos\left(dx+c\right)^{2}+5\right)\sin\left(dx+c\right)-5i\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqrt
(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I
*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2), x)
```

3.69 $\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=69

$$\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

```
[Out] 2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5bd}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^2}$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{3}{5}\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx}}{5\sqrt{\cos(c+dx)}}$$
$$= \frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5bd}$$

Mathematica [A]
time = 0.04, size = 62, normalized size = 0.90

$$\frac{\sqrt{b\cos(c+dx)}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(2(c+dx))\right)}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[b*Cos[c + d*x]]*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*Sin
[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(85) = 170$.
time = 0.08, size = 211, normalized size = 3.06

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-8*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
```

```
-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 88, normalized size = 1.28

$$\frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2), x)
```

3.70 $\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(b*
cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}b\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,d$$
$$= \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

Mathematica [A]
time = 0.04, size = 61, normalized size = 0.91

$$\frac{2(b\cos(c+dx))^{3/2}\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]],x]
[Out] (2*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[c + d*x]))/(3*b*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(83) = 166$.
time = 0.08, size = 188, normalized size = 2.81

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
```

```
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 76, normalized size = 1.13

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos(c+dx)}\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*cos(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)\sqrt{b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/2), x)
```

3.71 $\int \sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]],x]
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 38, normalized size = 1.00

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]],x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(60) = 120$.
time = 0.18, size = 142, normalized size = 3.74

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticE}(}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$
risch	$-\frac{i\sqrt{2}\,\sqrt{b\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\,\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*cos(d*x + c)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 63, normalized size = 1.66

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2),x, algorithm="fricas")

```
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{b\,\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2),x)
```

```
[Out] int((b*cos(c + d*x))^(1/2), x)
```

3.72 $\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=39

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 2721, 2720}

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\sec(c+dx)\,dx = b\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\left(b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 39, normalized size = 1.00

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(61) = 122$.
time = 0.10, size = 142, normalized size = 3.64

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 57, normalized size = 1.46

$$\frac{-i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos(c+dx)}\,\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\sqrt{b\,\cos(c+d\,x)}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x),x)
```

```
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x), x)
```

3.73 $\int \sqrt{b\cos(c+dx)}\,\sec^2(c+dx)\,dx$

Optimal. Leaf size=63

$$-\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/c
os(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
+ (2*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{b\cos(c+dx)}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.76

$$\frac{2b\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2,x]
```

```
[Out] (2*b*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d*S
qrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. 2(83) = 166.
time = 0.10, size = 196, normalized size = 3.11

method	result
default	$-\frac{2b\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2
*b)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
```

```
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 101, normalized size = 1.60

$$\frac{-i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^2,x)
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^2, x)
```

3.74 $\int \sqrt{b\cos(c+dx)}\,\sec^3(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]
]) + (2*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{b\cos(c+dx)}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}b\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\left(b\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&= \frac{2b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 49, normalized size = 0.70

$$\frac{2b\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3,x]
```

```
[Out] (2*b*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(86) = 172.
time = 0.10, size = 239, normalized size = 3.41

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
```

```
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b*(b*(2*cos(1/2*d*x+1/2*c)^
2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 100, normalized size = 1.43

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{3\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\cos(c+dx)}\,\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^3,x)
```

```
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^3, x)
```

3.75 $\int \sqrt{b\cos(c+dx)}\,\sec^4(c+dx)\,dx$

Optimal. Leaf size=95

$$-\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*b*sin(d*x+c)/d/(b*cos(d*x+c))
^(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]
) + (2*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{b\cos(c+dx)}\sec^4(c+dx)\,dx &= b^4\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{1}{5}\left(3b^2\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx \\
&= \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{3}{5}\int \sqrt{b\cos(c+dx)}\,dx \\
&= \frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{}}{5\sqrt{\cos(c+}} \\
&= -\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b\,\mathrm{si}}{5d\sqrt{b\,\mathrm{c}}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 69, normalized size = 0.73

$$\frac{2\sqrt{b\cos(c+dx)}\sec^2(c+dx)\left(-3\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{3}{2}\sin(2(c+dx))+\tan(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^4,x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(-3*Cos[c + d*x]^(3/2)*EllipticE[(c
+ d*x)/2, 2] + (3*Sin[2*(c + d*x)])/2 + Tan[c + d*x]))/(5*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(107) = 214$.
time = 0.13, size = 363, normalized size = 3.82

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*
c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d
*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*
x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 118, normalized size = 1.24

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\left(3\cos(dx+c)^{2}+1\right)\sin(dx+c)}{5d\cos(dx+c)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*x +
c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\cos(c+dx)}\,\sec^{4}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^4,x)
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^4, x)
```

3.76 $\int \sqrt{b\cos(c+dx)}\,\sec^5(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b^2*sin(d*x+c)/d/(b*cos(d*x
+c))^(3/2)+10/21*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5,x]
```

```
[Out] (10*b*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^2*Sin[c +
d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\,\sec^5(c+dx)\,dx = b^5\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(5b^3\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}(5b)\int \frac{1}{\sqrt{b\cos(c+}}$$
$$= \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^2\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(5b\sqrt{\cos(c+dx)}\right)\int}{21\sqrt{b\cos(}}$$
$$= \frac{10b\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^2\,\mathrm{s}}{21d(b\,\mathrm{co}}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.70

$$\frac{\sqrt{b\cos(c+dx)}\,\sec^3(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(10*Cos[c + d*x]^(5/2)*EllipticF[(c +
d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 395 vs. 2(110) = 220.
time = 0.11, size = 396, normalized size = 4.04

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+4
0*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))*b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 112, normalized size = 1.14

$$\frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\sqrt{b}\cos(dx+c)^4 \operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) + 2\sqrt{b\cos(dx+c)}\,(5\cos(dx+c)^2+3)\sin(dx+c)}{21\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5*co
s(d*x + c)^2 + 3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^5,x)
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^5, x)
```

3.77 $\int \sqrt{b\cos(c+dx)}\,\sec^6(c+dx)\,dx$

Optimal. Leaf size=123

$$-\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^3*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/2)+14/15*b*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^6,x]
```

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x
]]) + (2*b^5*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^3*Sin[c + d
*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{b\cos(c+dx)}\sec^6(c+dx)\,dx = b^6\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^4\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}\left(7b^2\right)\int \frac{1}{(b\cos(c+$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} -$$
$$= \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^3\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} -$$
$$= -\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b}{45d(b}$$

Mathematica [A]
time = 0.13, size = 79, normalized size = 0.64

$$\frac{\sqrt{b\cos(c+dx)}\sec^5(c+dx)\left(-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150\sin(c+dx)+91\sin(3(c+dx))+21\sin(5(c+dx))\right)}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^6,x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^5*(-336*Cos[c + d*x]^(9/2)*EllipticE[(c
+ d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x)]
))/(360*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(131) = 262.
time = 0.14, size = 412, normalized size = 3.35

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}{144b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1/144*cos(
1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 128, normalized size = 1.04

$$\frac{-21 i \sqrt{2}\,\sqrt{b}\,\cos\left(dx+c\right)^5 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+21 i \sqrt{2}\,\sqrt{b}\,\cos\left(dx+c\right)^5 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\left(21\cos\left(dx+c\right)^4+7\cos\left(dx+c\right)^2+5\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{45\, d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*co
s(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt(b*c
os(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(b*cos(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^6,x)
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^6, x)
```

3.78 $\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{30b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{30b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d}+\frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd}+\frac{2(b$$

```
[Out] 18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d*x
+c)/b^3/d+30/77*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+30/7
7*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^3d}+\frac{30b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77bd}+\frac{30b\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (30*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c +
d*x]]) + (30*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(77*b*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])
/(11*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(b\cos(c+dx))^{3/2}\,dx &= \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^4} \\
&= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^3d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^2} \\
&= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^3d} + \\
&= \frac{30b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \\
&= \frac{30b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77bd} + \\
&= \frac{30b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30b\sqrt{b\cos(c+dx)}\,\sin(c+d}{77d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 83, normalized size = 0.66

$$\frac{(b\cos(c+dx))^{3/2}\left(240F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(5(c+dx)))\right)}{616d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Cos
[c + d*x]^(3/2))
```

Maple [A]
time = 0.08, size = 236, normalized size = 1.87

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{77\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(448*co
s(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 102, normalized size = 0.81

$$\frac{-15i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(7b\cos(dx+c)^4+9b\cos(dx+c)^2+15b\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{77d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/77*(-15*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 15*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(7*b*cos(d*x + c)^4 + 9*b*cos(d*x + c)^2 + 15*b)*sqrt(
b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^4\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2), x)
```

3.79 $\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=95

$$\frac{14b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d}$$

```
[Out] 14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*x+c)
/b^2/d+14/15*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^2d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}+\frac{14bE\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*
x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*Cos[c + d*x]
)^(7/2)*Sin[c + d*x])/(9*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} +$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^2d} +$$
$$= \frac{14b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+d}{45d}$$

Mathematica [A]
time = 0.06, size = 75, normalized size = 0.79

$$\frac{(b\cos(c+dx))^{3/2}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(38\sin(2(c+dx))+5\sin(4(c+dx)))\right)}{180d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Cos[c + d*x])^(3/2),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(107) = 214.
time = 0.08, size = 223, normalized size = 2.35

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(160*co
s(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 103, normalized size = 1.08

$$\frac{21i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-21i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(5\,b\cos(dx+c)^{3}+7\,b\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(21*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b*co
s(d*x + c)^3 + 7*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^3, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2), x)
```

3.80 $\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd}$$

```
[Out] 2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b/d+10/21*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+10/21*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7bd}+\frac{10b\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^2}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + \frac{5}{7}\int (b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{10b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + $$
$$= \frac{10b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7bd} + $$
$$= \frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b\sqrt{b\cos(c+dx)}\,\sin(c+d}{21d}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 0.74

$$\frac{(b\cos(c+dx))^{3/2}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(23\sin(c+dx)+3\sin(3(c+dx)))\right)}{42d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(3/2),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*
(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.08, size = 210, normalized size = 2.14

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 91, normalized size = 0.93

$$\frac{-5i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(3b\cos(dx+c)^{2}+5b\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{21d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(3*b*cos(d*x + c)^2 + 5*b)*sqrt(b*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,(b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2), x)
```

3.81 $\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=67

$$\frac{6b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}+\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]
]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \cos(c+dx)(b\cos(c+dx))^{3/2}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b}\\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}(3b)\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+d}}{5\sqrt{\cos(c+dx)}}\\
&= \frac{6b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.97

$$
\frac{(b\cos(c+dx))^{5/2}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(2(c+dx))\right)}{5bd\cos^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[2*(c + d*x)]))/(5*b*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(83) = 166$.
time = 0.08, size = 213, normalized size = 3.18

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-8*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
```

```
c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 89, normalized size = 1.33

$$\frac{2\sqrt{b\cos(dx+c)}\,b\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,b^{\frac{3}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*b*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*b^(3/
2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) - 3*I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(3/2), x)
```

3.82 $\int (b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=70

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b*sin(d*x+c)
*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2720}

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{3/2}\,dx &= \frac{2b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d} + \frac{\left(b^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}} + \frac{2b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 0.83

$$
\frac{2(b\cos(c+dx))^{3/2}\left(F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[c + d*x]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(86) = 172$.
time = 0.07, size = 190, normalized size = 2.71

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 77, normalized size = 1.10

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\,b\sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] 1/3*(-I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(b*cos(d*x + c))*b*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(b\cos(c+dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2),x)
[Out] Integral((b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int(b\,\cos(c+d\,x))^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2),x)
[Out] int((b*cos(c + d*x))^(3/2), x)
```

3.83 $\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx$

Optimal. Leaf size=39

$$\frac{2b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 2721, 2719}

$$\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec(c+dx)\,dx = b\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{2b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(61) = 122$.
time = 0.08, size = 144, normalized size = 3.69

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticE}(}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\,d}$
risch	$-\frac{i\sqrt{2}\,b\sqrt{b\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d} - \frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 63, normalized size = 1.62

$$\frac{i\sqrt{2}\,b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,b^{\frac{3}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="fricas")
[Out] (I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(3/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x),x)
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x), x)
```

3.84 $\int (b\cos (c+dx))^{3/2}\sec ^2(c+dx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^2\sqrt{\cos (c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos (c+dx)}}$$

```
[Out] 2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 2721, 2720}

$$\frac{2b^2\sqrt{\cos (c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos (c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^2(c+dx)\,dx = b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\left(b^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [**A**]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.
time = 0.08, size = 144, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{Ellipti}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 57, normalized size = 1.39

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,b^{\frac{3}{2}}\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*b^(3/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^2,x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^2, x)
```

3.85 $\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=66

$$-\frac{2b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (-2*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]
) + (2*b^2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int (b\cos(c+dx))^{3/2}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - b\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 50, normalized size = 0.76

$$\frac{2b^2\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d
*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(86) = 172.
time = 0.09, size = 198, normalized size = 3.00

method	result
default	$-\frac{2b^2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\ \sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^2*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)
^2*b)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
```

```
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 102, normalized size = 1.55

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\sqrt{b\cos\left(dx+c\right)}\,b\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(3/2)*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*sqrt(b*cos(d*x + c))*b*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^3,x)
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^3, x)
```

3.86 $\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^2*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{3/2}\sec^4(c+dx)\,dx &= b^4\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}b^2\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\left(b^2\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&= \frac{2b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 51, normalized size = 0.71

$$\frac{2b^2\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4,x]
[Out] (2*b^2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.
time = 0.09, size = 241, normalized size = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\mathrm{co}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
```

```
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b^2*(b*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 101, normalized size = 1.40

$$\frac{-i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\sqrt{b\cos\left(dx+c\right)}\,b\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*b^(3/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*b^(3/2)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*b*sin(d*x
+ c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^4,x)
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^4, x)
```

3.87 $\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx$

Optimal. Leaf size=98

$$-\frac{6b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*b^2*sin(d*x+c)/d/(b*cos(d*x+c
))^(1/2)-6/5*b*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(si
n(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5,x]
```

```
[Out] (-6*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x
]]) + (2*b^4*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b^2*Sin[c + d*
x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^5(c+dx)\,dx = b^5\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}(3b^3)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}(3b)\int \sqrt{b\cos(c+dx)}$$
$$= \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sqrt{}}{5\sqrt{\cos(c+}}$$
$$= -\frac{6b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^4\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^2\sin}{5d\sqrt{b}}$$

Mathematica [A]
time = 0.07, size = 69, normalized size = 0.70

$$\frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^5,x]
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c
 + d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 363 vs. 2(110) = 220.
time = 0.11, size = 364, normalized size = 3.71

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*
d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 119, normalized size = 1.21

$$\frac{-3i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+3i\sqrt{2}\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(3\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{5\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*b^(3/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(3/2)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*b*cos(d*x + c)^2 + b)*sqrt(b*cos(d*x + c))*sin(d*x
+ c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^5,x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^5, x)
```

3.88 $\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b^3*sin(d*x+c)/d/(b*cos(d*x
+c))^(3/2)+10/21*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6,x]
```

```
[Out] (10*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (2*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^3*Sin[c
+ d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^6(c+dx)\,dx = b^6\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(5b^4\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}\left(5b^2\right)\int \frac{1}{\sqrt{b\cos(c+}}$$
$$= \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(5b^2\sqrt{\cos(c+dx)}\right)\int}{21\sqrt{b\cos(c}}$$
$$= \frac{10b^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^3\,\mathrm{s}}{21d(b\,\mathrm{cc}}$$

Mathematica [A]
time = 0.08, size = 69, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{3/2}\sec^4(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(10*Cos[c + d*x]^(5/2)*EllipticF[(c
+ d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(112) = 224$.
time = 0.10, size = 398, normalized size = 3.98

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\mathrm{si}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+4
0*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))*b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 115, normalized size = 1.15

$$\frac{-5i\sqrt{2}\,b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,b^{\frac{3}{2}}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(5b\cos(dx+c)^2+3b\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(3/2)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b*cos(d*x + c)^2 + 3*b)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^6,x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^6, x)
```

3.89 $\int (b\cos (c+dx))^{3/2}\sec ^7(c+dx)\,dx$

Optimal. Leaf size=126

$$-\frac{14b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^4*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/2)+14/15*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*b*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14bE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^7,x]
```

```
[Out] (-14*b*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d
*x]]) + (2*b^6*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^4*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b^2*Sin[c + d*x])/(15*d*Sqrt[b*C
os[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{3/2}\sec^7(c+dx)\,dx = b^7\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^5\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}\left(7b^3\right)\int \frac{1}{(b\cos(c+d$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} -$$
$$= \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^4\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^2\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}} -$$
$$= -\frac{14b\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^6\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b}{45d(b}$$

Mathematica [A]
time = 0.12, size = 79, normalized size = 0.63

$$\frac{(b\cos(c+dx))^{3/2}\sec^6(c+dx)\left(-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150\sin(c+dx)+91\sin(3(c+dx))+21\sin(5(c+dx))\right)}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^6*(-336*Cos[c + d*x]^(9/2)*EllipticE[(
c + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x
)]))/(360*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(134) = 268$.
time = 0.13, size = 414, normalized size = 3.29

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^2\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}{144b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-1/144*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(
1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^7, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 132, normalized size = 1.05

$$\frac{-21i\sqrt{2}\,b^{\frac{3}{2}}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,b^{\frac{3}{2}}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(21b\cos(dx+c)^4+7b\cos(dx+c)^2+5b\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/45*(-21*I*sqrt(2)*b^(3/2)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*b^(3/2)*co
s(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(21*b*cos(d*x + c)^4 + 7*b*cos(d*x + c)^2 + 5*b)*sq
rt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)*sec(d*x + c)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{3/2}}{\cos(c+d\,x)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^7,x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^7, x)
```

3.90 $\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=125

$$\frac{30b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{30b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d}+\frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d}+\frac{2}{}$$

```
[Out] 18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d*x+c
)/b^2/d+30/77*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+30/77*
b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{30b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^2d}+\frac{30b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{77d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (30*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c +
d*x]]) + (30*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*d) + (18*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(77*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])
/(11*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^3(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^3}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^2d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^2d} + \frac{1}{7}$$
$$= \frac{30b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} +$$
$$= \frac{30b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77d} +$$
$$= \frac{30b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30b^2\sqrt{b\cos(c+dx)}\,\sin(c+da}{77d}$$

Mathematica [A]
time = 0.09, size = 83, normalized size = 0.66

$$\frac{(b\cos(c+dx))^{5/2}\left(240F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,(290\sin(c+dx)+57\sin(3(c+dx))+7\sin(5(c+dx)))\right)}{616d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(b*Cos[c + d*x])^(5/2),x]
[Out] ((b*Cos[c + d*x])^(5/2)*(240*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(290*Sin[c + d*x] + 57*Sin[3*(c + d*x)] + 7*Sin[5*(c + d*x)])))/(616*d*Cos
[c + d*x]^(5/2))
```

Maple [A]
time = 0.08, size = 236, normalized size = 1.89

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^9\left(\frac{dx}{2}+\right.\right.\right.}{77\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(448*co
s(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 108, normalized size = 0.86

$$\frac{-15i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(7b^2\cos(dx+c)^4+9b^2\cos(dx+c)^2+15b^2\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{77d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/77*(-15*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c)) + 15*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c)) + 2*(7*b^2*cos(d*x + c)^4 + 9*b^2*cos(d*x + c)^2 + 15*b^2)
*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2), x)
```

3.91 $\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=98

$$\frac{14b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd}$$

```
[Out] 14/45*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*x+
c)/b/d+14/15*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9bd}+\frac{14b\sin(c+dx)(b\cos(c+dx))^{3/2}}{45d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (14*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c +
d*x]]) + (14*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*(b*Cos[c +
d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^2}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + \frac{7}{9}\int (b\cos(c+dx))^{5/2}\,dx$$
$$= \frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + $$
$$= \frac{14b(b\cos(c+dx))^{3/2}\sin(c+dx)}{45d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9bd} + $$
$$= \frac{14b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{14b(b\cos(c+dx))^{3/2}\sin(c+}{45d}$$

Mathematica [A]
time = 0.07, size = 75, normalized size = 0.77

$$\frac{(b\cos(c+dx))^{5/2}\left(168E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(38\sin(2(c+dx))+5\sin(4(c+dx)))\right)}{180d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(5/2),x]
[Out] ((b*Cos[c + d*x])^(5/2)*(168*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*(38*Sin[2*(c + d*x)] + 5*Sin[4*(c + d*x)])))/(180*d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(110) = 220.
time = 0.08, size = 223, normalized size = 2.28

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(160*co
s(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 107, normalized size = 1.09

$$\frac{21i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 21i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) + 2\left(5b^2\cos(dx+c)^3+7b^2\cos(dx+c)\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{45d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(21*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(5*b^2*
cos(d*x + c)^3 + 7*b^2*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2), x)
```

3.92 $\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=97

$$\frac{10b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

```
[Out] 2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+10/21*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/d/(b*cos(d*x+c))^(1/2)+10/21*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 2715, 2721, 2720}

$$\frac{10b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

[-1, n, 1] && IntegerQ[2*n]

Rubi steps

$$\int \cos(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(5b)\int (b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{10b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{2\ldots}$$
$$= \frac{10b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \ldots$$
$$= \frac{10b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} \ldots$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.78

$$\frac{b^2\sqrt{b\cos(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(23\sin(c+dx)+3\sin(3(c+dx)))\right)}{42d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(5/2),x]
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 210, normalized size = 2.16

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\ldots\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 95, normalized size = 0.98

$$\frac{-5i\,\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\,\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+2\,(3\,b^2\cos(dx+c)^2+5\,b^2)\,\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(3*b^2*cos(d*x + c)^2 + 5*b^2)*sqrt(b*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4848 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\ \left(b\,\cos(c+d\,x)\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(5/2), x)
```

3.93 $\int (b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=70

$$\frac{6b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

```
[Out] 2/5*b*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/d+6/5*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2715, 2721, 2719}

$$\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*
x]]) + (2*b*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\,dx = \frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{1}{5}\left(3b^2\right)\int \sqrt{b\cos(c+dx)}\,dx$$

$$= \frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d} + \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{5\sqrt{\cos(c+dx)}}$$

$$= \frac{6b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b(b\cos(c+dx))^{3/2}\sin(c+dx)}{5d}$$

Mathematica [**A**]
time = 0.01, size = 62, normalized size = 0.89

$$\frac{(b\cos(c+dx))^{5/2}\left(6E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(2(c+dx))\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(6*EllipticE[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*S
in[2*(c + d*x)]))/(5*d*Cos[c + d*x]^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(86) = 172.
time = 0.07, size = 213, normalized size = 3.04

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-8*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 91, normalized size = 1.30

$$\frac{2\sqrt{b\cos\left(dx+c\right)}b^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)+3i\sqrt{2}b^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3i\sqrt{2}b^{\frac{5}{2}}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*b^2*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*b^(
5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin
(d*x + c))) - 3*I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2),x)
[Out] int((b*cos(c + d*x))^(5/2), x)
```

3.94 $\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d
*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*b^2*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{5/2}\sec(c+dx)\,dx &= b\int (b\cos(c+dx))^{3/2}\,dx\\ &= \frac{2b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{1}{3}b^3\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{\left(b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx}}}{3\sqrt{b\cos(c+dx)}}\\ &= \frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3d}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 59, normalized size = 0.82

$$\frac{2b(b\cos(c+dx))^{3/2}\left(F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x],x]
```

```
[Out] (2*b*(b*Cos[c + d*x])^(3/2)*(EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]
*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(88) = 176$.
time = 0.08, size = 190, normalized size = 2.64

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
```

```
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 79, normalized size = 1.10

$$\frac{-i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,b^{\frac{5}{2}}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\,\sqrt{b\cos\left(dx+c\right)}\,b^{2}\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(b*cos(d*x + c))*b^2*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x),x)
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x), x)
```

3.95 $\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 2721, 2719}

$$\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^2(c+dx)\,dx = b^2\int \sqrt{b\cos(c+dx)}\,dx$$
$$= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int\sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{2b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{2b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.
time = 0.09, size = 144, normalized size = 3.51

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{EllipticE}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$
risch	$-\frac{i\sqrt{2}\,b^2\sqrt{b\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{d}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\ \sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 63, normalized size = 1.54

$$\frac{i\sqrt{2}\,b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,b^{\frac{5}{2}}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="fricas")
[Out] (I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*b^(5/2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^2,x)
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^2, x)
```

3.96 $\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx$

Optimal. Leaf size=41

$$\frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 2721, 2720}

$$\frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^3(c+dx)\,dx = b^3\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$
$$= \frac{\left(b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.93

$$\frac{2(b\cos(c+dx))^{5/2}F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*EllipticF[(c + d*x)/2, 2])/(d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.09, size = 144, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^3\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{Elliptic}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 57, normalized size = 1.39

$$\frac{-i\sqrt{2}\,b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\,\sqrt{2}\,b^{\frac{5}{2}}\text{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*b^(5/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^3,x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^3, x)
```

3.97 $\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx$

Optimal. Leaf size=68

$$-\frac{2b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (-2*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x
]]) + (2*b^3*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{5/2}\sec^4(c+dx)\,dx &= b^4\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - b^2\int \sqrt{b\cos(c+dx)}\,dx\\
&= \frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{2b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 50, normalized size = 0.74

$$\frac{2b^3\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^4,x]
```

```
[Out] (2*b^3*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d
*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.
time = 0.10, size = 198, normalized size = 2.91

method	result
default	$-\frac{2b^3\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\,(\mathrm{s}}}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\ \mathrm{s}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -2*b^3*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)
^2*b)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
```

```
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 104, normalized size = 1.53

$$\frac{-i\sqrt{2}\,b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,b^{\frac{5}{2}}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,b^{2}\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*b^(5/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*b^(5/2)*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*sqrt(b*cos(d*x + c))*b^2*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^4,x)
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^4, x)
```

3.98 $\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx$

Optimal. Leaf size=72

$$\frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*b^3*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int (b\cos(c+dx))^{5/2}\sec^5(c+dx)\,dx &= b^5\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}b^3\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\left(b^3\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&= \frac{2b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 51, normalized size = 0.71

$$\frac{2b^3\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5,x]
```

```
[Out] (2*b^3*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. 2(88) = 176.
time = 0.09, size = 241, normalized size = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\cdots\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
```

```
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b^3*(b*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 103, normalized size = 1.43

$$\frac{-i\sqrt{2}\, b^{\frac{5}{2}} \cos\left(dx+c\right)^{2} \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right)+i \sin\left(dx+c\right)\right)+i\sqrt{2}\, b^{\frac{5}{2}} \cos\left(dx+c\right)^{2} \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(dx+c\right)-i \sin\left(dx+c\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\, b^{2} \sin\left(dx+c\right)}{3\, d \cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*b^(5/2)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*b^2*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}{\cos\left(c+d\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^5,x)
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^5, x)
```

3.99 $\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx$

Optimal. Leaf size=100

$$-\frac{6b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}}+\frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*b^5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*b^3*sin(d*x+c)/d/(b*cos(d*x+c
))^(1/2)-6/5*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6,x]
```

```
[Out] (-6*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d
*x]]) + (2*b^5*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*b^3*Sin[c +
d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^6(c+dx)\,dx = b^6\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}\left(3b^4\right)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{1}{5}\left(3b^2\right)\int \sqrt{b\cos(c+d$$
$$= \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b^3\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)}{5\sqrt{\cos(c-}$$
$$= -\frac{6b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2b^5\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6b}{5d\sqrt{}}$$

Mathematica [A]
time = 0.10, size = 69, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(-12\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+7\sin(c+dx)+3\sin(3(c+dx))\right)}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^6,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(-12*Cos[c + d*x]^(5/2)*EllipticE[(c
 + d*x)/2, 2] + 7*Sin[c + d*x] + 3*Sin[3*(c + d*x)]))/(10*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(112) = 224$.
time = 0.12, size = 366, normalized size = 3.66

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^2\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/
2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 123, normalized size = 1.23

$$\frac{-3i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,b^{2}\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{5\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*b^(5/2)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*b^(5/2)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(3*b^2*cos(d*x + c)^2 + b^2)*sqrt(b*cos(d*x + c))*sin(
d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^6,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^6,x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^6, x)
```

3.100 $\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx$

Optimal. Leaf size=100

$$\frac{10b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*b^6*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b^4*sin(d*x+c)/d/(b*cos(d*x
+c))^(3/2)+10/21*b^3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7,x]
```

```
[Out] (10*b^3*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (2*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b^4*Sin[c
+ d*x])/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^7(c+dx)\,dx = b^7\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{1}{7}\left(5b^5\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{1}{21}\left(5b^3\right)\int \frac{1}{\sqrt{b\cos(c}}$$
$$= \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}} + \frac{\left(5b^3\sqrt{\cos(c+dx)}\right)}{21\sqrt{b\cos}}$$
$$= \frac{10b^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2b^6\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10b^4}{21d(b}$$

Mathematica [A]
time = 0.12, size = 69, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{5/2}\sec^5(c+dx)\left(10\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+5\sin(2(c+dx))+6\tan(c+dx)\right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7,x]
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(10*Cos[c + d*x]^(5/2)*EllipticF[(c
+ d*x)/2, 2] + 5*Sin[2*(c + d*x)] + 6*Tan[c + d*x]))/(21*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(112) = 224$.
time = 0.10, size = 398, normalized size = 3.98

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+4
0*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))*b^3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 119, normalized size = 1.19

$$\frac{-5i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+2\left(5\,b^{2}\cos\left(dx+c\right)^{2}+3\,b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{21\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*b^(5/2)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*(5*b^2*cos(d*x + c)^2 + 3*
b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^7,x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^7, x)
```

3.101 $\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx$

Optimal. Leaf size=128

$$-\frac{14b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}}+\frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*b^7*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^5*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/2)+14/15*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*b^2*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14b^2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^8,x]
```

```
[Out] (-14*b^2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c +
 d*x]]) + (2*b^7*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^5*Sin[c
 + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*b^3*Sin[c + d*x])/(15*d*Sqrt[b
*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{5/2}\sec^8(c+dx)\,dx = b^8\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^6\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{1}{15}\left(7b^4\right)\int \frac{1}{(b\cos(c+$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$
$$= \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b^5\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14b^3\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$
$$= -\frac{14b^2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2b^7\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{45d}$$

Mathematica [A]
time = 0.16, size = 79, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{5/2}\sec^7(c+dx)\left(-336\cos^{\frac{9}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+150\sin(c+dx)+91\sin(3(c+dx))+21\sin(5(c+dx))\right)}{360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^8,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^7*(-336*Cos[c + d*x]^(9/2)*EllipticE[(
c + d*x)/2, 2] + 150*Sin[c + d*x] + 91*Sin[3*(c + d*x)] + 21*Sin[5*(c + d*x
)]))/(360*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(136) = 272.
time = 0.13, size = 414, normalized size = 3.23

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, b^3\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\right.\right.\right.}}{144b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-1/144*co
s(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)
/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*
x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(
1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c
)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^8, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 138, normalized size = 1.08

$$\frac{-21i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^5 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))+21i\sqrt{2}\,b^{\frac{5}{2}}\cos\left(dx+c\right)^5 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))+2\left(21b^2\cos\left(dx+c\right)^4+7b^2\cos\left(dx+c\right)^2+5b^2\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{45d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/45*(-21*I*sqrt(2)*b^(5/2)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*b^(5/2)*co
s(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(21*b^2*cos(d*x + c)^4 + 7*b^2*cos(d*x + c)^2 + 5*b
^2)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*sec(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*sec(d*x+c)^8,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)*sec(d*x + c)^8, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^8,x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^8, x)
```

3.102 $\int (b\cos(c+dx))^{7/2}\,dx$

Optimal. Leaf size=98

$$\frac{10b^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^3\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d}$$

```
[Out] 2/7*b*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/d+10/21*b^4*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)/d/(b*cos(d*x+c))^(1/2)+10/21*b^3*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2715, 2721, 2720}

$$\frac{10b^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10b^3\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d}+\frac{2b\sin(c+dx)(b\cos(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (10*b^4*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c +
d*x]]) + (10*b^3*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(b*Cos[c
+ d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{7/2}\,dx = \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{7}(5b^2)\int (b\cos(c+dx))^{3/2}\,dx$$
$$= \frac{10b^3\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{1}{21}(5b^4)\int$$
$$= \frac{10b^3\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b(b\cos(c+dx))^{5/2}\sin(c+dx)}{7d} + \frac{\left(5b^4\sqrt{\cos(\right.}{}$$
$$= \frac{10b^4\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10b^3\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2b(b\cos}{}$$

Mathematica [A]
time = 0.01, size = 76, normalized size = 0.78

$$\frac{b^3\sqrt{b\cos(c+dx)}\left(20F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,(23\sin(c+dx)+3\sin(3(c+dx)))\right)}{42d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(7/2),x]
```

```
[Out] (b^3*Sqrt[b*Cos[c + d*x]]*(20*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]
]*(23*Sin[c + d*x] + 3*Sin[3*(c + d*x)])))/(42*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.07, size = 210, normalized size = 2.14

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,b^4\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^4*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(7/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 95, normalized size = 0.97

$$\frac{-5i\sqrt{2}\,b^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,b^{\frac{7}{2}}\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\left(3\,b^{3}\cos(dx+c)^{2}+5\,b^{3}\right)\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{21\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(7/2),x, algorithm="fricas")
[Out] 1/21*(-5*I*sqrt(2)*b^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(
d*x + c)) + 5*I*sqrt(2)*b^(7/2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I
*sin(d*x + c)) + 2*(3*b^3*cos(d*x + c)^2 + 5*b^3)*sqrt(b*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(7/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(7/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b\,\cos(c+d\,x))^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(7/2),x)
[Out] int((b*cos(c + d*x))^(7/2), x)
```

3.103 $\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=125

$$\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}}+\frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77bd}+\frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d}+\frac{2(b\cos$$

```
[Out] 18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d
*x+c)/b^5/d+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+30/77*
sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^5d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^3d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77bd}+\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*d*Sqrt[b*Cos[c + d*x]
]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b*d) + (18*(b*Cos[c + d*x])
^(5/2)*Sin[c + d*x])/(77*b^3*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(
11*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^6}$$

$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^4}$$

$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} + \frac{45\int (b\cos(c+$$

$$= \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77bd} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx}{11}$$

$$= \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77bd} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2(b\cos(c+dx}{11}$$

$$= \frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77bd} + \frac{18(b\cos(c+}{}$$

Mathematica [A]
time = 0.09, size = 73, normalized size = 0.58

$$\frac{480\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+347\sin(2(c+dx))+64\sin(4(c+dx))+7\sin(6(c+dx))}{1232d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.09, size = 233, normalized size = 1.86

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{77\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(448*cos(1/
2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040*c
os(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+15*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4
-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2
-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 101, normalized size = 0.81

$$\frac{2\left(7\cos\left(dx+c\right)^4+9\cos\left(dx+c\right)^2+15\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-15i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+15i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)}{77\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*sin
(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7319 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^6}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^6/(b*cos(c + d*x))^(1/2), x)
```

3.104 $\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d}$$

```
[Out] 14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*
x+c)/b^4/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^4d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^2d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*
x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*(b*Cos[c +
d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^5}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^3}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{7\int \sqrt{b\cos(c+}}{15b}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{\left(7\sqrt{b\cos(c+}\right.}{15b}$$
$$= \frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2(b\cos(c-}{}$$

Mathematica [A]
time = 0.07, size = 71, normalized size = 0.71

$$\frac{168\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right) + \cos(c+dx)(38\sin(2(c+dx)) + 5\sin(4(c+dx)))}{180d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2*
(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 220, normalized size = 2.20

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4\right.}{45\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*cos(1/
2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos(1
/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*co
s(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/
sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 104, normalized size = 1.04

$$\frac{2\,\left(5\,\cos\left(dx+c\right)^{3}+7\,\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+21i\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-21i\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{45\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^5/(b*cos(c + d*x))^(1/2), x)
```

3.105 $\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd}+\frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d}$$

```
[Out] 2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^3/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.04, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^3d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21bd}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]
]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*(b*Cos[c + d*x])^
(5/2)*Sin[c + d*x])/(7*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^4}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b^2}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{5}{21}\int \frac{1}{\sqrt{b\cos(c}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^3d} + \frac{\left(5\sqrt{\cos(c+dx}\right.}{21\sqrt{}}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21bd} + \frac{2(b\cos(c+}{}$$

Mathematica [A]
time = 0.06, size = 63, normalized size = 0.65

$$\frac{40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+26\sin(2(c+dx))+3\sin(4(c+dx))}{84d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
[Out] (40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3*
Sin[4*(c + d*x)])/(84*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 207, normalized size = 2.13

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-72\left(\right.\right.}{21\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*cos(1/2
*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/2*
d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*sin(
1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(
1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 91, normalized size = 0.94

$$\frac{2\sqrt{b\cos(dx+c)}\,(3\cos(dx+c)^2+5)\sin(dx+c) - 5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + 5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{21\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqrt
(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I
*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
/(b*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^4/(b*cos(c + d*x))^(1/2), x)
```

3.106 $\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}$$

```
[Out] 2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^2/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]
]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^3} \\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b} \\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}} \\
&= \frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 0.81

$$
\frac{6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)\sin(2(c+dx))}{5d\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
[Out] (6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c + d
*x)])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(88) = 176$.
time = 0.08, size = 210, normalized size = 2.92

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right.}{5\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\mathrm{s}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-8*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2
```

```
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2
*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2
*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 91, normalized size = 1.26

$$\frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(b*cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^3/(b*cos(c + d*x))^(1/2), x)
```

3.107 $\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

[-1, n, 1] && IntegerQ[2*n]

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b^2}\\
&= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}+\frac{1}{3}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\
&= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}}\\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3bd}
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 51, normalized size = 0.74

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sin(2(c+dx))}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(85) = 170.
time = 0.08, size = 187, normalized size = 2.71

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{}\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*sin(1/2*d
*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+(s
```

```
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 79, normalized size = 1.14

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{3\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(cos(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.14, size = 58, normalized size = 0.84

$$\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3\,d\,\sqrt{b\,\cos\left(c+d\,x\right)}}+\frac{2\,\sin\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{3\,b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(3*d*(b*cos(c + d*x))^(1
/2)) + (2*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(3*b*d)
```

3.108 $\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 2721, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b}$$
$$= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(63) = 126.
time = 0.09, size = 141, normalized size = 3.44

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticE}\left(\mathrm{co}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\,d}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,e^{-i(dx+c)}}{d\sqrt{b\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(d}\right.}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2
*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 66, normalized size = 1.61

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(1/2),x)
[Out] Integral(cos(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.15, size = 33, normalized size = 0.80

$$\frac{2\sqrt{\cos(c+d\,x)}\;\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\sqrt{b\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*ellipticE(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2
))
```

3.109 $\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2721, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 38, normalized size = 1.00

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 54, normalized size = 1.42

method	result	size
default	$\frac{2\sqrt{2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{am}^{-1}\left(\frac{dx}{2}+\frac{c}{2}\middle|\sqrt{2}\right)}{d\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)*I
nverseJacobiAM(1/2*d*x+1/2*c,2^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*cos(d*x + c)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 60, normalized size = 1.58

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(1/2),x)
[Out] Integral(1/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.19, size = 33, normalized size = 0.87

$$\frac{2\,\sqrt{\cos(c+d\,x)}\;\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d\,\sqrt{b\,\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(1/2),x)
[Out] (2*cos(c + d*x)^(1/2)*ellipticF(c/2 + (d*x)/2, 2))/(d*(b*cos(c + d*x))^(1/2
))
```

3.110 $\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=65

$$-\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/c
os(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]
) + (2*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b}\\ &= \frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.72

$$\frac{2\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(d*Sqr
t[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(85) = 170.
time = 0.08, size = 195, normalized size = 3.00

method	result
default	$-\frac{2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b
)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
```

```
2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 104, normalized size = 1.60

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{bd\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/2)), x)
```

3.111 $\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=67

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*b*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(
b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]])
 + (2*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx \\
&= \frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{1}{3}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx \\
&= \frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3\sqrt{b\cos(c+dx)}} \\
&= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 48, normalized size = 0.72

$$\frac{2\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\tan(c+dx)\right)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*d*Sqrt
[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(83) = 166$.
time = 0.09, size = 238, normalized size = 3.55

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{3\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
```

```
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(b*(2*cos(1/2*d*x+1/2*c)^2-
1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*
c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*
x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 103, normalized size = 1.54

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{3bd\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/2)), x)
```

3.112 $\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=97

$$-\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/d/(b*cos(d*x+c))^(
1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x
]]) + (2*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{1}{5}(3b)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b}\\
&= \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b\sqrt{\cos(c+dx)}}\\
&= -\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 65, normalized size = 0.67

$$\frac{-6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+6\sin(c+dx)+2\sec(c+dx)\tan(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/Sqrt[b*Cos[c + d*x]],x]
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(109) = 218$.
time = 0.12, size = 366, normalized size = 3.77

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*
d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 121, normalized size = 1.25

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\left(3\cos(dx+c)^2+1\right)\sin(dx+c)}{5\,bd\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*x +
c))/(b*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/2)), x)
```

3.113 $\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*b*sin(d*x+c)/d/(b*cos(d*x+c
))^(3/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]
]) + (2*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*b*Sin[c + d*x]
)/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^4\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}\left(5b^2\right)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{5}{21}\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx\\ &= \frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+d}}}{21\sqrt{b\cos(c+dx)}}\\ &= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10b\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.66

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\left(5+3\sec^2(c+dx)\right)\tan(c+dx)}{21d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2)
*Tan[c + d*x])/(21*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(107) = 214$.
time = 0.10, size = 395, normalized size = 4.16

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\right(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+4
0*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(
2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 115, normalized size = 1.21

$$\frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\,(5\cos(dx+c)^2+3)\sin(dx+c)}{21\,bd\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5*co
s(d*x + c)^2 + 3)*sin(d*x + c))/(b*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/sqrt(b*cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^4\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(1/2)), x)
```

3.114 $\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=125

$$-\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*b^4*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b^2*sin(d*x+c)/d/(b*cos(d*x
+c))^(5/2)+14/15*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(
d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d
*x]]) + (2*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b^2*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c
 + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= b^5\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx\\ &= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{1}{9}\left(7b^3\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{1}{15}(7b)\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{7\int \sqrt{b\cos}}{1}\\ &= \frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15d\sqrt{b\cos(c+dx)}}-\frac{\left(7\sqrt{b\cos(}\right.}{}\\ &= -\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}}+\frac{2b^4\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b^2\sin(c+dx}{45d(b\cos(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 77, normalized size = 0.62

$$\frac{-42\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+42\sin(c+dx)+2\sec(c+dx)\left(7+5\sec^2(c+dx)\right)\tan(c+dx)}{45d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Sec
[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 410 vs. 2(133) = 266.
time = 0.13, size = 411, normalized size = 3.29

method	result

default	$-\frac{\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{72b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1/72*cos(1/2*d
*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+
cos(1/2*d*x+1/2*c)^2)^5-7/90*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-28/15*sin(1/2
*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+
1/2*c)^2)^(1/2)+14/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-14/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^
(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 131, normalized size = 1.05

$$\frac{-21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(21\cos(dx+c)^4+7\cos(dx+c)^2+5\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{45\,bd\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*co
s(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt(b*c
os(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
[Out] Integral(sec(c + d*x)**5/sqrt(b*cos(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^5\,\sqrt{b\,\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^5*(b*cos(c + d*x))^(1/2)), x)
```

3.115 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=128

$$\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77bd\sqrt{b\cos(c+dx)}}+\frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^2d}+\frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d}+\frac{2(b\cos$$

```
[Out] 18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d
*x+c)/b^6/d+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+30/7
7*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^6d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^4d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77b^2d}+\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*b*d*Sqrt[b*Cos[c + d*
x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b^2*d) + (18*(b*Cos[c + d
*x])^(5/2)*Sin[c + d*x])/(77*b^4*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x
])/(11*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^7}$$

$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^6d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^5}$$

$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^6d} + \frac{45\int (b\cos(c$$

$$= \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+$$

$$= \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^4d} + \frac{2(b\cos(c+$$

$$= \frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{77bd\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^2d} + \frac{18(b\cos(c$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.59

$$\frac{480\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+347\sin(2(c+dx))+64\sin(4(c+dx))+7\sin(6(c+dx))}{1232bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.09, size = 236, normalized size = 1.84

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{77b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(448*cos(
1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-2040
*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+15
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)
^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)
^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 101, normalized size = 0.79

$$\frac{2\left(7\cos\left(dx+c\right)^4+9\cos\left(dx+c\right)^2+15\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)-15i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+15i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)}{77\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*sin
(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^7}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^7/(b*cos(c + d*x))^(3/2), x)
```

3.116 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d}$$

```
[Out] 14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*
x+c)/b^5/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^5d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^3d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c +
d*x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*(b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^6}$$

$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^4}$$

$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{7\int \sqrt{b\cos(c}}{15b}$$

$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^5d} + \frac{\left(7\sqrt{b\cos(c}\right.}{1}$$

$$= \frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^3d} + \frac{2(b\cos(}{}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.74

$$\frac{168\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + \cos(c+dx)(38\sin(2(c+dx)) + 5\sin(4(c+dx)))}{180bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(b*Cos[c + d*x])^(3/2),x]
[Out] (168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2*
(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.09, size = 223, normalized size = 2.23

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\right.}{45b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(160*cos(
1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*cos
(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*
cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2
)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 104, normalized size = 1.04

$$\frac{2\left(5\cos\left(dx+c\right)^{3}+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+21i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-21i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{45\,b^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^6}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^6/(b*cos(c + d*x))^(3/2), x)
```

3.117 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d}+\frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d}$$

```
[Out] 2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^4/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/b/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*
x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*(b*Cos[c + d*
x])^(5/2)*Sin[c + d*x])/(7*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^5}\\ &= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b^3}\\ &= \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{5\int \frac{1}{\sqrt{b\cos(c}}}{21b}\\ &= \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{\left(5\sqrt{\cos(c+}\right.}{21b}\\ &= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^2d} + \frac{2(b\cos(c}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.66

$$\frac{40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) + 26\sin(2(c+dx)) + 3\sin(4(c+dx))}{84bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(b*Cos[c + d*x])^(3/2),x]
[Out] (40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3*
Sin[4*(c + d*x)])/(84*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.08, size = 210, normalized size = 2.10

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right.}{21b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*cos(1
/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(1/
2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*si
n(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*co
s(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 91, normalized size = 0.91

$$\frac{2\sqrt{b\cos(dx+c)}\,(3\cos(dx+c)^2+5)\sin(dx+c)-5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))}{21\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqrt
(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I
*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
/(b^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^5/(b*cos(c + d*x))^(3/2), x)
```

3.118 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}$$

```
[Out] 2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^3/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^3d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*
x]]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^4}\\ &= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}\\ &= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}+\frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}}\\ &= \frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.85

$$\frac{6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)\sin(2(c+dx))}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c + d
*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.
time = 0.08, size = 213, normalized size = 2.96

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{5b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-8*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
```

```
-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*
(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*
(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 91, normalized size = 1.26

$$\frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4849 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4/(b*cos(c + d*x))^(3/2), x)
```

3.119 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(b*
cos(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b^3}\\ &= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}+\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b}\\ &= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.75

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*b*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(88) = 176.
time = 0.08, size = 190, normalized size = 2.64

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{}\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+
```

```
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)
)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 79, normalized size = 1.10

$$\frac{-i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{3\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3064 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3/(b*cos(c + d*x))^(3/2), x)
```

3.120 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 2721, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^2}$$
$$= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}}$$

Mathematica [**A**]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]
])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.10, size = 144, normalized size = 3.51

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticE}(\ldots}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}\ldots}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,e^{-i(dx+c)}}{db\sqrt{b\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e\ldots\right.}}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/b/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1
/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 66, normalized size = 1.61

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^2}{\left(b\,\cos\left(c+d\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b*cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2/(b*cos(c + d*x))^(3/2), x)
```

3.121 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {16, 2721, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b}$$
$$= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.08, size = 144, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{EllipticF}(}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c
)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/sin(
1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 60, normalized size = 1.46

$$\frac{-i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+i\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/(b^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(cos(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(c+d\,x)}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)/(b*cos(c + d*x))^(3/2), x)
```

3.122 $\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2
/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2721, 2719}

$$\frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x
]]) + (2*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^2}\\ &= \frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^2\sqrt{\cos(c+dx)}}\\ &= -\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

__Mathematica__ [A]
time = 0.01, size = 50, normalized size = 0.74

$$\frac{2\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-3/2),x]
```

```
[Out] (2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(b*d*S
qrt[b*Cos[c + d*x]])
```

__Maple__ [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.
time = 0.07, size = 198, normalized size = 2.91

method	result
default	$-\frac{2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/b*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2
*b)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*
d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2
)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(-3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 104, normalized size = 1.53

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] (-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(3/2),x)
[Out] Integral((b*cos(c + d*x))**(-3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(-3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(3/2),x)
[Out] int(1/(b*cos(c + d*x))^(3/2), x)
```

3.123 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*sin(d*x+c)/d/(b*cos(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(
b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {16, 2716, 2721, 2720}

$$\frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]
]) + (2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\ &= \frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b}\\ &= \frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3d(b\cos(c+dx))^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.02, size = 51, normalized size = 0.74

$$\frac{2\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\tan(c+dx)\right)}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*b*d*Sq
rt[b*Cos[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(85) = 170$.
time = 0.08, size = 241, normalized size = 3.49

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{3b\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/b*(b*(2*cos(1/2*d*x+1/2*c)^
2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/
2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*
d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 103, normalized size = 1.49

$$\frac{-i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right))+i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right))+2\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3b^{2}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b^2*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)\,\left(b\,\cos(c+d\,x)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(3/2)), x)
```

3.124 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=98

$$-\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*b*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/b/d/(b*cos(d*x+c))^(
1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2716, 2721, 2719}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d
*x]]) + (2*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{3}{5}\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx\\ &= \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}-\frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^2}\\ &= \frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}}-\frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+d}}{5b^2\sqrt{\cos(c+dx)}}\\ &= -\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}}+\frac{2b\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5bd\sqrt{b\cos(c+dx}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.69

$$\frac{-6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+6\sin(c+dx)+2\sec(c+dx)\tan(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(110) = 220$.
time = 0.11, size = 366, normalized size = 3.73

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\text{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/
2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 121, normalized size = 1.23

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,(3\cos(dx+c)^2+1)\sin(dx+c)}{5b^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*x +
c))/(b^2*d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(3/2)), x)
```

3.125 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=97

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*sin(d*x+c)/d/(b*cos(d*x+c))
^(3/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2716, 2721, 2720}

$$\frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*
x]]) + (2*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*Sin[c + d*x]
)/(21*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{1}{7}(5b)\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx\\
&= \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{5\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b}\\
&= \frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/2}}+\frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+\ldots}}}{21b\sqrt{b\cos(c+dx)}}\\
&= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21d(b\cos(c+dx))^{3/\ldots}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.68

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(5+3\sec^2(c+dx))\tan(c+dx)}{21bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2)
*Tan[c + d*x])/(21*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(109) = 218$.
time = 0.10, size = 398, normalized size = 4.10

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\ldots\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+4
0*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b
*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)
^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.15, size = 115, normalized size = 1.19

$$\frac{-5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\sqrt{b}\cos(dx+c)^4\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\left(5\cos(dx+c)^2+3\right)\sin(dx+c)}{21\,b^2d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5*co
s(d*x + c)^2 + 3)*sin(d*x + c))/(b^2*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(3/2)), x)
```

3.126 $\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=126

$$-\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}}+\frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*b^3*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*b*sin(d*x+c)/d/(b*cos(d*x+c
))^(5/2)+14/15*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(
d*x+c))^(1/2)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$\frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}}+\frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c +
 d*x]]) + (2*b^3*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*b*Sin[c +
 d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*b*d*Sqrt[b*Cos
[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = b^4\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}\left(7b^2\right)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{7}{15}\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx$$
$$= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}} - \frac{7\int \sqrt{b\,c}}{}$$
$$= \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15bd\sqrt{b\cos(c+dx)}} - \frac{\left(7\sqrt{b\,\mathrm{co}}\right.}{}$$
$$= -\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2b^3\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14b\sin(c+dx}{45d(b\cos(c+dx)}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.63

$$\frac{-42\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+42\sin(c+dx)+2\sec(c+dx)\left(7+5\sec^2(c+dx)\right)\tan(c+dx)}{45bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Sec
[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*b*d*Sqrt[b*Cos[c + d*x]]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. $2(134) = 268$.
time = 0.13, size = 414, normalized size = 3.29

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{144b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1/144*cos(
1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(
-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+
1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/
2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 131, normalized size = 1.04

$$\frac{-21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(21\cos(dx+c)^4+7\cos(dx+c)^2+5\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{45\,b^2d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*co
s(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt(b*c
os(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sec(c + d*x)**4/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(c+d\,x)^4\,(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^4*(b*cos(c + d*x))^(3/2)), x)
```

3.127 $\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=128

$$\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77b^2d\sqrt{b\cos(c+dx)}}+\frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^3d}+\frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d}+\frac{2(b\cos$$

```
[Out] 18/77*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+2/11*(b*cos(d*x+c))^(9/2)*sin(d
*x+c)/b^7/d+30/77*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+30
/77*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{9/2}}{11b^7d}+\frac{18\sin(c+dx)(b\cos(c+dx))^{5/2}}{77b^5d}+\frac{30\sin(c+dx)\sqrt{b\cos(c+dx)}}{77b^3d}+\frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (30*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(77*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (30*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(77*b^3*d) + (18*(b*Cos[c +
 d*x])^(5/2)*Sin[c + d*x])/(77*b^5*d) + (2*(b*Cos[c + d*x])^(9/2)*Sin[c + d
*x])/(11*b^7*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{11/2}\,dx}{b^8}$$
$$= \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^7d} + \frac{9\int (b\cos(c+dx))^{7/2}\,dx}{11b^6}$$
$$= \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^7d} + \frac{45\int (b\cos(c}{77}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+}{}$$
$$= \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^5d} + \frac{2(b\cos(c+}{}$$
$$= \frac{30\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{77b^2d\sqrt{b\cos(c+dx)}} + \frac{30\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{77b^3d} + \frac{18(b\cos(c}{}$$

Mathematica [A]
time = 0.04, size = 76, normalized size = 0.59

$$\frac{480\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+347\sin(2(c+dx))+64\sin(4(c+dx))+7\sin(6(c+dx))}{1232b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(b*Cos[c + d*x])^(5/2),x]
[Out] (480*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 347*Sin[2*(c + d*x)] +
64*Sin[4*(c + d*x)] + 7*Sin[6*(c + d*x)])/(1232*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.09, size = 236, normalized size = 1.84

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(448\left(\cos^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1568\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2384\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{77b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/77*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(448*co
s(1/2*d*x+1/2*c)^13-1568*cos(1/2*d*x+1/2*c)^11+2384*cos(1/2*d*x+1/2*c)^9-20
40*cos(1/2*d*x+1/2*c)^7+1084*cos(1/2*d*x+1/2*c)^5-370*cos(1/2*d*x+1/2*c)^3+
15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+62*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*
c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*
c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^8/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 101, normalized size = 0.79

$$\frac{2\left(7\cos\left(dx+c\right)^4+9\cos\left(dx+c\right)^2+15\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)-15i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+15i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)}{77\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/77*(2*(7*cos(d*x + c)^4 + 9*cos(d*x + c)^2 + 15)*sqrt(b*cos(d*x + c))*sin
(d*x + c) - 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 15*I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^8/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^8}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^8/(b*cos(c + d*x))^(5/2), x)
```

3.128 $\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15b^3d\sqrt{\cos(c+dx)}}+\frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d}+\frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d}$$

```
[Out] 14/45*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+2/9*(b*cos(d*x+c))^(7/2)*sin(d*
x+c)/b^6/d+14/15*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{7/2}}{9b^6d}+\frac{14\sin(c+dx)(b\cos(c+dx))^{3/2}}{45b^4d}+\frac{14E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c +
d*x]]) + (14*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*(b*Cos[c
+ d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{9/2}\,dx}{b^7}$$
$$= \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{7\int (b\cos(c+dx))^{5/2}\,dx}{9b^5}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{7\int \sqrt{b\cos(\ldots}}{15\ldots}$$
$$= \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{\left(7\sqrt{b\cos(c\ldots}\right.}{1\ldots}$$
$$= \frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{14(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2(b\cos(\ldots}{\ldots}$$

Mathematica [A]
time = 0.04, size = 74, normalized size = 0.74

$$\frac{168\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)(38\sin(2(c+dx))+5\sin(4(c+dx)))}{180b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(b*Cos[c + d*x])^(5/2),x]
[Out] (168*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*(38*Sin[2*
(c + d*x)] + 5*Sin[4*(c + d*x)]))/(180*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.09, size = 223, normalized size = 2.23

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160\left(\cos^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-480\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+616\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\ldots\right.}{45b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin\ldots\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(160*co
s(1/2*d*x+1/2*c)^11-480*cos(1/2*d*x+1/2*c)^9+616*cos(1/2*d*x+1/2*c)^7-432*c
os(1/2*d*x+1/2*c)^5+160*cos(1/2*d*x+1/2*c)^3-21*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2
4*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1
/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 104, normalized size = 1.04

$$\frac{2\left(5\cos\left(dx+c\right)^{3}+7\cos\left(dx+c\right)\right)\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)+21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)))-21i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)))}{45\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/45*(2*(5*cos(d*x + c)^3 + 7*cos(d*x + c))*sqrt(b*cos(d*x + c))*sin(d*x +
c) + 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^7/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^7}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^7/(b*cos(c + d*x))^(5/2), x)
```

3.129 $\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=100

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d}+\frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d}$$

```
[Out] 2/7*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/b^5/d+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
/b^2/d/(b*cos(d*x+c))^(1/2)+10/21*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}+\frac{10\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d}+\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (10*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(b*Cos[c +
d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int (b\cos(c+dx))^{7/2}\,dx}{b^6}$$
$$= \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{5\int (b\cos(c+dx))^{3/2}\,dx}{7b^4}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{5\int \frac{1}{\sqrt{b\cos(c}}}{21b^{}}$$
$$= \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{\left(5\sqrt{\cos(c+}\right.}{21b^{}}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{10\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{21b^3d} + \frac{2(b\cos(c-}{}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.66

$$\frac{40\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right) + 26\sin(2(c+dx)) + 3\sin(4(c+dx))}{84b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (40*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 26*Sin[2*(c + d*x)] + 3*
Sin[4*(c + d*x)])/(84*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.09, size = 210, normalized size = 2.10

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(48\left(\cos^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-120\left(\cos^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+128\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7\right.}{21b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*cos
(1/2*d*x+1/2*c)^9-120*cos(1/2*d*x+1/2*c)^7+128*cos(1/2*d*x+1/2*c)^5-72*cos(
1/2*d*x+1/2*c)^3+5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*cos(1/2*d*x+1/2*c))/(-b*(2*
sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*
cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.13, size = 91, normalized size = 0.91

$$\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,\left(3\,\cos\left(dx+c\right)^{2}+5\right)\sin\left(dx+c\right)-5i\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5i\,\sqrt{2}\,\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))}{21\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 5)*sin(d*x + c) - 5*I*sqrt
(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*I
*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))
/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^6/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^6}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^6/(b*cos(c + d*x))^(5/2), x)
```

3.130 $\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}$$

```
[Out] 2/5*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/b^4/d+6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/
2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2719}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^4d}+\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*
x]]) + (2*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int (b\cos(c+dx))^{5/2}\,dx}{b^5} \\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4 d} + \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^3} \\
&= \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4 d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{5b^3\sqrt{\cos(c+dx)}} \\
&= \frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3 d\sqrt{\cos(c+dx)}} + \frac{2(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4 d}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 61, normalized size = 0.85

$$
\frac{6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\cos(c+dx)\sin(2(c+dx))}{5b^2 d\sqrt{b\cos(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]*Sin[2*(c + d
*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.
time = 0.09, size = 213, normalized size = 2.96

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-8\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+8\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{5b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-8*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+8*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*
```

```
c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-
b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(
b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 91, normalized size = 1.26

$$\frac{2\sqrt{b\cos(dx+c)}\cos(dx+c)\sin(dx+c)+3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{5b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(2*sqrt(b*cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*I*sqrt(2)*sqrt(b)
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^5}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5/(b*cos(c + d*x))^(5/2), x)
```

3.131 $\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d}$$

```
[Out] 2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)+2/3*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2715, 2721, 2720}

$$\frac{2\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}+\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\int (b\cos(c+dx))^{3/2}\,dx}{b^4}\\ &= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d}+\frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}\\ &= \frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d}+\frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{3b^3d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.75

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])/(3*b^2*
d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(88) = 176.
time = 0.08, size = 190, normalized size = 2.64

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(4\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)+\sqrt{}\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*sin(1
/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c
```

```
)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^
2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 79, normalized size = 1.10

$$\frac{-i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\sqrt{b}\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^4}{\left(b\,\cos\left(c+d\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4/(b*cos(c + d*x))^(5/2), x)
```

3.132 $\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^3d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 2721, 2719}

$$\frac{2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^3}$$
$$= \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}$$
$$= \frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^3d\sqrt{\cos(c+dx)}}$$

Mathematica [**A**]
time = 0.01, size = 41, normalized size = 1.00

$$\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]
])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.09, size = 144, normalized size = 3.51

method	result
default	$\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticE}\Big(}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\,\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$
risch	$-\frac{i\left(e^{2i(dx+c)}+1\right)\sqrt{2}\,e^{-i(dx+c)}}{d\,b^2\sqrt{b\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}-\frac{i\left(-\frac{2\left(b\,e^{2i(dx+c)}+b\right)}{b\sqrt{e^{i(dx+c)}\left(b\,e^{2i(dx+c)}+b\right)}}+\frac{i\sqrt{-i\left(e^{i(dx+c)}+i\right)}\,\sqrt{2}\,\sqrt{i\,(}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))/b^2/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin
(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 66, normalized size = 1.61

$$\frac{i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
[Out] (I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - I*sqrt(2)*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 5989 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\cos\left(c+d\,x\right)}^3}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^3/(b*cos(c + d*x))^(5/2), x)
```

3.133 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=41

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {16, 2721, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x]
])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{b^2}$$
$$= \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 0.93

$$\frac{2\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2])/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(63) = 126.
time = 0.09, size = 144, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{EllipticF}(}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2
*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))/si
n(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 60, normalized size = 1.46

$$\frac{-i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\sin{(dx+c)})+i\sqrt{2}\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\sin{(dx+c)})}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
)) + I*sqrt(2)*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c)))/(b^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos{(c+d\,x)}^2}{{(b\,\cos{(c+d\,x)})}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2/(b*cos(c + d*x))^(5/2), x)
```

3.134 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-2*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b
^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {16, 2716, 2721, 2719}

$$\frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{2E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-2*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x
]]) + (2*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
```

```
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{b}$$
$$= \frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\int \sqrt{b\cos(c+dx)}\,dx}{b^3}$$
$$= \frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{b\cos(c+dx)}\int \sqrt{\cos(c+dx)}\,dx}{b^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{2\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 0.74

$$\frac{2\left(-\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
[Out] (2*(-(Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + Sin[c + d*x]))/(b^2*d
*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(88) = 176$.
time = 0.08, size = 198, normalized size = 2.91

method	result
default	$-\frac{2\left(-2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b+\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}\,\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}}{b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}\ \mathrm{s}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/b^2*(-2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)
^2*b)^(1/2)*sin(1/2*d*x+1/2*c)^2+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
```

```
x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/
2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1
/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 104, normalized size = 1.53

$$\frac{-i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\sqrt{b}\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\sin(dx+c)}{b^3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*sqrt(b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*sqrt(b)*cos(d*x + c)*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) + 2*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b*cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)/(b*cos(c + d*x))^(5/2), x)
```

3.135 $\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=72

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/3*sin(d*x+c)/b/d/(b*cos(d*x+c))^(3/2)+2/3*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2
/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2716, 2721, 2720}

$$\frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*
x]]) + (2*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx = \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{3b^2}$$
$$= \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}} + \frac{\sqrt{\cos(c+dx)}\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{3b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2\sin(c+dx)}{3bd(b\cos(c+dx))^{3/2}}$$

Mathematica [A]
time = 0.01, size = 51, normalized size = 0.71

$$\frac{2\left(\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\tan(c+dx)\right)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-5/2),x]
```

```
[Out] (2*(Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Tan[c + d*x]))/(3*b^2*d*
Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(88) = 176$.
time = 0.07, size = 241, normalized size = 3.35

method	result
default	$-\frac{2\left(-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\ \mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{3b^2\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}\right.\right.\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2-2*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/b^2*(b*(2*cos(1/2*d*x+1/2*c
)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+
1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/
2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 103, normalized size = 1.43

$$\frac{-i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,b^{3}d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*sqrt(b)*cos(d*x + c)^2*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*sin(d*x +
c))/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(c+dx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**(-5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(5/2),x)
[Out] int(1/(b*cos(c + d*x))^(5/2), x)
```

3.136 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=97

$$-\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*sin(d*x+c)/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(
1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {16, 2716, 2721, 2719}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/(5
*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b}$$
$$= \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^3}$$
$$= \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+}}{5b^3\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^2d\sqrt{b\cos(c+d}}$$

Mathematica [A]
time = 0.02, size = 68, normalized size = 0.70

$$\frac{-6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 6\sin(c+dx) + 2\sec(c+dx)\tan(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(5/2),x]
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(109) = 218.
time = 0.11, size = 366, normalized size = 3.77

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/
2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 121, normalized size = 1.25

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,(3\cos(dx+c)^2+1)\sin(dx+c)}{5b^3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*x +
c))/(b^3*d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(5/2)), x)
```

3.137 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=98

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}+\frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}$$

```
[Out] 2/7*b*sin(d*x+c)/d/(b*cos(d*x+c))^(7/2)+10/21*sin(d*x+c)/b/d/(b*cos(d*x+c))
^(3/2)+10/21*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2720}

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}}+\frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}}+\frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c +
d*x]]) + (2*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (10*Sin[c + d*x]
)/(21*b*d*(b*Cos[c + d*x])^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b^2\int \frac{1}{(b\cos(c+dx))^{9/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{5}{7}\int \frac{1}{(b\cos(c+dx))^{5/2}}\,dx$$
$$= \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{5\int \frac{1}{\sqrt{b\cos(c+dx)}}\,dx}{21b^2}$$
$$= \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{\left(5\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+}}}{21b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2b\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}} + \frac{10\sin(c+dx)}{21bd(b\cos(c+dx))^{3}}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.67

$$\frac{10\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right) + 2(5+3\sec^2(c+dx))\tan(c+dx)}{21b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (10*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5 + 3*Sec[c + d*x]^2)
*Tan[c + d*x])/(21*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(110) = 220$.
time = 0.09, size = 398, normalized size = 4.06

method	result
default	$-\frac{2\left(-40\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/21*(-40*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^6-40*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^6+60*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+4
0*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-30*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1
/2*d*x+1/2*c)^2-16*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))/b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(
-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*
c)^2-1)^3/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.12, size = 115, normalized size = 1.17

$$\frac{-5i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5i\sqrt{2}\sqrt{b}\cos\left(dx+c\right)^{4}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+2\sqrt{b\cos\left(dx+c\right)}\left(5\cos\left(dx+c\right)^{2}+3\right)\sin\left(dx+c\right)}{21\,b^{3}d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/21*(-5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*sqrt(b)*cos(d*x + c)^4*weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 2*sqrt(b*cos(d*x + c))*(5*co
s(d*x + c)^2 + 3)*sin(d*x + c))/(b^3*d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(5/2)), x)
```

3.138 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=125

$$-\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}+\frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/9*b^2*sin(d*x+c)/d/(b*cos(d*x+c))^(9/2)+14/45*sin(d*x+c)/d/(b*cos(d*x+c))
^(5/2)+14/15*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)-14/15*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*(b*cos(
d*x+c))^(1/2)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {16, 2716, 2721, 2719}

$$-\frac{14E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}}+\frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}}+\frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}}+\frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-14*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c +
 d*x]]) + (2*b^2*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (14*Sin[c + d
*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (14*Sin[c + d*x])/(15*b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = b^3\int \frac{1}{(b\cos(c+dx))^{11/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{1}{9}(7b)\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{7\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{15b}$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}} - \frac{7\int\sqrt{b}}{}$$
$$= \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx)}{45d(b\cos(c+dx))^{5/2}} + \frac{14\sin(c+dx)}{15b^2d\sqrt{b\cos(c+dx)}} - \frac{\left(7\sqrt{bc}\right.}{}$$
$$= -\frac{14\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2b^2\sin(c+dx)}{9d(b\cos(c+dx))^{9/2}} + \frac{14\sin(c+dx}{45d(b\cos(c+dx}$$

Mathematica [A]
time = 0.03, size = 80, normalized size = 0.64

$$\frac{-42\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 42\sin(c+dx) + 2\sec(c+dx)\left(7+5\sec^2(c+dx)\right)\tan(c+dx)}{45b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (-42*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 42*Sin[c + d*x] + 2*Sec
[c + d*x]*(7 + 5*Sec[c + d*x]^2)*Tan[c + d*x])/(45*b^2*d*Sqrt[b*Cos[c + d*x
]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(133) = 266.
time = 0.13, size = 414, normalized size = 3.31

method	result

default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-b\left(2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}}{144b\left(-\frac{1}{2}+\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-1/144*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^5-7/180*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(-1/2+cos(1/2*d*x+1/2*c)^2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 131, normalized size = 1.05

$$\frac{-21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\sqrt{b}\cos(dx+c)^5\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(21\cos(dx+c)^4+7\cos(dx+c)^2+5\right)\sqrt{b\cos(dx+c)}\sin(dx+c)}{45\,b^3d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] 1/45*(-21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*I*sqrt(2)*sqrt(b)*cos(d*x + c)^5*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(21*cos(d*x + c)^4 + 7*cos(d*x + c)^2 + 5)*sqrt(b*cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^5)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3006 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(5/2)), x)
```

3.139 $\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=100

$$-\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}+\frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

```
[Out] 2/5*sin(d*x+c)/b/d/(b*cos(d*x+c))^(5/2)+6/5*sin(d*x+c)/b^3/d/(b*cos(d*x+c))
^(1/2)-6/5*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))*(b*cos(d*x+c))^(1/2)/b^4/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2716, 2721, 2719}

$$-\frac{6E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}}+\frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}+\frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sqrt[Cos[c + d
*x]]) + (2*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/2)) + (6*Sin[c + d*x])/
(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2721

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n, Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && LtQ
[-1, n, 1] && IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{7/2}}\,dx = \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{3\int \frac{1}{(b\cos(c+dx))^{3/2}}\,dx}{5b^2}$$
$$= \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{3\int \sqrt{b\cos(c+dx)}\,dx}{5b^4}$$
$$= \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sqrt{\cos(c+}}{5b^4\sqrt{\cos(c+dx)}}$$
$$= -\frac{6\sqrt{b\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{6\sin(c+dx)}{5b^3d\sqrt{b\cos(c+}}$$

Mathematica [A]
time = 0.01, size = 68, normalized size = 0.68

$$\frac{-6\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right) + 6\sin(c+dx) + 2\sec(c+dx)\tan(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-7/2),x]
```

```
[Out] (-6*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*Sin[c + d*x] + 2*Sec[c
 + d*x]*Tan[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(112) = 224$.
time = 0.10, size = 366, normalized size = 3.66

method	result
default	$-\frac{2\sqrt{b\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
```

```
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4*b+sin(1/
2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-7/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 121, normalized size = 1.21

$$\frac{-3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\sqrt{b}\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\sqrt{b\cos(dx+c)}\,(3\cos(dx+c)^2+1)\sin(dx+c)}{5b^4d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] 1/5*(-3*I*sqrt(2)*sqrt(b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*sqrt(b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 1)*sin(d*x +
c))/(b^4*d*cos(d*x + c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(-7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\,\cos\left(c+d\,x\right)\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(7/2),x)
```

```
[Out] int(1/(b*cos(c + d*x))^(7/2), x)
```

3.140 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d}+\frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d}$$

```
[Out] 1/4*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*x*(b*cos(d*x+c))
^(1/2)/cos(d*x+c)^(1/2)+3/8*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2
)/d
```

Rubi [A]
time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (3*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \text{co}}{4\sqrt{\cos(c+dx}}$$

$$= \frac{3\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} + \frac{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c}}{4d}$$

$$= \frac{3x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d} +$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.56

$$\frac{\sqrt{b\cos(c+dx)}\,(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[b*Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]
))/(32*d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.81, size = 62, normalized size = 0.63

method	result
default	$\frac{\sqrt{b\cos(dx+c)}\,(2\sin(dx+c)(\cos^3(dx+c))+3\sin(dx+c)\cos(dx+c)+3dx+3c)}{8d\sqrt{\cos(dx+c)}}$
risch	$\frac{3\sqrt{b\cos(dx+c)}\,(\sqrt{\cos}\,(dx+c))\mathrm{e}^{i(dx+c)}x}{4(\mathrm{e}^{2i(dx+c)}+1)} - \frac{i\sqrt{b\cos(dx+c)}\,(\sqrt{\cos}\,(dx+c))\mathrm{e}^{5i(dx+c)}}{32(\mathrm{e}^{2i(dx+c)}+1)d} + \frac{i\sqrt{b\cos(dx+}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(b*cos(d*x+c))^(1/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+
c)+3*d*x+3*c)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.60, size = 49, normalized size = 0.50

$$\frac{\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A]
time = 0.43, size = 176, normalized size = 1.80

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,\left(2\,\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{16\,d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,\left(2\,\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) + 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*(sqrt(b*cos(d*x + c))*(2*
cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arctan(sqrt
(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0(3*sqrt(sageVAR
```

Mupad [B]
time = 1.26, size = 75, normalized size = 0.77

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3*
d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) + 1
))
```

3.141 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*sin(d*x+c)^3*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\,\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 45, normalized size = 0.64

$$\frac{\sqrt{b\cos(c+dx)}\,(5+\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[Cos[c
+ d*x]])
```

Maple [A]
time = 0.16, size = 40, normalized size = 0.57

method	result
default	$\frac{(\cos^2(dx+c)+2)\sin(dx+c)\sqrt{b\cos(dx+c)}}{3d\sqrt{\cos(dx+c)}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{4i(dx+c)}}{12(e^{2i(dx+c)}+1)d}-\frac{3i\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}}{4(e^{2i(dx+c)}+1)d}+\frac{3i\sqrt{b\cos(d}}{4(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.59, size = 42, normalized size = 0.60

$$\frac{\sqrt{b}\left(\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))/d
```

Fricas [A]
time = 0.38, size = 39, normalized size = 0.56

$$\frac{\sqrt{b\cos(dx+c)}\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{3\,d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(d*sqrt(cos(d*x
+ c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 71047 vs. 2(60) = 120.
time = 6.75, size = 71047, normalized size = 1014.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] -1/96*(3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^3*tan(1/3*c)^6*tan(c) - 24*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^4*tan(c)^2 - 18*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c)^2 + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^6*tan(c)^2 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 48*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^6 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 72*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^4*tan(c) - 72*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
```

```
1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 24*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*
tan(1/3*c)^6*tan(c) + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c) - 72*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c) + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) - 72*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
*tan(c) + 24*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2
- 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 +
27*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan
(c)^2 + 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2
 + 108*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2
+ 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 54*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^4 + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 27*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4
+ 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^4 - 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 54*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 - 108*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 48*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*t
an(1/3*c)^6 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2...
```

Mupad [B]
time = 0.74, size = 57, normalized size = 0.81

$$\frac{\sqrt{\cos(c+d\,x)}\ \sqrt{b\,\cos(c+d\,x)}\ (10\,\sin(2\,c+2\,d\,x)+\sin(4\,c+4\,d\,x))}{12\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c +
 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.142 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*sin(d*x+c)*cos(d*x+c)^(1/2)
*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\sqrt{b\cos(c+dx)}\int 1\,dx}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.71

$$\frac{\sqrt{b\cos(c+dx)}\,(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[b*Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[Cos[c + d
*x]])
```

Maple [A]
time = 0.15, size = 42, normalized size = 0.67

method	result
default	$\frac{\sqrt{b\cos(dx+c)}\,(\sin(dx+c)\cos(dx+c)+dx+c)}{2d\sqrt{\cos(dx+c)}}$
risch	$\frac{\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{i(dx+c)}x}{e^{2i(dx+c)}+1} - \frac{i\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{3i(dx+c)}}{4\left(e^{2i(dx+c)}+1\right)d} + \frac{i\sqrt{b\cos(dx+c}}{4(e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(sin(d*x+c)*cos(d*x+c)+d*x+c)/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.58, size = 25, normalized size = 0.40

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [**A**]
time = 0.43, size = 150, normalized size = 2.38

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)*log
(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)
^(3/2))))/d]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(56) = 112$.
time = 41.86, size = 121, normalized size = 1.92

$$\begin{cases} 0 & \text{for}\ c = -dx + \frac{\pi}{2} \vee c = -dx + \frac{3\pi}{2} \\ x\sqrt{b\cos\left(c\right)}\,\cos^{\frac{3}{2}}\left(c\right) & \text{for}\ d = 0 \\ \frac{x\sqrt{b\cos\left(c+dx\right)}\,\sin^{2}\left(c+dx\right)}{2\sqrt{\cos\left(c+dx\right)}} + \frac{x\sqrt{b\cos\left(c+dx\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)}{2} + \frac{\sqrt{b\cos\left(c+dx\right)}\,\sin\left(c+dx\right)\sqrt{\cos\left(c+dx\right)}}{2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((0, Eq(c, -d*x + pi/2) | Eq(c, -d*x + 3*pi/2)), (x*sqrt(b*cos(c))
*cos(c)**(3/2), Eq(d, 0)), (x*sqrt(b*cos(c + d*x))*sin(c + d*x)**2/(2*sqrt(
cos(c + d*x))) + x*sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)/2 + sqrt(b*cos(
c + d*x))*sin(c + d*x)*sqrt(cos(c + d*x))/(2*d), True))
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0(s
qrt(sageVARb)*sageVARd*sageVARx*tan((sageVARc+sageVARd*sageVARx)/2)^4+2*sqr
t(sageVARb)*sage
```

Mupad [B]
time = 0.69, size = 62, normalized size = 0.98

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x)
 + 4*d*x*cos(c + d*x)))/(4*d*(cos(2*c + 2*d*x) + 1))
```

3.143 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\sin(dx+c)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$	29
risch	$-\frac{i\sqrt{b\cos\left(dx+c\right)}\,\left(\sqrt{\cos}\,(dx+c)\right)\mathrm{e}^{2i(dx+c)}}{\left(\mathrm{e}^{2i(dx+c)}+1\right)d}+\frac{i\sqrt{b\cos\left(dx+c\right)}\,\left(\sqrt{\cos}\,(dx+c)\right)}{\left(\mathrm{e}^{2i(dx+c)}+1\right)d}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Maxima [A]
time = 0.59, size = 13, normalized size = 0.41

$$\frac{\sqrt{b}\,\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(b)*sin(d*x + c)/d
```

Fricas [A]
time = 0.46, size = 28, normalized size = 0.88

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

time = 1.83, size = 60, normalized size = 1.88

$$\begin{cases} x\sqrt{b\cos(c)}\,\sqrt{\cos(c)} & \text{for } d=0 \\ 0 & \text{for } c=-dx+\frac{\pi}{2} \vee c=-dx+\frac{3\pi}{2} \\ \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(b*cos(c))*sqrt(cos(c)), Eq(d, 0)), (0, Eq(c, -d*x + pi/2)
 | Eq(c, -d*x + 3*pi/2)), (sqrt(b*cos(c + d*x))*sin(c + d*x)/(d*sqrt(cos(c
+ d*x))), True))
```

Giac [A]
time = 0.74, size = 31, normalized size = 0.97

$$\frac{2\sqrt{b}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b)*tan(1/2*d*x + 1/2*c)/(d*tan(1/2*d*x + 1/2*c)^2 + d)
```

Mupad [B]
time = 0.39, size = 44, normalized size = 1.38

$$\frac{\sqrt{\cos(c+d\,x)}\,\sin(2\,c+2\,d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(d*(cos(2*c +
2*d*x) + 1))
```

3.144 $\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{17, 8\}$

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int 1\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Maple [A]
time = 0.10, size = 28, normalized size = 1.17

method	result	size
risch	$\frac{x\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}$	21
default	$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)*(d*x+c)
```

Maxima [A]
time = 0.53, size = 26, normalized size = 1.08

$$\frac{2\sqrt{b}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A]
time = 0.46, size = 94, normalized size = 3.92

$$\left[\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{2\,d},\ \frac{\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt
(cos(d*x + c))*sin(d*x + c) - b)/d, sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin
(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

Sympy [A]
time = 0.87, size = 22, normalized size = 0.92

$$\frac{x\sqrt{b\cos\left(c+dx\right)}}{\sqrt{\cos\left(c+dx\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
[Out] x*sqrt(b*cos(c + d*x))/sqrt(cos(c + d*x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.10, size = 20, normalized size = 0.83

$$\frac{x\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(1/2),x)
[Out] (x*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2)
```

3.145 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Maple [A]
time = 0.14, size = 42, normalized size = 1.27

method	result	size
default	$-\frac{2\,\mathrm{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$	42
risch	$-\frac{\sqrt{b\cos\left(dx+c\right)}\,\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}+\frac{\sqrt{b\cos\left(dx+c\right)}\,\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{\sqrt{\cos\left(dx+c\right)}\,d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*arctanh((-1+cos(d*x+c))/sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.
time = 0.60, size = 65, normalized size = 1.97

$$\frac{\sqrt{b}\left(\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

Fricas [A]
time = 0.44, size = 113, normalized size = 3.42

$$\left[\frac{\sqrt{b}\,\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2\,d},-\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(b)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b)*
arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))/d
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(3/2), x)
```

3.146 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\sqrt{b\cos(c+dx)}\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(5/2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.14, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\cos(dx+c)^{\frac{3}{2}}}$	29
risch	$\frac{2i\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.57, size = 54, normalized size = 1.69

$$\frac{2\sqrt{b}\,\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 0.39, size = 28, normalized size = 0.88

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{b\cos\left(c+dx\right)}}{\cos^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(b*cos(c + d*x))/cos(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 0.70, size = 59, normalized size = 1.84

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(d*c
os(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.147 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*arctanh(sin(d*x+
c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 3853, 3855}

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (Sq
rt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\,\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+dx)}\,\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.72

$$\frac{\sqrt{b\cos(c+dx)}\,\left(\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)\right)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])
)/(2*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.19, size = 104, normalized size = 1.44

method	result
default	$-\frac{\left(\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)\right)\sqrt{b\cos(dx+c)}}{2d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\sqrt{b\cos(dx+c)}\,\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}-\frac{\sqrt{b\cos(dx+c)}\,\ln\left(e^{i(dx+c)}-i\right.}{2\sqrt{\cos(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(60) = 120.
time = 0.59, size = 661, normalized size = 9.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*
x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.42, size = 201, normalized size = 2.79

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4d\cos\left(dx+c\right)^{3}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c
)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\,\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(7/2), x)
```

3.148 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*sin(d*x+c)^3*(b*cos(
d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 3852}

$$\frac{\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{9}{2}}(c+dx)}\,dx &= \frac{\sqrt{b\cos(c+dx)}\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= -\frac{\sqrt{b\cos(c+dx)}\,\mathrm{Subst}\left(\int (1+x^2)\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\ &= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.64

$$\frac{\sqrt{b\cos(c+dx)}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(9/2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[Cos[c + d*
x]])
```

Maple [A]
time = 0.15, size = 42, normalized size = 0.60

method	result	size
default	$\frac{\left(2\left(\cos^2(dx+c)\right)+1\right)\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{3d\cos(dx+c)^{\frac{7}{2}}}$	42
risch	$\frac{4i\sqrt{b\cos\left(dx+c\right)}\,\left(3\,\mathrm{e}^{2i(dx+c)}+1\right)}{3\sqrt{\cos\left(dx+c\right)}\,d\left(\mathrm{e}^{2i(dx+c)}+1\right)^3}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(60) = 120.
time = 0.61, size = 294, normalized size = 4.20

$$\frac{4\left((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\sin(4dx+4c)-3\cos(6dx+6c)\sin(2dx+2c)-9\cos(4dx+4c)\sin(2dx+2c)\right)\sqrt{b}}{3\left(2(3\cos(4dx+4c)+3\cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3\cos(2dx+2c)+1)\cos(4dx+4c)+9\cos(4dx+4c)^2+9\cos(2dx+2c)^2+6(\sin(4dx+4c)+\sin(2dx+2c))\sin(6dx+6c)+\sin(6dx+6c)^2+9\sin(4dx+4c)^2+18\sin(4dx+4c)\sin(2dx+2c)+9\sin(2dx+2c)^2+6\cos(2dx+2c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) +
1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x +
4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*
x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 +
6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.36, size = 41, normalized size = 0.59

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(d*cos(d*x + c
)^(7/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

Mupad [**B**]
time = 1.86, size = 128, normalized size = 1.83

$$\frac{2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6\,\sin\left(4\,c+4\,d\,x\right)+\sin\left(6\,c+6\,d\,x\right)+10{}\mathrm{i}\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] (2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos
(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*d*
x) + 10i))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d*x
) + cos(6*c + 6*d*x) + 10))
```

3.149 $\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/4*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+3/8*sin(d*x+c)*(b*co
s(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+3/8*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(
1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 3853, 3855}

$$\frac{3\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(11/2),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\sqrt{b\cos(c+dx)}\int \sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(3\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$
$$= \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(3\sqrt{b\cos(c+dx)}\right.}{8\sqrt{\cos(c}}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3\sqrt{b\cos(c}}{8d\,c}$$

Mathematica [A]
time = 0.07, size = 66, normalized size = 0.62

$$\frac{\sqrt{b\cos(c+dx)}\left(3\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2+3\cos^2(c+dx))\sin(c+dx)\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]/Cos[c + d*x]^(11/2),x]
[Out] (Sqrt[b*Cos[c + d*x]]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[
c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.23, size = 121, normalized size = 1.13

method	result
default	$\frac{\left(3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx+\right.}{8d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{i\sqrt{b\cos(dx+c)}\,(3\,e^{7i(dx+c)}+11\,e^{5i(dx+c)}-11\,e^{3i(dx+c)}-3\,e^{i(dx+c)})}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}-\frac{3\sqrt{b\cos(dx+c)}\,\ln(e^{i(dx+c)}-i)}{8\sqrt{\cos(dx+c)}\,d}+\frac{3\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(89) = 178$.
time = 0.67, size = 1656, normalized size = 15.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos
(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x
+ 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2
 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d
*x + 2*c) + 1)*d)
```

Fricas [A]
time = 0.43, size = 227, normalized size = 2.12

$$\left[\frac{3 \sqrt{b} \cos\left(dx+c\right)^{5} \log\left(-\frac{b \cos\left(dx+c\right)^{3} - 2 \sqrt{b \cos\left(dx+c\right)} \sqrt{b} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) - 2 b \cos\left(dx+c\right)}{\cos\left(dx+c\right)^{3}}\right) + 2 \sqrt{b \cos\left(dx+c\right)} \left(3 \cos\left(dx+c\right)^{2} + 2\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{16 d \cos\left(dx+c\right)^{5}}, -\frac{3 \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos\left(dx+c\right)} \sqrt{-b} \sin\left(dx+c\right)}{b \sqrt{\cos\left(dx+c\right)}}\right) \cos\left(dx+c\right)^{5} - \sqrt{b \cos\left(dx+c\right)} \left(3 \cos\left(dx+c\right)^{2} + 2\right) \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{8 d \cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x + c)
)*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b*cos
(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\cos(c+dx)}}{\cos(c+dx)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(11/2),x)
[Out] int((b*cos(c + d*x))^(1/2)/cos(c + d*x)^(11/2), x)
```

3.150 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=101

$$\frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{3b\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{b\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

```
[Out] 1/4*b*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b*x*(b*cos(d*x
+c))^(1/2)/cos(d*x+c)^(1/2)+3/8*b*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(d*x+c)
)^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*b*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*b*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*Cos[c + d*x]^(5/2)*Sqrt[b*
Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int}{4\sqrt{\cos(c+}}$$
$$= \frac{3b\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d} + \frac{b\cos^{\frac{5}{2}}(c+dx)\sqrt{b\,c}}{}$$
$$= \frac{3bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3b\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.54

$$\frac{(b\cos(c+dx))^{3/2}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(32*d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.16, size = 62, normalized size = 0.61

method	result
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}(2\sin(dx+c)(\cos^3(dx+c))+3\sin(dx+c)\cos(dx+c)+3dx+3c)}{8d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{3b\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{i(dx+c)}x}{4(e^{2i(dx+c)}+1)} - \frac{ib\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{5i(dx+c)}}{32(e^{2i(dx+c)}+1)d} + \frac{ib\sqrt{b\cos(dx}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(b*cos(d*x+c))^(3/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+
c)+3*d*x+3*c)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.58, size = 53, normalized size = 0.52

$$\frac{\left(12\,(dx+c)b+b\sin\left(4\,dx+4\,c\right)+8\,b\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A]
time = 0.40, size = 183, normalized size = 1.81

$$\left[\frac{2\,\left(2\,b\cos\left(dx+c\right)^{2}+3\,b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\sqrt{-b}\,b\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{16\,d},\ \frac{\left(2\,b\cos\left(dx+c\right)^{2}+3\,b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(2*b*cos(d*x + c)^2 + 3*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) + 3*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c)
)*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((2*b*cos(d*x + c)^
2 + 3*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*b^(3/2)*a
rctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0sageVARb*(3*sqr
```

Mupad [B]
time = 1.05, size = 76, normalized size = 0.75

$$\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c +
3*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x) +
 1))
```

3.151 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b*sin(d*x+c)^3*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2713}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}\,dx&=\frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\&=-\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\&=\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 45, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{3/2}(5+\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Cos[c + d
*x]^(3/2))
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.56

method	result
default	$\frac{(\cos^2(dx+c)+2)\sin(dx+c)(b\cos(dx+c))^{\frac{3}{2}}}{3d\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{ib\sqrt{b\cos\left(dx+c\right)}\left(\sqrt{\cos}\left(dx+c\right)\right)e^{4i(dx+c)}}{12\left(e^{2i(dx+c)}+1\right)d}-\frac{3ib\sqrt{b\cos\left(dx+c\right)}\left(\sqrt{\cos}\left(dx+c\right)\right)e^{2i(dx+c)}}{4\left(e^{2i(dx+c)}+1\right)d}+\frac{3ib\sqrt{b\cos}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.58, size = 45, normalized size = 0.62

$$\frac{\left(b\sin\left(3\,dx+3\,c\right)+9\,b\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
+ 3*c))))*sqrt(b)/d
```

Fricas [A]
time = 0.36, size = 43, normalized size = 0.60

$$\frac{\left(b\cos\left(dx+c\right)^{2}+2\,b\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(
d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71048 vs. $2(62) = 124$.
time = 6.60, size = 71048, normalized size = 986.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] -1/96*(3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*sqrt
(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^3*tan(1/3*c)^6*tan(c) - 24*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*sqrt(b)*d*x^
4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^4*tan(c)^2 - 18*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c)^2 + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*sqrt(b)*d*x^4*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^6*tan(c)^2 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*
c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 48*sqrt(
b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^6 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*sqrt(b)*d*x^4*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 72*
sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^4*tan(c) - 72*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
```

1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 24*sqrt(b)
*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*
tan(1/3*c)^6*tan(c) + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c) - 72*sqrt(b)*d*x^4*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c) + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) - 72*sqrt(b)*d*x^4*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
*tan(c) + 24*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2
- 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c
)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 +
27*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan
(c)^2 + 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2
 + 108*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2
*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2
+ 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*s
qrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 54*sq
rt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^4 + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 27*sqrt(b)*d*x^4*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4
+ 54*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^4 - 3*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 54*sqrt(b)*d*x^4*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 - 108*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 144*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 48*sqrt(b)*
d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*t
an(1/3*c)^6 - 9*sqrt(b)*d*x^4*tan(1/2*d*x + 1/2...

Mupad [B]
time = 0.70, size = 58, normalized size = 0.81

$$\frac{b\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2),x)
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c
 + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.152 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\,dx$

Optimal. Leaf size=65

$$\frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/2*b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b*sin(d*x+c)*cos(d*x+c)^(
1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (b*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*Sqrt[Cos[c + d*x]]*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{bx\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.69

$$\frac{(b\cos(c+dx))^{3/2}(2(c+dx)+\sin(2(c+dx)))}{4d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x]
^(3/2))
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.65

method	result
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}(\sin(dx+c)\cos(dx+c)+dx+c)}{2d\cos(dx+c)^{\frac{3}{2}}}$
risch	$\frac{b\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)e^{i(dx+c)}x}{e^{2i(dx+c)}+1} - \frac{ib\sqrt{b\cos(dx+c)}\,\left(\sqrt{\cos}\,(dx+c)\right)e^{3i(dx+c)}}{4(e^{2i(dx+c)}+1)d} + \frac{ib\sqrt{b\cos(dx+}}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(sin(d*x+c)*cos(d*x+c)+d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.61, size = 28, normalized size = 0.43

$$\frac{(2\,(dx+c)b+b\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A]
time = 0.42, size = 153, normalized size = 2.35

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,b\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{-b}\,b\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,b\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)*b
*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c)
)*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(
d*x + c) + b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2))))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(c+dx\right)\right)^{\frac{3}{2}}\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0sa
geVARb*(sqrt(sageVARb)*sageVARd*sageVARx*tan((sageVARc+sageVARd*sageVARx)/2
)^4+2*sqrt(sageV
```

Mupad [B]
time = 0.53, size = 63, normalized size = 0.97

$$\frac{b\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (b*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*
x) + 4*d*x*cos(c + d*x)))/(4*d*(cos(2*c + 2*d*x) + 1))
```

3.153 $\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=33

$$\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\sqrt{\cos(c+dx)}}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.97

$$\frac{(b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Sqrt[Cos[c + d*x]],x]
[Out] ((b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.88

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{d\cos(dx+c)^{\frac{3}{2}}}$	29
risch	$\frac{b\sin(dx+c)\sqrt{b\cos(dx+c)}}{d\sqrt{\cos(dx+c)}}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(3/2)
```

Maxima [A]
time = 0.57, size = 13, normalized size = 0.39

$$\frac{b^{\frac{3}{2}}\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
[Out] b^(3/2)*sin(d*x + c)/d
```

Fricas [A]
time = 0.38, size = 29, normalized size = 0.88

$$\frac{\sqrt{b\cos(dx+c)}\,b\sin(dx+c)}{d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [A]
time = 20.25, size = 46, normalized size = 1.39

$$\begin{cases} \frac{(b\cos(c+dx))^{\frac{3}{2}}\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} & \text{for } d \neq 0 \\ \frac{x(b\cos(c))^{\frac{3}{2}}}{\sqrt{\cos(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Piecewise(((b*cos(c + d*x))**(3/2)*sin(c + d*x)/(d*cos(c + d*x)**(3/2)), Ne
(d, 0)), (x*(b*cos(c))**(3/2)/sqrt(cos(c)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.24, size = 29, normalized size = 0.88

$$\frac{b\sin(c+d\,x)\,\sqrt{b\cos(c+d\,x)}}{d\,\sqrt{\cos(c+d\,x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (b*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(d*cos(c + d*x)^(1/2))
```

3.154 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] b*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{17, 8\}$

$$\frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{bx\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.96

$$\frac{x(b\cos(c+dx))^{3/2}}{\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (x*(b*Cos[c + d*x])^(3/2))/Cos[c + d*x]^(3/2)
```

Maple [A]
time = 0.09, size = 28, normalized size = 1.12

method	result	size
risch	$\frac{bx\sqrt{b\cos\left(dx+c\right)}}{\sqrt{\cos\left(dx+c\right)}}$	22
default	$\frac{\left(b\cos(dx+c)\right)^{\frac{3}{2}}\left(dx+c\right)}{d\cos(dx+c)^{\frac{3}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)*(d*x+c)
```

Maxima [A]
time = 0.51, size = 26, normalized size = 1.04

$$\frac{2\,b^{\frac{3}{2}}\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A]
time = 0.43, size = 95, normalized size = 3.80

$$\left[\frac{\sqrt{-b}\,b\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{2\,d},\ \frac{b^{\frac{3}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sq
rt(cos(d*x + c))*sin(d*x + c) - b)/d, b^(3/2)*arctan(sqrt(b*cos(d*x + c))*s
in(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

Sympy [A]
time = 12.93, size = 22, normalized size = 0.88

$$\frac{x(b\cos{(c+dx)})^{\frac{3}{2}}}{\cos^{\frac{3}{2}}{(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] x*(b*cos(c + d*x))**(3/2)/cos(c + d*x)**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 0.09, size = 21, normalized size = 0.84

$$\frac{b\,x\,\sqrt{b\,\cos{(c+d\,x)}}}{\sqrt{\cos{(c+d\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (b*x*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2)
```

3.155 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{b\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.97

$$\frac{\tanh^{-1}(\sin(c+dx))(b\cos(c+dx))^{3/2}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Cos[c + d*x])^(3/2))/(d*Cos[c + d*x]^(3/2))
```

Maple [A]
time = 0.11, size = 42, normalized size = 1.24

method	result	size
default	$-\frac{2(b\cos(dx+c))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)}{d\cos(dx+c)^{\frac{3}{2}}}$	42
risch	$\frac{b\sqrt{b\cos(dx+c)}\,\ln(e^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{b\cos(dx+c)}\,\ln(e^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(b*cos(d*x+c))^(3/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))/cos(d*x+c)^(3
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(30) = 60$.
time = 0.63, size = 68, normalized size = 2.00

$$\frac{\left(b\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-b\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(co
s(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A]
time = 0.41, size = 114, normalized size = 3.35

$$\left[\frac{b^{\frac{3}{2}}\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,d},\ -\frac{\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(3/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b)*
b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/d]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3004 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2),x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(5/2), x)
```

3.156 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=33

$$\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{17, 3852, 8\}$

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.97

$$\frac{(b\cos(c+dx))^{3/2}\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(7/2),x]
[Out] ((b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.88

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{d\cos(dx+c)^{\frac{5}{2}}}$	29
risch	$\frac{2ib\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.57, size = 54, normalized size = 1.64

$$\frac{2\,b^{\frac{3}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 2*b^(3/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 0.36, size = 29, normalized size = 0.88

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,b\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 0.50, size = 60, normalized size = 1.82

$$\frac{b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] (b*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(d
*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.157 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b*arctanh(sin(
d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 3853, 3855}

$$\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (
b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$

$$= \frac{b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [A]
time = 0.03, size = 52, normalized size = 0.70

$$\frac{(b\cos(c+dx))^{3/2}\left(\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)\right)}{2d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x
]))/(2*d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.15, size = 104, normalized size = 1.41

method	result
default	$-\frac{\left(\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)\right)(b\cos(dx+c))^{\frac{3}{2}}}{2d\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}+\frac{b\sqrt{b\cos(dx+c)}\,\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}-\frac{b\sqrt{b\cos(dx+c)}\,\ln\left(e^{i(dx+c)}\right.}{2\sqrt{\cos(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(b*cos(d*x+c))^(3/
2)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 691 vs. 2(62) = 124.
time = 0.59, size = 691, normalized size = 9.34

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c
))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*
c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*c
os(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (
b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d
*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*co
s(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*s
qrt(b)/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 0.47, size = 204, normalized size = 2.76

$$\left[\frac{b^{\frac{3}{2}} \cos\left(dx+c\right)^{3} \log\left(-\frac{b \cos(dx+c)^{3}-2 \sqrt{b \cos\left(dx+c\right)} \sqrt{b} \sqrt{\cos\left(dx+c\right)} \sin(dx+c)-2 b \cos(dx+c)}{\cos(dx+c)^{3}}\right) + 2 \sqrt{b \cos\left(dx+c\right)} b \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{4 d \cos\left(dx+c\right)^{3}}, -\frac{\sqrt{-b} b \arctan\left(\frac{\sqrt{b \cos\left(dx+c\right)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos\left(dx+c\right)}}\right) \cos\left(dx+c\right)^{3} - \sqrt{b \cos\left(dx+c\right)} b \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right)}{2 d \cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(3/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
 c)^3), -1/2*(sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*b*sqrt(cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{3/2}}{\cos\left(c+d\,x\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(9/2), x)
```

3.158 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=72

$$\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b*sin(d*x+c)^3*(b*
cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3852}

$$\frac{b\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\left(b\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int (1+x^2)\,dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.62

$$\frac{(b\cos(c+dx))^{3/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(11/2),x]
[Out] ((b*Cos[c + d*x])^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Cos[c + d*x]^
(3/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.58

method	result	size
default	$\frac{\left(2\left(\cos^2(dx+c)\right)+1\right)(b\cos(dx+c))^{\frac{3}{2}}\sin(dx+c)}{3d\cos(dx+c)^{\frac{9}{2}}}$	42
risch	$\frac{4ib\sqrt{b\cos\left(dx+c\right)}\left(3\,e^{2i(dx+c)}+1\right)}{3\sqrt{\cos\left(dx+c\right)}\,d\left(e^{2i(dx+c)}+1\right)^3}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(9/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. 2(62) = 124.
time = 0.59, size = 299, normalized size = 4.15

$$-\frac{4\left(3\,b\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)-\left(3\,b\cos\left(2\,dx+2\,c\right)+b\right)\sin\left(6\,dx+6\,c\right)-3\left(3\,b\cos\left(2\,dx+2\,c\right)+b\right)\sin\left(4\,dx+4\,c\right)\right)\sqrt{b}}{3\left(2\left(3\cos\left(4\,dx+4\,c\right)+3\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^2+6\left(3\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\cos\left(4\,dx+4\,c\right)^2+9\cos\left(2\,dx+2\,c\right)^2+6\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^2+9\sin\left(4\,dx+4\,c\right)^2+18\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\sin\left(2\,dx+2\,c\right)^2+6\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*x + 4*c)*sin(2*d*
x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3*(3*b*cos(2*d*x +
 2*c) + b)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin
(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9
*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x +
2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.41, size = 42, normalized size = 0.58

$$\frac{\left(2\,b\cos\left(dx+c\right)^{2}+b\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*cos(d*x + c)^2 + b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c)^(7/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2), x)
```

Mupad [**B**]
time = 1.14, size = 129, normalized size = 1.79

$$\frac{2\,b\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6\,\sin\left(4\,c+4\,d\,x\right)+\sin\left(6\,c+6\,d\,x\right)+10{}\mathrm{i}\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(11/2),x)
```

```
[Out] (2*b*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + c
os(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*
d*x) + 10i))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d
*x) + cos(6*c + 6*d*x) + 10))
```

3.159 $\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{3b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/4*b*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+3/8*b*sin(d*x+c)*(
b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+3/8*b*arctanh(sin(d*x+c))*(b*cos(d*x
+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.03, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{3b\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (3*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) +
 (b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*b*Sqrt
[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b\sqrt{b\cos(c+dx)}\right)\int \sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(3b\sqrt{b\cos(c+\right.}{8\sqrt{c}}$$

$$= \frac{3b\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b\sqrt{b\,c}}{}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 0.61

$$\frac{b\sqrt{b\cos(c+dx)}\,\left(3\tanh^{-1}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(c+dx)\right)}{8d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Co
s[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))
```

Maple [A]
time = 0.20, size = 121, normalized size = 1.10

method	result
default	$\frac{\left(3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx+\right.}{8d\cos(dx+c)^{\frac{11}{2}}}$
risch	$-\frac{ib\sqrt{b\cos(dx+c)}\,(3\,e^{7i(dx+c)}+11\,e^{5i(dx+c)}-11\,e^{3i(dx+c)}-3\,e^{i(dx+c)})}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4} - \frac{3b\sqrt{b\cos(dx+c)}\,\ln(e^{i(dx+c)}-i)}{8\sqrt{\cos(dx+c)}\,d} + \frac{3b\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(92) = 184$.

time = 0.69, size = 1742, normalized size = 15.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
 + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4
*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b
*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3
*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2
+ 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2
+ 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*s
in(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*co
s(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*
d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2
*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3*
(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^2 +
 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^2 +
 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b*si
n(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(2*d
*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*x +
 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x + 4*
c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) + 2*
b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*
(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4
4*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*c
os(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

```
 + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4
*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*
c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*c
os(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x +
2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*s
in(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
 + 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.41, size = 234, normalized size = 2.13

$$\left[\frac{3\,b^{\frac{3}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(3\,b\cos\left(dx+c\right)^{2}+2\,b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^{5}}, -\frac{3\,\sqrt{-b}\,b\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(3\,b\cos\left(dx+c\right)^{2}+2\,b\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*(3*b*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*b*arctan(sqrt(b*cos(d*
x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (3*b
*cos(d*x + c)^2 + 2*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+dx))^{3/2}}{\cos(c+dx)^{13/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(13/2),x)
[Out] int((b*cos(c + d*x))^(3/2)/cos(c + d*x)^(13/2), x)
```

3.160 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=116

$$\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{2b^2\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{b\cos(c+dx)}\,\sin^5(c+dx)}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-2/3*b^2*sin(d*x+c)^3
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/5*b^2*sin(d*x+c)^5*(b*cos(d*x+c)
)^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{b^2\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}-\frac{2b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (2*b^2*Sqr
t[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[b*Co
s[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\text{Subst}\left(\int(1-2x^2+x^4)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{2b^2\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} +$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.50

$$\frac{(b\cos(c+dx))^{5/2}(150\sin(c+dx)+25\sin(3(c+dx))+3\sin(5(c+dx)))}{240d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(150*Sin[c + d*x] + 25*Sin[3*(c + d*x)] + 3*Sin[5*(
c + d*x)]))/(240*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.15, size = 52, normalized size = 0.45

method	result
default	$\frac{\left(3\left(\cos^4(dx+c)\right)+4\left(\cos^2(dx+c)\right)+8\right)\sin(dx+c)(b\cos(dx+c))^{\frac{5}{2}}}{15d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{6i(dx+c)}}{80\left(e^{2i(dx+c)}+1\right)d} - \frac{5ib^2\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}}{8\left(e^{2i(dx+c)}+1\right)d} + \frac{5ib^2\sqrt{b\,c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d*(3*cos(d*x+c)^4+4*cos(d*x+c)^2+8)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/co
s(d*x+c)^(5/2)
```

Maxima [A]
time = 0.58, size = 77, normalized size = 0.66

$$\frac{\left(3\,b^2\sin\left(5\,dx+5\,c\right)+25\,b^2\sin\left(\frac{3}{5}\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)+150\,b^2\sin\left(\frac{1}{5}\arctan\left(\sin\left(5\,dx+5\,c\right),\cos\left(5\,dx+5\,c\right)\right)\right)\right)\sqrt{b}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), co
s(5*d*x + 5*c))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*
c))))*sqrt(b)/d
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.53

$$\frac{\left(3\,b^{2}\cos\left(dx+c\right)^{4}+4\,b^{2}\cos\left(dx+c\right)^{2}+8\,b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b^2*cos(d*x + c)^4 + 4*b^2*cos(d*x + c)^2 + 8*b^2)*sqrt(b*cos(d*x +
 c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 1.35, size = 73, normalized size = 0.63

$$\frac{b^{2}\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(175\,\sin\left(2\,c+2\,d\,x\right)+28\,\sin\left(4\,c+4\,d\,x\right)+3\,\sin\left(6\,c+6\,d\,x\right)\right)}{240\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(175*sin(2*c + 2*d*x) + 28*s
in(4*c + 4*d*x) + 3*sin(6*c + 6*d*x)))/(240*d*(cos(2*c + 2*d*x) + 1))
```

3.161 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{3b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{8d}+\frac{b^2\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\sin(c+dx)}{4d}$$

```
[Out] 1/4*b^2*cos(d*x+c)^(5/2)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d+3/8*b^2*x*(b*cos
(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+3/8*b^2*sin(d*x+c)*cos(d*x+c)^(1/2)*(b*cos(
d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}{4d}+\frac{3b^2\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (3*b^2*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (3*b^2*Sqrt[Cos[c +
 d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]^(5/2)*S
qrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d} + \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)}{4\sqrt{\cos(c+}}$$

$$= \frac{3b^2\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d} + \frac{b^2\cos^{\frac{5}{2}}(c+dx)\sqrt{b\,c}}{}$$

$$= \frac{3b^2x\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{3b^2\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx}{8d}$$

Mathematica [A]
time = 0.05, size = 55, normalized size = 0.51

$$\frac{(b\cos(c+dx))^{5/2}(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x
)]))/(32*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.16, size = 62, normalized size = 0.58

method	result
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}(2\sin(dx+c)(\cos^3(dx+c))+3\sin(dx+c)\cos(dx+c)+3dx+3c)}{8d\cos(dx+c)^{\frac{5}{2}}}$
risch	$\frac{3b^2\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{i(dx+c)}x}{4(e^{2i(dx+c)}+1)} - \frac{ib^2\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{5i(dx+c)}}{32(e^{2i(dx+c)}+1)d} + \frac{ib^2\sqrt{b\cos(d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(b*cos(d*x+c))^(5/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+
c)+3*d*x+3*c)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.57, size = 59, normalized size = 0.55

$$\frac{\left(12\,(dx+c)b^2+b^2\sin\left(4\,dx+4\,c\right)+8\,b^2\sin\left(\frac{1}{2}\,\arctan\left(\sin\left(4\,dx+4\,c\right),\cos\left(4\,dx+4\,c\right)\right)\right)\right)\sqrt{b}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4
*d*x + 4*c), cos(4*d*x + 4*c))))*sqrt(b)/d
```

Fricas [A]
time = 0.40, size = 193, normalized size = 1.80

$$\left[\frac{3\sqrt{-b}\,b^2\log\left(2\,b\cos\left(dx+c\right)^2-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)+2\,\left(2\,b^2\cos\left(dx+c\right)^2+3\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{16\,d}, \frac{3\,b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)+\left(2\,b^2\cos\left(dx+c\right)^2+3\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(
-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*b^2*cos(d*x + c)^2 + 3*b^2)
*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1/8*(3*b^(5/2)*ar
ctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + (2*b
^2*cos(d*x + c)^2 + 3*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c))/d]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Simpl
ification assuming sageVARc near 0Simplification assuming sageVARc near 0Si
mplification assuming sageVARc near 0Simplification assuming sageVARc near
0(3*sageVARb^2*s
```

Mupad [B]
time = 1.03, size = 78, normalized size = 0.73

$$\frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c
+ 3*d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*d*(cos(2*c + 2*d*x)
 + 1))
```

3.162 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/3*b^2*sin(d*x+c)^3
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {17, 2713}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\
&= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}}\\
&= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-\frac{b^2\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 45, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{5/2}(5+\cos(2(c+dx)))\sin(c+dx)}{6d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Cos[c + d
*x]^(5/2))
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.53

method	result
default	$\frac{(\cos^2(dx+c)+2)\sin(dx+c)(b\cos(dx+c))^{\frac{5}{2}}}{3d\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{4i(dx+c)}}{12(e^{2i(dx+c)}+1)d}-\frac{3ib^2\sqrt{b\cos(dx+c)}\left(\sqrt{\cos}(dx+c)\right)e^{2i(dx+c)}}{4(e^{2i(dx+c)}+1)d}+\frac{3ib^2\sqrt{b\cos}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.57, size = 49, normalized size = 0.64

$$\frac{\left(b^2\sin\left(3\,dx+3\,c\right)+9\,b^2\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)\right)\sqrt{b}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))))*sqrt(b)/d
```

Fricas [A]
time = 0.40, size = 47, normalized size = 0.62

$$\frac{\left(b^2\cos\left(dx+c\right)^2+2\,b^2\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(
cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 71047 vs. $2(66) = 132$.
time = 7.54, size = 71047, normalized size = 934.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] -1/96*(3*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1
/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 3*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 24*b^(5
/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*
c)^3*tan(1/3*c)^6*tan(c) - 24*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*
d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*b^(5/2)*d*x^
4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan
(1/3*c)^4*tan(c)^2 - 18*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x +
1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 48*b^(5/2)*d*x^4*t
an(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c)^2 + 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*
c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 18*b^(5/2)*d*x^4*tan(1
/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)
^6*tan(c)^2 - 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6
*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4 + 18*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*
c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 48*b^(5/
2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c
)^3*tan(1/3*c)^6 - 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6
*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 + 18*b^(5/2)*d*x^4*tan(1/2*d*x +
 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6 - 72*
b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x +
1/2*c)^3*tan(1/3*c)^4*tan(c) - 72*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
```

1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c) + 24*b^(5/2)
*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*
tan(1/3*c)^6*tan(c) + 144*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c) - 72*b^(5/2)*d*x^4*t
an(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/
3*c)^6*tan(c) + 144*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*
c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c) - 72*b^(5/2)*d*x^4*tan(1/2
*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6
*tan(c) + 24*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c) + 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2
*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2*tan(c)^2
- 54*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^2*tan(1/3*c)^4*tan(c)^2 - 144*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c
)^3*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4*tan(c)^2 +
27*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x
 + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 - 54*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^2
*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4*tan(c)^2 + 3*b
^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(1/3*c)^6*tan
(c)^2 + 48*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)*tan(1/3*c)^6*tan(c)^2 - 54*b^(5/2)*d*x^4*tan(1/2*d*x + 1/
2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2
 + 108*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2
*d*x + 1/2*c)^2*tan(1/3*c)^6*tan(c)^2 - 144*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2
*c)^3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2
+ 48*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x
 + 1/2*c)^3*tan(1/3*c)^6*tan(c)^2 + 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*
tan(1/2*d*x + 1/6*c)^2*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 54*b
^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1
/2*c)^4*tan(1/3*c)^6*tan(c)^2 + 3*b^(5/2)*d*x^4*tan(1/2*d*x + 1/6*c)^6*tan(
-1/2*d*x + 1/2*c)^4*tan(1/3*c)^6*tan(c)^2 - 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1
/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^2 + 54*b^
(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/
2*c)^2*tan(1/3*c)^4 + 144*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(1/2*d*x
+ 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^4 - 27*b^(5/2)*d*x^4*tan(1/2*
d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^4*tan(1/3*c)^4
+ 54*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d
*x + 1/2*c)^4*tan(1/3*c)^4 - 3*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^4*tan(1/2
*d*x + 1/6*c)^6*tan(1/3*c)^6 - 48*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^3*tan(
1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)*tan(1/3*c)^6 + 54*b^(5/2)*d*x^4*ta
n(1/2*d*x + 1/2*c)^4*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^2*tan(1/3
*c)^6 - 108*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^2*tan(1/2*d*x + 1/6*c)^6*tan
(-1/2*d*x + 1/2*c)^2*tan(1/3*c)^6 + 144*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2*c)^
3*tan(1/2*d*x + 1/6*c)^4*tan(-1/2*d*x + 1/2*c)^3*tan(1/3*c)^6 - 48*b^(5/2)*
d*x^4*tan(1/2*d*x + 1/2*c)*tan(1/2*d*x + 1/6*c)^6*tan(-1/2*d*x + 1/2*c)^3*t
an(1/3*c)^6 - 9*b^(5/2)*d*x^4*tan(1/2*d*x + 1/2...

Mupad [B]
time = 0.62, size = 60, normalized size = 0.79

$$\frac{b^2\,\sqrt{\cos\left(c+d\,x\right)}\,\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2),x)
[Out] (b^2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4
*c + 4*d*x)))/(12*d*(cos(2*c + 2*d*x) + 1))
```

3.163 $\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=69

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d}$$

```
[Out] 1/2*b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)+1/2*b^2*sin(d*x+c)*cos(d*x+
c)^(1/2)*(b*cos(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}}+\frac{b^2\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b^2*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*Sqrt[Cos[c + d*x
]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}}{\sqrt{\cos(c+dx)}}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d} + \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2x\sqrt{b\cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}\sin(c+dx)}{2d}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.65

$$\frac{(b\cos(c+dx))^{5/2}(2(c+dx)+\sin(2(c+dx)))}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x]
^(5/2))
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.61

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}(\sin(dx+c)\cos(dx+c)+dx+c)}{2d\cos(dx+c)^{\frac{5}{2}}}$	42
risch	$\frac{b^2x\sqrt{b\cos(dx+c)}}{2\sqrt{\cos(dx+c)}} + \frac{b^2\sqrt{b\cos(dx+c)}\sin(2dx+2c)}{4\sqrt{\cos(dx+c)}\,d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(sin(d*x+c)*cos(d*x+c)+d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.56, size = 32, normalized size = 0.46

$$\frac{(2\,(dx+c)b^2 + b^2\sin(2\,dx+2\,c))\sqrt{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*sqrt(b)/d
```

Fricas [A]
time = 0.42, size = 159, normalized size = 2.30

$$\left[\frac{2\sqrt{b\cos\left(dx+c\right)}\,b^{2}\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{-b}\,b^{2}\log\left(2\,b\cos\left(dx+c\right)^{2}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,b^{2}\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+b^{\frac{5}{2}}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{\sqrt{b}\,\cos\left(dx+c\right)^{\frac{3}{2}}}\right)}{2\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(-b)
*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x
+ c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c)
)*sin(d*x + c) + b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*
cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 0.44, size = 40, normalized size = 0.58

$$\frac{b^2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(2\,c+2\,d\,x\right)+2\,d\,x\right)}{4\,d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(sin(2*c + 2*d*x) + 2*d*x))/(4*d*cos(c + d*x)^(
1/2))
```

3.164 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \cos(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 32, normalized size = 0.91

$$\frac{(b\cos(c+dx))^{5/2}\sin(c+dx)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(3/2),x]
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.11, size = 29, normalized size = 0.83

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{d\cos(dx+c)^{\frac{5}{2}}}$	29
risch	$\frac{b^{2}\sin(dx+c)\sqrt{b\cos\left(dx+c\right)}}{d\sqrt{\cos\left(dx+c\right)}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(5/2)
```

Maxima [A]
time = 0.59, size = 13, normalized size = 0.37

$$\frac{b^{\frac{5}{2}}\sin\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
[Out] b^(5/2)*sin(d*x + c)/d
```

Fricas [A]
time = 0.35, size = 31, normalized size = 0.89

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,b^{2}\sin\left(dx+c\right)}{d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b^2*sin(d*x + c)/(d*sqrt(cos(d*x + c)))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 0.32, size = 31, normalized size = 0.89

$$\frac{b^2 \sin\left(c+d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{d\,\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(3/2),x)
[Out] (b^2*sin(c + d*x)*(b*cos(c + d*x))^(1/2))/(d*cos(c + d*x)^(1/2))
```

3.165 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*x*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int 1\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2x\sqrt{b\cos(c+dx)}}{\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.89

$$\frac{x(b\cos(c+dx))^{5/2}}{\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(5/2),x]
```

```
[Out] (x*(b*Cos[c + d*x])^(5/2))/Cos[c + d*x]^(5/2)
```

Maple [A]
time = 0.09, size = 28, normalized size = 1.04

method	result	size
risch	$\frac{b^{2} x \sqrt{b \cos \left(d x+c\right)}}{\sqrt{\cos \left(d x+c\right)}}$	24
default	$\frac{\left(b \cos \left(d x+c\right)\right)^{\frac{5}{2}} \left(d x+c\right)}{d \cos \left(d x+c\right)^{\frac{5}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)*(d*x+c)
```

Maxima [A]
time = 0.52, size = 26, normalized size = 0.96

$$\frac{2 b^{\frac{5}{2}} \arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/d
```

Fricas [A]
time = 0.40, size = 97, normalized size = 3.59

$$\left[\frac{\sqrt{-b}\, b^{2} \log\left(2 b \cos\left(dx+c\right)^{2} - 2 \sqrt{b \cos\left(dx+c\right)} \sqrt{-b} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) - b\right)}{2 d}, \frac{b^{\frac{5}{2}} \arctan\left(\frac{\sqrt{b \cos\left(dx+c\right)} \sin\left(dx+c\right)}{\sqrt{b} \cos\left(dx+c\right)^{\frac{3}{2}}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*
sqrt(cos(d*x + c))*sin(d*x + c) - b)/d, b^(5/2)*arctan(sqrt(b*cos(d*x + c))
*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/d]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6189 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x)
```

Mupad [B]
time = 0.09, size = 23, normalized size = 0.85

$$\frac{b^2\,x\,\sqrt{b\,\cos\left(c+d\,x\right)}}{\sqrt{\cos\left(c+d\,x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(5/2),x)
[Out] (b^2*x*(b*cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2)
```

3.166 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

```
[Out] b^2*arctanh(sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{b^2\sqrt{b\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{7}{2}}(c+dx)}\,dx &= \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{\sqrt{\cos(c+dx)}}\\ &= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.92

$$\frac{\tanh^{-1}(\sin(c+dx))(b\cos(c+dx))^{5/2}}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(7/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*(b*Cos[c + d*x])^(5/2))/(d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 1.17

method	result	size
default	$-\frac{2(b\cos(dx+c))^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)}{d\cos(dx+c)^{\frac{5}{2}}}$	42
risch	$\frac{b^2\sqrt{b\cos(dx+c)}\,\ln(\mathrm{e}^{i(dx+c)}+i)}{\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{b\cos(dx+c)}\,\ln(\mathrm{e}^{i(dx+c)}-i)}{\sqrt{\cos(dx+c)}\,d}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*(b*cos(d*x+c))^(5/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))/cos(d*x+c)^(5
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(32) = 64.
time = 0.59, size = 72, normalized size = 2.00

$$\frac{\left(b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-b^2\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)\right)\sqrt{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*lo
g(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*sqrt(b)/d
```

Fricas [A]
time = 0.43, size = 116, normalized size = 3.22

$$\left[\frac{b^{\frac{5}{2}}\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,d},\,-\frac{\sqrt{-b}\,b^2\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/2*b^(5/2)*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(c
os(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/d, -sqrt(-b)*
b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))
))/d]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{5/2}}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2),x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(7/2), x)
```

3.167 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^2(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.91

$$\frac{(b\cos(c+dx))^{5/2}\sin(c+dx)}{d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(9/2),x]
[Out] ((b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(d*Cos[c + d*x]^(7/2))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.83

method	result	size
default	$\frac{(b\cos(dx+c))^{\frac{5}{2}}\sin(dx+c)}{d\cos(dx+c)^{\frac{7}{2}}}$	29
risch	$\frac{2ib^2\sqrt{b\cos(dx+c)}}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(7/2)
```

Maxima [A]
time = 0.64, size = 54, normalized size = 1.54

$$\frac{2\,b^{\frac{5}{2}}\sin(2\,dx+2\,c)}{\left(\cos(2\,dx+2\,c)^2+\sin(2\,dx+2\,c)^2+2\,\cos(2\,dx+2\,c)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 2*b^(5/2)*sin(2*d*x + 2*c)/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.37, size = 31, normalized size = 0.89

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,b^{2}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*b^2*sin(d*x + c)/(d*cos(d*x + c)^(3/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2), x)
```

Mupad [**B**]
time = 0.42, size = 62, normalized size = 1.77

$$\frac{b^{2}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(9/2),x)
```

```
[Out] (b^2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/
(d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.168 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/2*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/2*b^2*arctanh(
sin(d*x+c))*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{2d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) +
 (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{11}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec(c+dx)\,dx}{2\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)}$$

Mathematica [**A**]
time = 0.03, size = 52, normalized size = 0.67

$$\frac{(b\cos(c+dx))^{5/2}\left(\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)\right)}{2d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(11/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x
]))/(2*d*Cos[c + d*x]^(9/2))
```

Maple [**A**]
time = 0.15, size = 104, normalized size = 1.33

method	result
default	$-\frac{\left(\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)\right)(b\cos(dx+c))^{\frac{5}{2}}}{2d\cos(dx+c)^{\frac{9}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{\sqrt{\cos(dx+c)}\,d\left(e^{2i(dx+c)}+1\right)^2}+\frac{b^2\sqrt{b\cos(dx+c)}\,\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{\cos(dx+c)}\,d}-\frac{b^2\sqrt{b\cos(dx+c)}\,\ln(e^{i(dx}}{2\sqrt{\cos(dx+c)}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*(b*cos(d*x+c))^(5/
2)/cos(d*x+c)^(9/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 747 vs. 2(66) = 132.
time = 0.60, size = 747, normalized size = 9.58

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d
*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(
4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*
d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2
 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2
*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2
*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x
 + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(2*cos(2
*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)
^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
 + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 0.43, size = 210, normalized size = 2.69

$$\left[\frac{b^{\frac{5}{2}}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}b^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4d\cos\left(dx+c\right)^{3}},-\frac{\sqrt{-b}b^{2}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}b^{2}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/4*(b^(5/2)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3), -1/2*(sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x +
 c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*b^2*sqrt(
cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2),x)
```

```
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(11/2), x)
```

3.169 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)}+\frac{b^2\sqrt{b\cos(c+dx)}\,\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

```
[Out] b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/3*b^2*sin(d*x+c)^3
*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3852}

$$\frac{b^2\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\cos^{\frac{7}{2}}(c+dx)}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b^2*Sqrt[
b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{13}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^4(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= -\frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\operatorname{Subst}\left(\int (1+x^2)\;dx, x, -\tan(c+dx)\right)}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{b\cos(c+dx)}\;\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)} + \frac{b^2\sqrt{b\cos(c+dx)}\;\sin^3(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.59

$$\frac{(b\cos(c+dx))^{5/2}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(13/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Cos[c + d*x]^
(5/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.55

method	result	size
default	$\frac{\left(2\left(\cos^2(dx+c)\right)+1\right)\left(b\cos(dx+c)\right)^{\frac{5}{2}}\sin(dx+c)}{3d\cos(dx+c)^{\frac{11}{2}}}$	42
risch	$\frac{4ib^2\sqrt{b\cos\left(dx+c\right)}\;\left(3\,e^{2i(dx+c)}+1\right)}{3\sqrt{\cos\left(dx+c\right)}\;d\left(e^{2i(dx+c)}+1\right)^3}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(2*cos(d*x+c)^2+1)*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(11/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(66) = 132.
time = 0.61, size = 311, normalized size = 4.09

$$-\frac{4\left(3\,b^2\cos\left(6\,dx+6\,c\right)\sin\left(2\,dx+2\,c\right)+9\,b^2\cos\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)-\left(3\,b^2\cos\left(2\,dx+2\,c\right)+b^2\right)\sin\left(6\,dx+6\,c\right)-3\left(3\,b^2\cos\left(2\,dx+2\,c\right)+b^2\right)\sin\left(4\,dx+4\,c\right)\right)\sqrt{b}}{3\left(2\left(3\,\cos\left(4\,dx+4\,c\right)+3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(6\,dx+6\,c\right)+\cos\left(6\,dx+6\,c\right)^2+6\left(3\,\cos\left(2\,dx+2\,c\right)+1\right)\cos\left(4\,dx+4\,c\right)+9\,\cos\left(4\,dx+4\,c\right)^2+9\,\cos\left(2\,dx+2\,c\right)^2+6\left(\sin\left(4\,dx+4\,c\right)+\sin\left(2\,dx+2\,c\right)\right)\sin\left(6\,dx+6\,c\right)+\sin\left(6\,dx+6\,c\right)^2+9\,\sin\left(4\,dx+4\,c\right)^2+18\,\sin\left(4\,dx+4\,c\right)\sin\left(2\,dx+2\,c\right)+9\,\sin\left(2\,dx+2\,c\right)^2+6\,\cos\left(2\,dx+2\,c\right)+1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="maxima")
```

```
[Out] -4/3*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*c
os(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*sqrt(b)/((2*(3*cos(4*d*x + 4*c) +
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.38, size = 46, normalized size = 0.61

$$\frac{\left(2\,b^{2}\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b^2*cos(d*x + c)^2 + b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^(7/2))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2), x)
```

Mupad [**B**]
time = 1.18, size = 131, normalized size = 1.72

$$\frac{2\,b^{2}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6\,\sin\left(4\,c+4\,d\,x\right)+\sin\left(6\,c+6\,d\,x\right)+10\mathrm{i}\right)}{3\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(13/2),x)
```

```
[Out] (2*b^2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i +
 cos(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c +
6*d*x) + 10i))/(3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4
*d*x) + cos(6*c + 6*d*x) + 10))
```

3.170 $\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{3b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}+\frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)}$$

```
[Out] 1/4*b^2*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(9/2)+3/8*b^2*sin(d*x+
c)*(b*cos(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+3/8*b^2*arctanh(sin(d*x+c))*(b*c
os(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3853, 3855}

$$\frac{3b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{8d\cos^{\frac{5}{2}}(c+dx)}+\frac{b^2\sin(c+dx)\sqrt{b\cos(c+dx)}}{4d\cos^{\frac{9}{2}}(c+dx)}+\frac{3b^2\sqrt{b\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(15/2),x]
```

```
[Out] (3*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]])
 + (b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (3*b^
2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^{5/2}}{\cos^{\frac{15}{2}}(c+dx)}\,dx = \frac{\left(b^2\sqrt{b\cos(c+dx)}\right)\int \sec^5(c+dx)\,dx}{\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{\left(3b^2\sqrt{b\cos(c+dx)}\right)\int \sec^3(c+dx)\,dx}{4\sqrt{\cos(c+dx)}}$$
$$= \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)} + \frac{\left(3b^2\sqrt{b\cos(c\right.}}{8\sqrt{}}$$
$$= \frac{3b^2\tanh^{-1}(\sin(c+dx))\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}} + \frac{b^2\sqrt{b\cos(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{9}{2}}(c+dx)} + \frac{3b^2\sqrt{}}{}$$

Mathematica [A]
time = 0.06, size = 66, normalized size = 0.57

$$\frac{(b\cos(c+dx))^{5/2}\left(3\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2+3\cos^2(c+dx))\sin(c+dx)\right)}{8d\cos^{\frac{13}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)/Cos[c + d*x]^(15/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Co
s[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/2))
```

Maple [A]
time = 0.20, size = 121, normalized size = 1.04

method	result
default	$\frac{\left(3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx+\right.}{8d\cos(dx+c)^{\frac{13}{2}}}$
risch	$-\frac{ib^2\sqrt{b\cos(dx+c)}\,(3\,e^{7i(dx+c)}+11\,e^{5i(dx+c)}-11\,e^{3i(dx+c)}-3\,e^{i(dx+c)})}{4\sqrt{\cos(dx+c)}\,d(e^{2i(dx+c)}+1)^4}+\frac{3b^2\sqrt{b\cos(dx+c)}\,\ln(e^{i(dx+c)}+i)}{8\sqrt{\cos(dx+c)}\,d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1914 vs. 2(98) = 196.
time = 0.66, size = 1914, normalized size = 16.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x
+ 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(
4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^
2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c)
+ 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x
 + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin
(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 4
8*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2
*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c
) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4
*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x
+ 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*
x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x +
 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d
*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*s
in(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 +
2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c)
 + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*
c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x + 8
*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^2*c
os(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*c
os(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
```

```
 + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*
c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*co
s(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*sqrt(b)/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*
cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x
+ 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*
c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) +
 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 3
6*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x
+ 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [**A**]
time = 0.42, size = 244, normalized size = 2.10

$$\left[\frac{3\,b^{\frac{5}{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\left(3\,b^{2}\cos\left(dx+c\right)^{2}+2\,b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{16\,d\cos\left(dx+c\right)^{5}},\ -\frac{3\,\sqrt{-b}\,b^{2}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\left(3\,b^{2}\cos\left(dx+c\right)^{2}+2\,b^{2}\right)\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{8\,d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*(3*b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*b^2*arctan(sqrt(b*
cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5
- (3*b^2*cos(d*x + c)^2 + 2*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)/cos(d*x+c)**(15/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)/cos(d*x+c)^(15/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\cos(c+d\,x))^{5/2}}{\cos(c+d\,x)^{15/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(15/2),x)
[Out] int((b*cos(c + d*x))^(5/2)/cos(c + d*x)^(15/2), x)
```

3.171 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{2\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\,\sin^5(c+dx)}{5d\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-2/3*sin(d*x+c)^3*cos(d*x
+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)+1/5*sin(d*x+c)^5*cos(d*x+c)^(1/2)/d/(b*cos
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{17, 2713\}$

$$\frac{\sin^5(c+dx)\sqrt{\cos(c+dx)}}{5d\sqrt{b\cos(c+dx)}}-\frac{2\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (2*Sqrt[Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]]) + (Sqrt[Cos[c + d*x]]*Si
n[c + d*x]^5)/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int (1-2x^2+x^4)\,dx, x, -\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{2\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\,\sin^5(}{5d\sqrt{b\cos(c+d}$$

Mathematica [A]
time = 0.05, size = 58, normalized size = 0.54

$$\frac{\sqrt{\cos(c+dx)}\,(150\sin(c+dx)+25\sin(3(c+dx))+3\sin(5(c+dx)))}{240d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(150*Sin[c + d*x] + 25*Sin[3*(c + d*x)] + 3*Sin[5*(c + d*x)]))/(240*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.16, size = 52, normalized size = 0.49

method	result	size
default	$\frac{(3(\cos^4(dx+c))+4(\cos^2(dx+c))+8)\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{15d\sqrt{b\cos(dx+c)}}$	52
risch	$\frac{5\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{8d\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(5dx+5c)}{80\sqrt{b\cos(dx+c)}\,d} + \frac{5\left(\sqrt{\cos}\,(dx+c)\right)\sin(3dx+3c)}{48\sqrt{b\cos(dx+c)}\,d}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/d*(3*cos(d*x+c)^4+4*cos(d*x+c)^2+8)*sin(d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.58, size = 68, normalized size = 0.64

$$\frac{3\sin(5\,dx+5\,c)+25\sin\left(\frac{3}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)+150\sin\left(\frac{1}{5}\arctan(\sin(5\,dx+5\,c),\cos(5\,dx+5\,c))\right)}{240\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/240*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x
+ 5*c))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))/(sqrt(
b)*d)
```

Fricas [A]
time = 0.37, size = 54, normalized size = 0.50

$$\frac{\left(3\,\cos\left(dx+c\right)^{4}+4\,\cos\left(dx+c\right)^{2}+8\right)\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\,bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(d*x + c)^4 + 4*cos(d*x + c)^2 + 8)*sqrt(b*cos(d*x + c))*sin(d*x
 + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 1.18, size = 73, normalized size = 0.68

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(175\,\sin\left(2\,c+2\,d\,x\right)+28\,\sin\left(4\,c+4\,d\,x\right)+3\,\sin\left(6\,c+6\,d\,x\right)\right)}{240\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(175*sin(2*c + 2*d*x) + 28*sin(4
*c + 4*d*x) + 3*sin(6*c + 6*d*x)))/(240*b*d*(cos(2*c + 2*d*x) + 1))
```

3.172 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

```
[Out] 3/8*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/4*cos(d*x+c)^(7/2)
*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+3/8*x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1
/2)
```

Rubi [A]

time = 0.02, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 2715, 8}

$$\frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4d\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c + d*x])
/(4*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4\sqrt{b\cos(c+dx)}}$$
$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8\sqrt{b\cos(c+dx)}}$$
$$= \frac{3x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 55, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}\,(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.17, size = 62, normalized size = 0.63

method	result	size
default	$\frac{(2\sin(dx+c)(\cos^3(dx+c))+3\sin(dx+c)\cos(dx+c)+3dx+3c)\left(\sqrt{\cos}\,(dx+c)\right)}{8d\sqrt{b\cos(dx+c)}}$	62
risch	$\frac{3x\left(\sqrt{\cos}\,(dx+c)\right)}{8\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(4dx+4c)}{32\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(2dx+2c)}{4\sqrt{b\cos(dx+c)}\,d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*(2*sin(d*x+c)*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+c)+3*d*x+3*c)*cos(d*x
+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.60, size = 49, normalized size = 0.50

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(sqrt(b)*d)
```

Fricas [**A**]
time = 0.41, size = 182, normalized size = 1.86

$$\left[\frac{2\sqrt{b\cos\left(dx+c\right)}\left(2\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\sqrt{-b}\log\left(2b\cos\left(dx+c\right)^{2}+2\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{16bd}, \frac{\sqrt{b\cos\left(dx+c\right)}\left(2\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{8bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/8*(sqrt(b*cos(d*x + c))
*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arctan(
sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/sqrt(b*cos(d*x + c)), x)
```

Mupad [**B**]
time = 1.06, size = 78, normalized size = 0.80

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(b*cos(c + d*x))^(1/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3*
d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*b*d*(cos(2*c + 2*d*x) +
 1))
```

3.173 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*cos(d*x
+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[c +
d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int\left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.64

$$\frac{\sqrt{\cos(c+dx)}\,(5+\cos(2(c+dx)))\sin(c+dx)}{6d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[b*Cos[c
+ d*x]])
```

Maple [A]
time = 0.13, size = 40, normalized size = 0.57

method	result	size
default	$\frac{(\cos^2(dx+c)+2)\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{3d\sqrt{b\cos(dx+c)}}$	40
risch	$\frac{3\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{4d\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(3dx+3c)}{12\sqrt{b\cos(dx+c)}\,d}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.57, size = 42, normalized size = 0.60

$$\frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c)\right)\right)}{12\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(sqrt(b)*d)
```

Fricas [A]
time = 0.39, size = 42, normalized size = 0.60

$$\frac{\sqrt{b\cos\left(dx+c\right)}\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b*d*sqrt(cos(d*
x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.64, size = 60, normalized size = 0.86

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c +
 4*d*x)))/(12*b*d*(cos(2*c + 2*d*x) + 1))
```

3.174 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=63

$$\frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/2)+1/2*x*cos(d*x+c)^(1/
2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {17, 2715, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{2\sqrt{b\cos(c+dx)}}$$
$$= \frac{x\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.71

$$\frac{\sqrt{\cos(c+dx)}\,(2(c+dx)+\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Cos[c + d
*x]])
```

Maple [A]
time = 0.14, size = 42, normalized size = 0.67

method	result	size
default	$\frac{(\sin(dx+c)\cos(dx+c)+dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{2d\sqrt{b\cos\left(dx+c\right)}}$	42
risch	$\frac{x\left(\sqrt{\cos}\,(dx+c)\right)}{2\sqrt{b\cos\left(dx+c\right)}}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(2dx+2c)}{4\sqrt{b\cos\left(dx+c\right)}\,d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*(sin(d*x+c)*cos(d*x+c)+d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.58, size = 25, normalized size = 0.40

$$\frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(sqrt(b)*d)
```

Fricas [A]
time = 0.41, size = 157, normalized size = 2.49

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,bd},\ \frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*log
(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x
+ c)^(3/2))))/(b*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.65, size = 65, normalized size = 1.03

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,b\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x)
 + 4*d*x*cos(c + d*x)))/(4*b*d*(cos(2*c + 2*d*x) + 1))
```

3.175 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 1.00

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/Sqrt[b*Cos[c + d*x]],x]
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{d\sqrt{b\cos\left(dx+c\right)}}$	29
risch	$\frac{\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{d\sqrt{b\cos\left(dx+c\right)}}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Maxima [A]
time = 0.56, size = 13, normalized size = 0.41

$$\frac{\sin\left(dx+c\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] sin(d*x + c)/(sqrt(b)*d)
```

Fricas [A]
time = 0.38, size = 31, normalized size = 0.97

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{bd\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [A]
time = 24.21, size = 46, normalized size = 1.44

$$\begin{cases} \frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}} & \text{for } d \neq 0 \\ \frac{x\cos^{\frac{3}{2}}(c)}{\sqrt{b\cos(c)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Piecewise((sin(c + d*x)*sqrt(cos(c + d*x))/(d*sqrt(b*cos(c + d*x))), Ne(d,
0)), (x*cos(c)**(3/2)/sqrt(b*cos(c)), True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.36, size = 47, normalized size = 1.47

$$\frac{\sqrt{\cos(c+d\,x)}\,\sin(2\,c+2\,d\,x)\,\sqrt{b\,\cos(c+d\,x)}}{b\,d\,(\cos(2\,c+2\,d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(b*d*(cos(2*c
+ 2*d*x) + 1))
```

3.176 $\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=24

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

```
[Out] x*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 1.00

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]]
```

Maple [A]
time = 0.10, size = 28, normalized size = 1.17

method	result	size
risch	$\frac{x\left(\sqrt{\cos}\,(dx+c)\right)}{\sqrt{b\cos\left(dx+c\right)}}$	21
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(dx+c)}{d\sqrt{b\cos\left(dx+c\right)}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)*(d*x+c)
```

Maxima [A]
time = 0.52, size = 26, normalized size = 1.08

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A]
time = 0.42, size = 97, normalized size = 4.04

$$\left[-\frac{\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{2\,bd},\ \frac{\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{\sqrt{b}\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/(b*d), arctan(sqrt(b*cos(d*x + c))*sin(d*
x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(sqrt(b)*d)]
```

Sympy [A]
time = 1.09, size = 22, normalized size = 0.92

$$\frac{x\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] x*sqrt(cos(c + d*x))/sqrt(b*cos(c + d*x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)
```

Mupad [B]
time = 0.26, size = 37, normalized size = 1.54

$$\frac{2\,x\cos\left(c+d\,x\right)^{3/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(1/2),x)
```

```
[Out] (2*x*cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2))/(b*(cos(2*c + 2*d*x) + 1))
```

3.177 $\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {18, 3855}

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.13, size = 42, normalized size = 1.27

method	result	size
default	$-\frac{2\left(\sqrt{\cos}\,(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)}{d\sqrt{b\cos\left(dx+c\right)}}$	42
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}+i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}-i\right)}{\sqrt{b\cos\left(dx+c\right)}\,d}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] -2/d*cos(d*x+c)^(1/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))/(b*cos(d*x+c))^(1
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(29) = 58$.
time = 0.56, size = 65, normalized size = 1.97

$$\frac{\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\,\sin\left(dx+c\right)+1\right)}{2\,\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(sqrt(b)*d)
```

Fricas [A]
time = 0.44, size = 116, normalized size = 3.52

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^{3}-2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{b}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)}{2\,\sqrt{b}\,d},\ -\frac{\sqrt{-b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)}{bd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(sqrt(b)*d), -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/(b*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\cos{(c+dx)}}\,\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*cos(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.178 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3852, 8}

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{d\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 1.00

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.13, size = 29, normalized size = 0.91

method	result	size
default	$\frac{\sin(dx+c)}{d\sqrt{\cos(dx+c)}\sqrt{b\cos(dx+c)}}$	29
risch	$\frac{i e^{-i(dx+c)}}{\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\, d}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] sin(d*x+c)/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(28) = 56$.
time = 0.56, size = 59, normalized size = 1.84

$$\frac{2\sqrt{b}\sin(2\,dx+2\,c)}{\left(b\cos(2\,dx+2\,c)^2 + b\sin(2\,dx+2\,c)^2 + 2\,b\cos(2\,dx+2\,c) + b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 +
2*b*cos(2*d*x + 2*c) + b)*d)
```

Fricas [A]
time = 0.42, size = 31, normalized size = 0.97

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{bd\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\cos\left(c+dx\right)}\,\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(b*cos(c + d*x))*cos(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 0.54, size = 62, normalized size = 1.94

$$\frac{\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,1{}\mathrm{i}+\sin\left(2\,c+2\,d\,x\right)+1{}\mathrm{i}\right)}{b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(b*d
*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.179 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=72

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*arctanh(sin(d*x+
c))*cos(d*x+c)^(1/2)/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3853, 3855}

$$\frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + Sin
[c + d*x]/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Mathematica [**A**]
time = 0.02, size = 52, normalized size = 0.72

$$\frac{\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/
2)*Sqrt[b*Cos[c + d*x]])
```

Maple [**A**]
time = 0.16, size = 104, normalized size = 1.44

method	result
default	$-\frac{\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)}{2d\sqrt{b\cos(dx+c)}\,\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i\left(e^{2i(dx+c)}-1\right)}{2\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,\left(e^{2i(dx+c)}+1\right)d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}+i\right)}{2\sqrt{b\cos(dx+c)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx\right.}}{2\sqrt{b\cos(dx+c)}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))/(b*cos(d*x+c))^(1/
2)/cos(d*x+c)^(3/2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(60) = 120.
time = 0.60, size = 661, normalized size = 9.18

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
 + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) +
1)*sqrt(b)*d)
```

Fricas [A]
time = 0.39, size = 207, normalized size = 2.88

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4bd\cos\left(dx+c\right)^{3}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2bd\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x +
 c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.180 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*sin(d*x+c)^3/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+sin(d*x+c)/d/cos(d
*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 3852}

$$\frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] Sin[c + d*x]/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]^3/(
3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx&=\frac{\sqrt{\cos(c+dx)}\,\int \sec^4(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}\\&=-\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int (1+x^2)\,dx,x,-\tan(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}\\&=\frac{\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.64

$$\frac{\sqrt{\cos(c+dx)}\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A]
time = 0.14, size = 42, normalized size = 0.60

method	result	size
default	$\frac{\sin(dx+c)\left(2\left(\cos^2(dx+c)\right)+1\right)}{3d\sqrt{b\cos\left(dx+c\right)}\,\cos(dx+c)^{\frac{5}{2}}}$	42
risch	$\frac{2i(4\cos(dx+c)+2i\sin(dx+c))}{3\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)^2 d}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(60) = 120$.
time = 0.58, size = 294, normalized size = 4.20

$$\frac{4\left((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\sin(4dx+4c)-3\cos(6dx+6c)\sin(2dx+2c)-9\cos(4dx+4c)\sin(2dx+2c)\right)}{3\left(2(3\cos(4dx+4c)+3\cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3\cos(2dx+2c)+1)\cos(4dx+4c)+9\cos(4dx+4c)^2+9\cos(2dx+2c)^2+6(\sin(4dx+4c)+\sin(2dx+2c))\sin(6dx+6c)+\sin(6dx+6c)^2+9\sin(4dx+4c)^2+18\sin(4dx+4c)\sin(2dx+2c)+9\sin(2dx+2c)^2+6\cos(2dx+2c)+1\right)\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6
*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4
*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + s
in(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)
^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*
d*x + 2*c) + 1)*sqrt(b)*d)
```

Fricas [A]
time = 0.38, size = 44, normalized size = 0.63

$$\frac{\sqrt{b\cos\left(dx+c\right)}\left(2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{3\,bd\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b*d*cos(d*x +
 c)^(7/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x)
```

Mupad [B]
time = 1.37, size = 131, normalized size = 1.87

$$\frac{2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6\,\sin\left(4\,c+4\,d\,x\right)+\sin\left(6\,c+6\,d\,x\right)+10{}\mathrm{i}\right)}{3\,b\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] (2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos
(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*d*
x) + 10i))/(3*b*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4*d
*x) + cos(6*c + 6*d*x) + 10))
```

3.181 $\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=107

$$\frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+3/8*sin(d*x+c)/d/cos
(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+3/8*arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)
/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {18, 3853, 3855}

$$\frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\,\tanh^{-1}(\sin(c+dx))}{8d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + S
in[c + d*x]/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[c + d*x]
)/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)\,dx}{\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \sec^3(c+dx)}{4\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$
$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+d}}$$

Mathematica [A]
time = 0.04, size = 66, normalized size = 0.62

$$\frac{3\tanh^{-1}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d*
x]))/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.21, size = 121, normalized size = 1.13

method	result
default	$\frac{3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx-}{8d\sqrt{b\cos(dx+c)}\cos(dx+c)^{\frac{7}{2}}}$
risch	$-\frac{i(3e^{6i(dx+c)}+11e^{4i(dx+c)}-11e^{2i(dx+c)}-3)}{8\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)^3 d} - \frac{3\left(\sqrt{\cos}(dx+c)\right)\ln(e^{i(dx+c)}-i)}{8\sqrt{b\cos(dx+c)}\,d} + \frac{3\left(\sqrt{\cos}(dx+c)\right)\ln(}{8\sqrt{b\cos(dx-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1656 vs. $2(89) = 178$.

time = 0.58, size = 1656, normalized size = 15.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))))/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1
)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d
*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x +
 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2
 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d
*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c
) + 1)*sqrt(b)*d)
```

Fricas [**A**]
time = 0.42, size = 233, normalized size = 2.18

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{16bd\cos\left(dx+c\right)^{5}}, -\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8bd\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(b*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x +
c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b*c
os(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*c
os(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(9/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos (c+d\,x)^{9/2}\,\sqrt{b\,\cos (c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)),x)
[Out] int(1/(cos(c + d*x)^(9/2)*(b*cos(c + d*x))^(1/2)), x)
```

3.182 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 3/8*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/4*cos(d*x+c)^(7/
2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+3/8*x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+
c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/2)
*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c + d
*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b\sqrt{b\cos(c+dx)}}$$
$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b\sqrt{b\cos(c+dx)}}$$
$$= \frac{3x\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.51

$$\frac{\cos^{\frac{3}{2}}(c+dx)(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(3/2),x]
[Out] (Cos[c + d*x]^(3/2)*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.16, size = 62, normalized size = 0.58

method	result	size
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(2\sin(dx+c)(\cos^3(dx+c))+3\sin(dx+c)\cos(dx+c)+3dx+3c)}{8d(b\cos(dx+c))^{\frac{3}{2}}}$	62
risch	$\frac{3x\left(\sqrt{\cos}\,(dx+c)\right)}{8b\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(4dx+4c)}{32b\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*cos(d*x+c)^(3/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+c)+3
*d*x+3*c)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.59, size = 49, normalized size = 0.46

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\,\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(3/2)*d)
```

Fricas [A]
time = 0.43, size = 182, normalized size = 1.70

$$\left[\frac{2\sqrt{b\cos\left(dx+c\right)}\left(2\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\sqrt{-b}\log\left(2b\cos\left(dx+c\right)^{2}+2\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{16b^{2}d}, \frac{\sqrt{b\cos\left(dx+c\right)}\left(2\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{8b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/8*(sqrt(b*cos(d*x + c
))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.98, size = 78, normalized size = 0.73

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3*
d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*b^2*d*(cos(2*c + 2*d*x)
 + 1))
```

3.183 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*cos(d
*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[c
+ d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} - \frac{\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.59

$$\frac{\cos^{\frac{3}{2}}(c+dx)(5+\cos(2(c+dx)))\sin(c+dx)}{6d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b*Cos[c + d*
x])^(3/2))
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.53

method	result	size
default	$\frac{(\cos^2(dx+c)+2)\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{3d(b\cos(dx+c))^{\frac{3}{2}}}$	40
risch	$\frac{3\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{4bd\sqrt{b\cos\left(dx+c\right)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(3dx+3c)}{12b\sqrt{b\cos\left(dx+c\right)}\,d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*(cos(d*x+c)^2+2)*cos(d*x+c)^(3/2)*sin(d*x+c)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.58, size = 42, normalized size = 0.55

$$\frac{\sin\left(3\,dx+3\,c\right)+9\,\sin\left(\frac{1}{3}\,\arctan\left(\sin\left(3\,dx+3\,c\right),\cos\left(3\,dx+3\,c\right)\right)\right)}{12\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(3/2)*d)
```

Fricas [A]
time = 0.38, size = 42, normalized size = 0.55

$$\frac{\sqrt{b\cos\left(dx+c\right)}\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,b^{2}d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b^2*d*sqrt(cos(
d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.70, size = 60, normalized size = 0.79

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c +
 4*d*x)))/(12*b^2*d*(cos(2*c + 2*d*x) + 1))
```

3.184 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(1/2)+1/2*x*cos(d*x+c)^(
1/2)/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*Sin
[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{2b\sqrt{b\cos(c+dx)}}$$
$$= \frac{x\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 45, normalized size = 0.65

$$\frac{\cos^{\frac{3}{2}}(c+dx)(2(c+dx)+\sin(2(c+dx)))}{4d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(3/2),x]
[Out] (Cos[c + d*x]^(3/2)*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*d*(b*Cos[c + d*x])
^(3/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.61

method	result	size
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(\sin(dx+c)\cos(dx+c)+dx+c)}{2d(b\cos(dx+c))^{\frac{3}{2}}}$	42
risch	$\frac{x\left(\sqrt{\cos}(dx+c)\right)}{2b\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}(dx+c)\right)\sin(2dx+2c)}{4b\sqrt{b\cos(dx+c)}\,d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*cos(d*x+c)^(3/2)*(sin(d*x+c)*cos(d*x+c)+d*x+c)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.58, size = 25, normalized size = 0.36

$$\frac{2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)}{4\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(3/2)*d)

Fricas [A]
time = 0.40, size = 157, normalized size = 2.28

$$\left[\frac{2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{4\,b^{2}d},\frac{\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{b}\,\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{2\,b^{2}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x)

Mupad [B]
time = 0.59, size = 65, normalized size = 0.94

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+4\,d\,x\,\cos\left(c+d\,x\right)\right)}{4\,b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^(7/2)/(b*cos(c + d*x))^(3/2),x)

[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x) + 4*d*x*cos(c + d*x)))/(4*b^2*d*(cos(2*c + 2*d*x) + 1))

3.185 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\int \cos(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 32, normalized size = 0.91

$$\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.11, size = 29, normalized size = 0.83

method	result	size
default	$\frac{\sin(dx+c)\left(\cos^{\frac{3}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{3}{2}}}$	29
risch	$\frac{\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{bd\sqrt{b\cos\left(dx+c\right)}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*sin(d*x+c)*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [A]
time = 0.59, size = 13, normalized size = 0.37

$$\frac{\sin\left(dx+c\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(3/2)*d)
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.89

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{b^2d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.32, size = 47, normalized size = 1.34

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(2\,c+2\,d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b^2\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(b^2*d*(cos(2*
c + 2*d*x) + 1))
```

3.186 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

```
[Out] x*cos(d*x+c)^(1/2)/b/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.89

$$\frac{x\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(3/2),x]
[Out] (x*Cos[c + d*x]^(3/2))/(b*Cos[c + d*x])^(3/2)
```

Maple [A]
time = 0.10, size = 28, normalized size = 1.04

method	result	size
risch	$\frac{x\left(\sqrt{\cos}\,(dx+c)\right)}{b\sqrt{b\cos\left(dx+c\right)}}$	24
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)(dx+c)}{d(b\cos(dx+c))^{\frac{3}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/d*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)*(d*x+c)
```

Maxima [A]
time = 0.50, size = 26, normalized size = 0.96

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A]
time = 0.40, size = 97, normalized size = 3.59

$$\left[-\frac{\sqrt{-b}\,\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{2\,b^{2}d},\ \frac{\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{b^{\frac{3}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
[Out] [-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/(b^2*d), arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(b^(3/2)*d)]
```

Sympy [A]
time = 14.53, size = 22, normalized size = 0.81

$$\frac{x\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] x*cos(c + d*x)**(3/2)/(b*cos(c + d*x))**(3/2)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x)
```

Mupad [B]
time = 0.28, size = 37, normalized size = 1.37

$$\frac{2\,x\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b^2\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] (2*x*cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2))/(b^2*(cos(2*c + 2*d*x) + 1)
)
```

3.187 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(3/2),x]
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.92

$$\frac{\tanh^{-1}(\sin(c+dx))\cos^{\frac{3}{2}}(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(3/2),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^(3/2))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.13, size = 42, normalized size = 1.17

method	result	size
default	$-\frac{2\left(\cos^{\frac{3}{2}}(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)}{d(b\cos(dx+c))^{\frac{3}{2}}}$	42
risch	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}-i\right)}{b\sqrt{b\cos\left(dx+c\right)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}+i\right)}{b\sqrt{b\cos\left(dx+c\right)}\,d}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*cos(d*x+c)^(3/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))/(b*cos(d*x+c))^(3
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(32) = 64.
time = 0.58, size = 65, normalized size = 1.81

$$\frac{\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)}{2\,b^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(3/2)*d)
```

Fricas [A]
time = 0.41, size = 116, normalized size = 3.22

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2\,b^{\frac{3}{2}}d},\ -\frac{\sqrt{-b}\ \arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)}{b^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(b^(3/2)*d), -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/(b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(sqrt(cos(c + d*x))/(b*cos(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\cos(c+d\,x)}}{(b\,\cos(c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(3/2), x)
```

3.188 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

[Out] sin(d*x+c)/b/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3852, 8}

$$\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]

[Out] Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}(\int 1\,dx, x, -\tan(c+dx))}{bd\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

<u>**Mathematica**</u> [A]
time = 0.01, size = 32, normalized size = 0.91

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(3/2))
```

<u>**Maple**</u> [A]
time = 0.12, size = 29, normalized size = 0.83

method	result	size
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(dx+c)}{d(b\cos(dx+c))^{\frac{3}{2}}}$	29
risch	$\frac{i\mathrm{e}^{-i(dx+c)}}{b\sqrt{b\cos(dx+c)}\,\sqrt{\cos(dx+c)}\,d}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
[Out] 1/d*cos(d*x+c)^(1/2)*sin(d*x+c)/(b*cos(d*x+c))^(3/2)
```

<u>**Maxima**</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(31) = 62$.
time = 0.58, size = 67, normalized size = 1.91

$$\frac{2\sqrt{b}\,\sin(2\,dx+2\,c)}{\left(b^2\cos(2\,dx+2\,c)^2+b^2\sin(2\,dx+2\,c)^2+2\,b^2\cos(2\,dx+2\,c)+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^
2 + 2*b^2*cos(2*d*x + 2*c) + b^2)*d)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 0.89

$$\frac{\sqrt{b\cos(dx+c)}\,\sin(dx+c)}{b^2d\cos(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^(3/2))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(b\cos(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((b*cos(c + d*x))**(3/2)*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))), x)
```

Mupad [B]
time = 0.51, size = 62, normalized size = 1.77

$$\frac{\sqrt{b\,\cos(c+d\,x)}\,\left(\cos(2\,c+2\,d\,x)\,1\mathrm{i}+\sin(2\,c+2\,d\,x)+1\mathrm{i}\right)}{b^2\,d\,\sqrt{\cos(c+d\,x)}\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] ((b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*1i + sin(2*c + 2*d*x) + 1i))/(b^2
*d*cos(c + d*x)^(1/2)*(cos(2*c + 2*d*x) + 1))
```

3.189 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*arctanh(sin(d*
x+c))*cos(d*x+c)^(1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3853, 3855}

$$\frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + S
in[c + d*x]/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{2b\sqrt{b\cos(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.67

$$\frac{\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x
]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.15, size = 104, normalized size = 1.33

method	result
default	$-\frac{(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-(\cos^2(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)}{2d(b\cos(dx+c))^{\frac{3}{2}}\sqrt{\cos(dx+c)}}$
risch	$-\frac{i(e^{2i(dx+c)}-1)}{2b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)d}+\frac{(\sqrt{\cos}(dx+c))\ln(e^{i(dx+c)}+i)}{2b\sqrt{b\cos(dx+c)}\,d}-\frac{(\sqrt{\cos}(dx+c))\ln(e^{i(d}}{2b\sqrt{b\cos(dx+c}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))/(b*cos(d*x+c))^(3/
2)/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 670 vs. 2(66) = 132.
time = 0.58, size = 670, normalized size = 8.59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)
^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin
(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2
*d*x + 2*c) + b)*sqrt(b)*d)
```

Fricas [A]
time = 0.42, size = 207, normalized size = 2.65

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2\,b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\,b^{2}d\cos\left(dx+c\right)^{3}},\ -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2\,b^{2}d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x
 + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(c+dx\right)\right)^{\frac{3}{2}}\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(3/2),x)
[Out] Integral(1/((b*cos(c + d*x))**(3/2)*cos(c + d*x)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{3/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.190 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*sin(d*x+c)^3/b/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+sin(d*x+c)/b/d/c
os(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {18, 3852}

$$\frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] Sin[c + d*x]/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]^3
/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{bd\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.59

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*(b*Cos[c + d*x])^
(3/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.55

method	result	size
default	$\frac{\sin(dx+c)\left(2\left(\cos^2(dx+c)\right)+1\right)}{3d(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{3}{2}}}$	42
risch	$\frac{2i(4\cos(dx+c)+2i\sin(dx+c))}{3b\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)^2 d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(66) = 132.
time = 0.57, size = 311, normalized size = 4.09

$$\frac{4\left((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\sin(4dx+4c)-3\cos(6dx+6c)\sin(2dx+2c)-9\cos(4dx+4c)\sin(2dx+2c)\right)}{3\left(b\cos(6dx+6c)^2+9b\cos(4dx+4c)^2+9b\cos(2dx+2c)^2+b\sin(6dx+6c)^2+9b\sin(4dx+4c)^2+18b\sin(4dx+4c)\sin(2dx+2c)+9b\sin(2dx+2c)^2+2(3b\cos(4dx+4c)+3b\cos(2dx+2c)+b)\cos(6dx+6c)+6(3b\cos(2dx+2c)+b)\cos(4dx+4c)+6b\cos(2dx+2c)+6(b\sin(4dx+4c)+b\sin(2dx+2c))\sin(6dx+6c)+b\right)\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*c
os(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x
 + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2
*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) +
b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)*d)
```

Fricas [A]
time = 0.38, size = 44, normalized size = 0.58

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(2\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{3\,b^{2}d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b^2*d*cos(d*x
 + c)^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [B]
time = 1.26, size = 131, normalized size = 1.72

$$\frac{2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6\,\sin\left(4\,c+4\,d\,x\right)+\sin\left(6\,c+6\,d\,x\right)+10{}\mathrm{i}\right)}{3\,b^2\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(3/2)),x)
```

```
[Out] (2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos
(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*d*
x) + 10i))/(3*b^2*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4
*d*x) + cos(6*c + 6*d*x) + 10))
```

3.191 $\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*sin(d*x+c)/b/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+3/8*sin(d*x+c)/b/d
/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+3/8*arctanh(sin(d*x+c))*cos(d*x+c)^(
1/2)/b/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3853, 3855}

$$\frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c + d*x]]) +
 Sin[c + d*x]/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[c +
d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^{3/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)\,dx}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \sec^3(c+d$$

$$= \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8bd\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4bd\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.57

$$\frac{3\tanh^{-1}(\sin(c+dx))\cos^4(c+dx)+(2+3\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d*
x]])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A]
time = 0.19, size = 121, normalized size = 1.04

method	result
default	$\frac{3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx-}{8d(b\cos(dx+c))^{\frac{3}{2}}\cos(dx+c)^{\frac{5}{2}}}$
risch	$-\frac{i(3e^{6i(dx+c)}+11e^{4i(dx+c)}-11e^{2i(dx+c)}-3)}{8b\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)^3 d}-\frac{3\left(\sqrt{\cos}(dx+c)\right)\ln(e^{i(dx+c)}-i)}{8b\sqrt{b\cos(dx+c)}\,d}+\frac{3\left(\sqrt{\cos}(dx+c)\right)\ln}{8b\sqrt{b\cos(dx}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1679 vs. $2(98) = 196$.
time = 0.61, size = 1679, normalized size = 14.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))))/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4
*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6
*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(
4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d
*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*
c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)*d)
```

Fricas [A]
time = 0.44, size = 233, normalized size = 2.01

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{16b^{2}d\cos\left(dx+c\right)^{5}}, -\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8b^{2}d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(b^2*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x
+ c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b
*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2
*d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(7/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos (c+d\,x)^{7/2}\,(b\,\cos (c+d\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)),x)
[Out] int(1/(cos(c + d*x)^(7/2)*(b*cos(c + d*x))^(3/2)), x)
```

3.192 $\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=107

$$\frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 3/8*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/4*cos(d*x+c)^(
7/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+3/8*x*cos(d*x+c)^(1/2)/b^2/(b*co
s(d*x+c))^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(13/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (3*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (3*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(7/2)*Sin[c
 + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{13}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \cos^2(c+dx)\,dx}{4b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int 1\,dx}{8b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{3x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b\cos(c+dx)}} + \frac{3\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 0.54

$$\frac{\sqrt{\cos(c+dx)}\,(12(c+dx)+8\sin(2(c+dx))+\sin(4(c+dx)))}{32b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(13/2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (Sqrt[Cos[c + d*x]]*(12*(c + d*x) + 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))
/(32*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.16, size = 62, normalized size = 0.58

method	result	size
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(2\sin(dx+c)(\cos^3(dx+c))+3\sin(dx+c)\cos(dx+c)+3dx+3c)}{8d(b\cos(dx+c))^{\frac{5}{2}}}$	62
risch	$\frac{3x\left(\sqrt{\cos}\,(dx+c)\right)}{8b^2\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(4dx+4c)}{32b^2\sqrt{b\cos(dx+c)}\,d} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(2dx+2c)}{4b^2\sqrt{b\cos(dx+c)}\,d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/8/d*cos(d*x+c)^(5/2)*(2*sin(d*x+c)*cos(d*x+c)^3+3*sin(d*x+c)*cos(d*x+c)+3
*d*x+3*c)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.58, size = 49, normalized size = 0.46

$$\frac{12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin\left(\frac{1}{2}\arctan(\sin(4\,dx+4\,c),\cos(4\,dx+4\,c))\right)}{32\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c)
, cos(4*d*x + 4*c))))/(b^(5/2)*d)
```

Fricas [A]
time = 0.44, size = 182, normalized size = 1.70

$$\left[\frac{2\sqrt{b\cos\left(dx+c\right)}\left(2\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-3\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-b\right)}{16\,b^{3}d},\ \frac{\sqrt{b\cos\left(dx+c\right)}\left(2\cos\left(dx+c\right)^{2}+3\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\sqrt{b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{8\,b^{3}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin
(d*x + c) - 3*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt
(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/8*(sqrt(b*cos(d*x + c
))*(2*cos(d*x + c)^2 + 3)*sqrt(cos(d*x + c))*sin(d*x + c) + 3*sqrt(b)*arcta
n(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(13/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(13/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(13/2)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.00, size = 78, normalized size = 0.73

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(8\,\sin\left(c+d\,x\right)+9\,\sin\left(3\,c+3\,d\,x\right)+\sin\left(5\,c+5\,d\,x\right)+24\,d\,x\,\cos\left(c+d\,x\right)\right)}{32\,b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(13/2)/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(8*sin(c + d*x) + 9*sin(3*c + 3*
d*x) + sin(5*c + 5*d*x) + 24*d*x*cos(c + d*x)))/(32*b^3*d*(cos(2*c + 2*d*x)
 + 1))
```

3.193 $\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}-\frac{\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)-1/3*sin(d*x+c)^3*cos
(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2713}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}-\frac{\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (Sqrt[Cos[
c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{11}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\mathrm{Subst}\left(\int (1-x^2)\,dx, x, -\sin(c+dx)\right)}{b^2 d\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2 d\sqrt{b\cos(c+dx)}} - \frac{\sqrt{\cos(c+dx)}\,\sin^3(c+dx)}{3b^2 d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.63

$$\frac{\sqrt{\cos(c+dx)}\,(5+\cos(2(c+dx)))\sin(c+dx)}{6b^2 d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (Sqrt[Cos[c + d*x]]*(5 + Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*d*Sqrt[b*Co
s[c + d*x]])
```

Maple [A]
time = 0.12, size = 40, normalized size = 0.53

method	result	size
default	$\frac{(\cos^2(dx+c)+2)\sin(dx+c)\left(\cos^{\frac{5}{2}}(dx+c)\right)}{3d(b\cos(dx+c))^{\frac{5}{2}}}$	40
risch	$\frac{3\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{4b^2 d\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(3dx+3c)}{12b^2\sqrt{b\cos(dx+c)}\,d}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/3/d*(cos(d*x+c)^2+2)*sin(d*x+c)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.60, size = 42, normalized size = 0.55

$$\frac{\sin(3\,dx+3\,c)+9\,\sin\left(\frac{1}{3}\,\arctan(\sin(3\,dx+3\,c),\cos(3\,dx+3\,c))\right)}{12\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))))/(b^(5/2)*d)
```

Fricas [**A**]
time = 0.40, size = 42, normalized size = 0.55

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(\cos\left(dx+c\right)^{2}+2\right)\sin\left(dx+c\right)}{3\,b^{3}d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(cos(d*x + c)^2 + 2)*sin(d*x + c)/(b^3*d*sqrt(cos(
d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(11/2)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**B**]
time = 0.60, size = 60, normalized size = 0.79

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(10\,\sin\left(2\,c+2\,d\,x\right)+\sin\left(4\,c+4\,d\,x\right)\right)}{12\,b^{3}\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(10*sin(2*c + 2*d*x) + sin(4*c +
 4*d*x)))/(12*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.194 $\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=69

$$\frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^(3/2)*sin(d*x+c)/b^2/d/(b*cos(d*x+c))^(1/2)+1/2*x*cos(d*x+c)
^(1/2)/b^2/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 2715, 8}

$$\frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (Cos[c + d*x]^(3/2)*S
in[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \cos^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{2b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{x\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 48, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\,(2(c+dx)+\sin(2(c+dx)))}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (Sqrt[Cos[c + d*x]]*(2*(c + d*x) + Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Cos[c
 + d*x]])
```

Maple [A]
time = 0.13, size = 42, normalized size = 0.61

method	result	size
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(\sin(dx+c)\cos(dx+c)+dx+c)}{2d(b\cos(dx+c))^{\frac{5}{2}}}$	42
risch	$\frac{x\left(\sqrt{\cos}\,(dx+c)\right)}{2b^2\sqrt{b\cos(dx+c)}} + \frac{\left(\sqrt{\cos}\,(dx+c)\right)\sin(2dx+2c)}{4b^2\sqrt{b\cos(dx+c)}\,d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/2/d*cos(d*x+c)^(5/2)*(sin(d*x+c)*cos(d*x+c)+d*x+c)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.58, size = 25, normalized size = 0.36

$$\frac{2\,dx+2\,c+\sin(2\,dx+2\,c)}{4\,b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))/(b^(5/2)*d)
```

Fricas [A]
time = 0.41, size = 157, normalized size = 2.28

$$\left[\frac{2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)-\sqrt{-b}\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)}{4b^3d}, \frac{\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sin(dx+c)}{\sqrt{b}\cos(dx+c)^{\frac{3}{2}}}\right)}{2b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - sqrt(-b)*log
(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*si
n(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c) + sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2))))/(b^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.61, size = 65, normalized size = 0.94

$$\frac{\sqrt{\cos(c+d\,x)}\,\sqrt{b\,\cos(c+d\,x)}\,\left(\sin(c+d\,x)+\sin(3\,c+3\,d\,x)+4\,d\,x\,\cos(c+d\,x)\right)}{4\,b^3\,d\,\left(\cos(2\,c+2\,d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(sin(c + d*x) + sin(3*c + 3*d*x)
 + 4*d*x*cos(c + d*x)))/(4*b^3*d*(cos(2*c + 2*d*x) + 1))
```

3.195 $\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

```
[Out] sin(d*x+c)*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 2717}

$$\frac{\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{7}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\,\int \cos(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 35, normalized size = 1.00

$$\frac{\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.83

method	result	size
default	$\frac{\sin(dx+c)\left(\cos^{\frac{5}{2}}(dx+c)\right)}{d(b\cos(dx+c))^{\frac{5}{2}}}$	29
risch	$\frac{\sin(dx+c)\left(\sqrt{\cos}\,(dx+c)\right)}{b^2 d\sqrt{b\cos\left(dx+c\right)}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*sin(d*x+c)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [A]
time = 0.58, size = 13, normalized size = 0.37

$$\frac{\sin\left(dx+c\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] sin(d*x + c)/(b^(5/2)*d)
```

Fricas [A]
time = 0.39, size = 31, normalized size = 0.89

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{b^3d\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.41, size = 47, normalized size = 1.34

$$\frac{\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(2\,c+2\,d\,x\right)\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b^3\,d\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (cos(c + d*x)^(1/2)*sin(2*c + 2*d*x)*(b*cos(c + d*x))^(1/2))/(b^3*d*(cos(2*
c + 2*d*x) + 1))
```

3.196 $\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=27

$$\frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

```
[Out] x*cos(d*x+c)^(1/2)/b^2/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = $\{17, 8\}$

$$\frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int 1\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{x\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 24, normalized size = 0.89

$$\frac{x\cos^{\frac{5}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)/(b*Cos[c + d*x])^(5/2),x]
```

```
[Out] (x*Cos[c + d*x]^(5/2))/(b*Cos[c + d*x])^(5/2)
```

Maple [A]
time = 0.10, size = 28, normalized size = 1.04

method	result	size
risch	$\frac{x\left(\sqrt{\cos}\,(dx+c)\right)}{b^{2}\sqrt{b\cos\left(dx+c\right)}}$	24
default	$\frac{\left(\cos^{\frac{5}{2}}(dx+c)\right)(dx+c)}{d(b\cos(dx+c))^{\frac{5}{2}}}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)*(d*x+c)
```

Maxima [A]
time = 0.57, size = 26, normalized size = 0.96

$$\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A]
time = 0.39, size = 97, normalized size = 3.59

$$\left[-\frac{\sqrt{-b}\log\left(2\,b\cos\left(dx+c\right)^{2}+2\,\sqrt{b\cos\left(dx+c\right)}\,\sqrt{-b}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-b\right)}{2\,b^{3}d},\ \frac{\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin(dx+c)}{\sqrt{b}\,\cos(dx+c)^{\frac{3}{2}}}\right)}{b^{\frac{5}{2}}d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqr
t(cos(d*x + c))*sin(d*x + c) - b)/(b^3*d), arctan(sqrt(b*cos(d*x + c))*sin(
d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))/(b^(5/2)*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [B]
time = 0.28, size = 37, normalized size = 1.37

$$\frac{2\,x\cos\left(c+d\,x\right)^{3/2}\,\sqrt{b\,\cos\left(c+d\,x\right)}}{b^3\,\left(\cos\left(2\,c+2\,d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)/(b*cos(c + d*x))^(5/2),x)
[Out] (2*x*cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(1/2))/(b^3*(cos(2*c + 2*d*x) + 1)
)
```

3.197 $\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

[Out] arctanh(sin(d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {17, 3855}

$$\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(5/2),x]

[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^{\frac{3}{2}}(c+dx)}{(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 0.92

$$\frac{\tanh^{-1}(\sin(c+dx))\cos^{\frac{5}{2}}(c+dx)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)/(b*Cos[c + d*x])^(5/2),x]
[Out] (ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^(5/2))/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 1.17

method	result	size
default	$-\frac{2\left(\cos^{\frac{5}{2}}(dx+c)\right)\operatorname{arctanh}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right)}{d(b\cos(dx+c))^{\frac{5}{2}}}$	42
risch	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}+i\right)}{b^2\sqrt{b\cos(dx+c)}\,d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}-i\right)}{b^2\sqrt{b\cos(dx+c)}\,d}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] -2/d*cos(d*x+c)^(5/2)*arctanh((-1+cos(d*x+c))/sin(d*x+c))/(b*cos(d*x+c))^(5
/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(32) = 64$.
time = 0.60, size = 65, normalized size = 1.81

$$\frac{\log\left(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1\right)-\log\left(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1\right)}{2b^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
[Out] 1/2*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*
x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/(b^(5/2)*d)
```

Fricas [A]
time = 0.42, size = 116, normalized size = 3.22

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)}{2b^{\frac{5}{2}}d}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)}{b^3d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3)/(b^(5/2)*d), -sqrt(-b
)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))
/(b^3*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)/(b*cos(c + d*x))^(5/2), x)
```

3.198 $\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

[Out] sin(d*x+c)/b^2/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)

Rubi [A]
time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {17, 3852, 8}

$$\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(5/2),x]

[Out] Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m + 1/2
)*b^(n - 1/2)*(Sqrt[b*v]/Sqrt[a*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}}{(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^2(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{b^2 d\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{b^2 d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}}$$

Mathematica [A]
time = 0.01, size = 32, normalized size = 0.91

$$\frac{\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]/(b*Cos[c + d*x])^(5/2),x]
[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.83

method	result	size
default	$\frac{\left(\cos^{\frac{3}{2}}(dx+c)\right)\sin(dx+c)}{d(b\cos(dx+c))^{\frac{5}{2}}}$	29
risch	$\frac{2i\left(\sqrt{\cos}\,(dx+c)\right)}{b^2\sqrt{b\cos\left(dx+c\right)}\,d\left(e^{2i(dx+c)}+1\right)}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
[Out] 1/d*cos(d*x+c)^(3/2)*sin(d*x+c)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. 2(31) = 62.
time = 0.56, size = 67, normalized size = 1.91

$$\frac{2\sqrt{b}\,\sin\left(2\,dx+2\,c\right)}{\left(b^3\cos\left(2\,dx+2\,c\right)^2+b^3\sin\left(2\,dx+2\,c\right)^2+2\,b^3\cos\left(2\,dx+2\,c\right)+b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b)*sin(2*d*x + 2*c)/((b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^
2 + 2*b^3*cos(2*d*x + 2*c) + b^3)*d)
```

Fricas [A]
time = 0.40, size = 31, normalized size = 0.89

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{b^{3}d\cos\left(dx+c\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^(3/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x)
```

Mupad [B]
time = 1.07, size = 87, normalized size = 2.49

$$\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\sin\left(c+d\,x\right)+\sin\left(3\,c+3\,d\,x\right)+\cos\left(c+d\,x\right)\,3\mathrm{i}+\cos\left(3\,c+3\,d\,x\right)\,1\mathrm{i}\right)}{b^3\,d\,\left(4\,\cos\left(2\,c+2\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)/(b*cos(c + d*x))^(5/2),x)
```

```
[Out] (2*cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(1/2)*(cos(c + d*x)*3i + sin(c + d*x
) + cos(3*c + 3*d*x)*1i + sin(3*c + 3*d*x)))/(b^3*d*(4*cos(2*c + 2*d*x) + c
os(4*c + 4*d*x) + 3))
```

3.199 $\int \frac{1}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/2*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+1/2*arctanh(sin(
d*x+c))*cos(d*x+c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3853, 3855}

$$\frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) +
 Sin[c + d*x]/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^3(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)}\int \sec(c+dx)\,d}{2b^2\sqrt{b\cos(c+dx)}}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{2b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+}$$

Mathematica [A]
time = 0.03, size = 55, normalized size = 0.71

$$\frac{\sqrt{b\cos(c+dx)}\left(\tanh^{-1}(\sin(c+dx))\cos^2(c+dx)+\sin(c+dx)\right)}{2b^3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + Sin[c + d*x])
)/(2*b^3*d*Cos[c + d*x]^(5/2))
```

Maple [A]
time = 0.16, size = 104, normalized size = 1.33

method	result
default	$-\frac{\left(\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)-\left(\cos^2(dx+c)\right)\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-\sin(dx+c)\right)\left(\sqrt{\cos}\,(dx+c)\right)}{2d(b\cos(dx+c))^{\frac{5}{2}}}$
risch	$-\frac{i\left(e^{2i(dx+c)}-1\right)}{2b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\left(e^{2i(dx+c)}+1\right)d}-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln\left(e^{i(dx+c)}-i\right)}{2b^2\sqrt{b\cos(dx+c)}\,d}+\frac{\left(\sqrt{\cos}\,(dx+c)\right)\ln(e^{i(}}{2b^2\sqrt{b\cos(dx+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*(cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-cos(d*x+c)^
2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-sin(d*x+c))*cos(d*x+c)^(1/2)/(
b*cos(d*x+c))^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 688 vs. 2(66) = 132.
time = 0.59, size = 688, normalized size = 8.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
 + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*
cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d
*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 +
4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x +
2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d
*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b)*d)
```

Fricas [A]
time = 0.42, size = 207, normalized size = 2.65

$$\left[\frac{\sqrt{b}\cos\left(dx+c\right)^{3}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4b^{3}d\cos\left(dx+c\right)^{3}}, -\frac{\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{3}-\sqrt{b\cos\left(dx+c\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{2b^{3}d\cos\left(dx+c\right)^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c)
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x
 + c)^3), -1/2*(sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/
(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c))/(b^3*d*cos(d*x + c)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(5/2)*sqrt(cos(d*x + c))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] int(1/(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.200 $\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/3*sin(d*x+c)^3/b^2/d/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2)+sin(d*x+c)/b^2
/d/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {18, 3852}

$$\frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] Sin[c + d*x]/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + Sin[c + d*x]
^3/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx &= \frac{\sqrt{\cos(c+dx)}\int \sec^4(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}\\ &= -\frac{\sqrt{\cos(c+dx)}\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{b^2d\sqrt{b\cos(c+dx)}}\\ &= \frac{\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\cos(c+dx)}}+\frac{\sin^3(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 0.59

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(Tan[c + d*x] + Tan[c + d*x]^3/3))/(d*(b*Cos[c + d*x])^
(5/2))
```

Maple [A]
time = 0.12, size = 42, normalized size = 0.55

method	result	size
default	$\frac{\sin(dx+c)\left(2\left(\cos^2(dx+c)\right)+1\right)}{3d(b\cos(dx+c))^{\frac{5}{2}}\sqrt{\cos\left(dx+c\right)}}$	42
risch	$\frac{2i(4\cos(dx+c)+2i\sin(dx+c))}{3b^2\sqrt{b\cos\left(dx+c\right)}\,\sqrt{\cos\left(dx+c\right)}\,\left(e^{2i(dx+c)}+1\right)^2 d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/d*sin(d*x+c)*(2*cos(d*x+c)^2+1)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(66) = 132$.
time = 0.57, size = 343, normalized size = 4.51

$$\frac{4\left((3\cos(2dx+2c)+1)\sin(6dx+6c)+3(3\cos(2dx+2c)+1)\sin(4dx+4c)-3\cos(6dx+6c)\sin(2dx+2c)-9\cos(4dx+4c)\sin(2dx+2c)\right)}{3\left(b^2\cos(6dx+6c)^2+9b^2\cos(4dx+4c)^2+9b^2\cos(2dx+2c)^2+b^2\sin(6dx+6c)^2+9b^2\sin(4dx+4c)^2+18b^2\sin(4dx+4c)\sin(2dx+2c)+9b^2\sin(2dx+2c)^2+6b^2\cos(2dx+2c)+b^2+2(3b^2\cos(4dx+4c)+3b^2\cos(2dx+2c)+b^2)\cos(6dx+6c)+6(3b^2\cos(2dx+2c)+b^2)\cos(4dx+4c)+6(b^2\sin(4dx+4c)+b^2\sin(2dx+2c))\sin(6dx+6c)\right)\sqrt{b}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 4/3*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)
*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c
)*sin(2*d*x + 2*c))/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9
*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2
 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x +
2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*s
qrt(b)*d)
```

Fricas [A]
time = 0.38, size = 44, normalized size = 0.58

$$\frac{\sqrt{b\cos\left(dx+c\right)}\,\left(2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{3\,b^{3}d\cos\left(dx+c\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(b*cos(d*x + c))*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(b^3*d*cos(d*x
 + c)^(7/2))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)), x)
```

Mupad [B]
time = 1.29, size = 131, normalized size = 1.72

$$\frac{2\,\sqrt{b\,\cos\left(c+d\,x\right)}\,\left(\cos\left(2\,c+2\,d\,x\right)\,15{}\mathrm{i}+\cos\left(4\,c+4\,d\,x\right)\,6{}\mathrm{i}+\cos\left(6\,c+6\,d\,x\right)\,1{}\mathrm{i}+9\,\sin\left(2\,c+2\,d\,x\right)+6\,\sin\left(4\,c+4\,d\,x\right)+\sin\left(6\,c+6\,d\,x\right)+10{}\mathrm{i}\right)}{3\,b^{3}\,d\,\sqrt{\cos\left(c+d\,x\right)}\,\left(15\,\cos\left(2\,c+2\,d\,x\right)+6\,\cos\left(4\,c+4\,d\,x\right)+\cos\left(6\,c+6\,d\,x\right)+10\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^(5/2)),x)
```

```
[Out] (2*(b*cos(c + d*x))^(1/2)*(cos(2*c + 2*d*x)*15i + cos(4*c + 4*d*x)*6i + cos
(6*c + 6*d*x)*1i + 9*sin(2*c + 2*d*x) + 6*sin(4*c + 4*d*x) + sin(6*c + 6*d*
x) + 10i))/(3*b^3*d*cos(c + d*x)^(1/2)*(15*cos(2*c + 2*d*x) + 6*cos(4*c + 4
*d*x) + cos(6*c + 6*d*x) + 10))
```

3.201 $\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=116

$$\frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

```
[Out] 1/4*sin(d*x+c)/b^2/d/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)+3/8*sin(d*x+c)/b
^2/d/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)+3/8*arctanh(sin(d*x+c))*cos(d*x+
c)^(1/2)/b^2/d/(b*cos(d*x+c))^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {18, 3853, 3855}

$$\frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}}+\frac{3\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[c + d*x]])
 + Sin[c + d*x]/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + (3*Sin[
c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])
```

Rule 18

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[a^(m - 1/2
)*b^(n + 1/2)*(Sqrt[a*v]/Sqrt[b*v]), Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] &&  !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^{5/2}}\,dx = \frac{\sqrt{\cos(c+dx)}\int \sec^5(c+dx)\,dx}{b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{\left(3\sqrt{\cos(c+dx)}\right)\int \sec^3(c+$$

$$= \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{3\sin(c+dx)}{8b^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+}$$

$$= \frac{3\tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{8b^2d\sqrt{b\cos(c+dx)}} + \frac{\sin(c+dx)}{4b^2d\cos^{\frac{7}{2}}(c+dx)\sqrt{b\cos(c-}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.57

$$\frac{3\tanh^{-1}(\sin(c+dx))\cos^4(c+dx) + (2+3\cos^2(c+dx))\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2 + 3*Cos[c + d*x]^2)*Sin[c + d*
x]])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A]
time = 0.19, size = 121, normalized size = 1.04

method	result
default	$\frac{3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)-\sin(dx+c)}{\sin(dx+c)}\right)-3(\cos^4(dx+c))\ln\left(-\frac{-1+\cos(dx+c)+\sin(dx+c)}{\sin(dx+c)}\right)+3(\cos^2(dx+c))\sin(dx+c)+2\sin(dx-}{8d(b\cos(dx+c))^{\frac{5}{2}}\cos(dx+c)^{\frac{3}{2}}}$
risch	$-\frac{i(3e^{6i(dx+c)}+11e^{4i(dx+c)}-11e^{2i(dx+c)}-3)}{8b^2\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}(e^{2i(dx+c)}+1)^3d} + \frac{3(\sqrt{\cos}(dx+c))\ln(e^{i(dx+c)}+i)}{8b^2\sqrt{b\cos(dx+c)}\,d} - \frac{3(\sqrt{\cos}(dx+c))}{8b^2\sqrt{b\cos(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d*(3*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-3*cos(d*x+
c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*cos(d*x+c)^2*sin(d*x+c)+2
*sin(d*x+c))/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. $2(98) = 196$.

time = 0.63, size = 1729, normalized size = 14.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/16*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*s
in(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*
(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x +
 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
 + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^
2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*c
os(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4
*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x
 + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3
*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6
*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*s
in(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 3
*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(
8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2
*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(
2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)
*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(
4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c)
 + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
```

```
c))))/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d
*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*s
in(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 +
2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c)
 + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*
c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x +
4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c))*sqrt(b)*d)
```

Fricas [**A**]
time = 0.41, size = 233, normalized size = 2.01

$$\left[\frac{3\sqrt{b}\cos\left(dx+c\right)^{5}\log\left(-\frac{b\cos(dx+c)^{3}-2\sqrt{b\cos\left(dx+c\right)}\sqrt{b}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^{3}}\right)+2\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{16\,b^{3}d\cos\left(dx+c\right)^{5}},-\frac{3\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos\left(dx+c\right)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos\left(dx+c\right)}}\right)\cos\left(dx+c\right)^{5}-\sqrt{b\cos\left(dx+c\right)}\left(3\cos\left(dx+c\right)^{2}+2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8\,b^{3}d\cos\left(dx+c\right)^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x +
 c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x +
c)^3) + 2*sqrt(b*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*si
n(d*x + c))/(b^3*d*cos(d*x + c)^5), -1/8*(3*sqrt(-b)*arctan(sqrt(b*cos(d*x
+ c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - sqrt(b
*cos(d*x + c))*(3*cos(d*x + c)^2 + 2)*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3
*d*cos(d*x + c)^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(5/2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(c+d\,x\right)^{5/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)),x)
[Out] int(1/(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^(5/2)), x)
```

3.202 $\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(4+3m),\frac{1}{6}(10+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(4+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 2/3+1/2*m],[5/3+1/
2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(4+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+4);\frac{1}{6}(3m+10);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (4 +
 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx = \frac{\sqrt[3]{b\cos(c+dx)}\int \cos^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}$$
$$= -\frac{3\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{1}{6}(10+3m);\cos^2\right.}{d(4+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 1.00

$$-\frac{\cos^{1+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{4}{3}+m\right),\frac{1}{2}\left(\frac{10}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{4}{3}+m\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*Hypergeometric2
F1[1/2, (4/3 + m)/2, (10/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d
*(4/3 + m)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos\left(c+dx\right)}\cos^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3),x)
[Out] Integral((b*cos(c + d*x))**(1/3)*cos(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(1/3), x)
```

3.203 $\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/10*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*
x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(10*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx &= \frac{\int (b\cos(c+dx))^{7/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 1.09

$$-\frac{3\cos^2(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 5/3, 8/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(10*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3), x)
```

3.204 $\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x
+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)\sqrt[3]{b\cos(c+dx)}\,dx &= \frac{\int (b\cos(c+dx))^{4/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 1.00

$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3),x]
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*b*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x)
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos(c+dx)}\,\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(1/3), x)
```

3.205 $\int \sqrt[3]{b\cos(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt[3]{b\cos(c+dx)}\,dx = -\frac{3(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.95

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\,\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(d x +c \right)\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b \cos{\left(c + d x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\cos(c+d\,x))^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/3),x)
[Out] int((b*cos(c + d*x))^(1/3), x)
```

3.206 $\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx$

Optimal. Leaf size=53

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)
/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x],x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx &= b\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\ &= -\frac{3\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.02

$$-\frac{3b\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x],x]
```

```
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{1}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos(c+dx)}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{1/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/3)/cos(c + d*x),x)
```

```
[Out] int((b*cos(c + d*x))^(1/3)/cos(c + d*x), x)
```

3.207 $\int \sqrt[3]{b\cos(c+dx)}\,\sec^2(c+dx)\,dx$

Optimal. Leaf size=56

$$\frac{3b\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*b*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))
^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}\,\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 56, normalized size = 1.00

$$\frac{3b\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}} \left(\sec^2\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos\left(c+dx\right)}\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c)**2,x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,\cos\left(c+d\,x\right))^{1/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^2,x)
```

```
[Out] int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^2, x)
```

3.208 $\int \sqrt[3]{b\cos(c+dx)}\,\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*b^2*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(5/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d
*(b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{b\cos(c+dx)}\,\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.09

$$\frac{3\sqrt[3]{b\cos(c+dx)}\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(1/3)*Sec[c + d*x]^3,x]
[Out] (3*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Co
s[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{1}{3}}\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x)
[Out] int((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\cos(c+dx)}\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*sec(d*x+c)**3,x)
```

```
[Out] Integral((b*cos(c + d*x))**(1/3)*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^3,x)
```

```
[Out] int((b*cos(c + d*x))^(1/3)/cos(c + d*x)^3, x)
```

3.209 $\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=82

$$-\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(5+3m),\frac{1}{6}(11+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*cos(d*x+c)^(1+m)*(b*cos(d*x+c))^(2/3)*hypergeom([1/2, 5/6+1/2*m],[11/6+1
/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(5+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+5);\frac{1}{6}(3m+11);\cos^2(c+dx)\right)}{d(3m+5)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5 +
 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*Sqrt[Sin[
c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{2/3}\,dx = \frac{(b\cos(c+dx))^{2/3}\int \cos^{\frac{2}{3}+m}(c+dx)\,dx}{\cos^{\frac{2}{3}}(c+dx)}$$
$$= -\frac{3\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5+3m);\frac{1}{6}(11+3m);\right.}{d(5+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 1.00

$$-\frac{\cos^{1+m}(c+dx)(b\cos(c+dx))^{2/3}\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{5}{3}+m\right),\frac{1}{2}\left(\frac{11}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{5}{3}+m\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*Hypergeometric2
F1[1/2, (5/3 + m)/2, (11/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d
*(5/3 + m)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(b\cos(dx+c)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos{(c+dx)})^{\frac{2}{3}}\cos^m{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3),x)
[Out] Integral((b*cos(c + d*x))**(2/3)*cos(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(2/3), x)
```

3.210 $\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{11/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/11*(b*cos(d*x+c))^(11/3)*hypergeom([1/2, 11/6],[17/6],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)}{11b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\cos(c+dx))^{2/3}\,dx &= \frac{\int (b\cos(c+dx))^{8/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{11/3}\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\cos^2(c+dx)\right)\sin(c+dx)}{11b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 1.09

$$-\frac{3\cos^2(c+dx)(b\cos(c+dx))^{2/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{11}{6},\frac{17}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{11d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 11/6, 17/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(11*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3), x)
```

3.211 $\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/8*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{2/3}\,dx &= \frac{\int (b\cos(c+dx))^{5/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.03, size = 58, normalized size = 1.00

$$-\frac{3(b\cos(c+dx))^{5/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(8*b*d)
```

Maple [**F**]
time = 0.04, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(2/3), x)
```

3.212 $\int (b\cos(c+dx))^{2/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{2/3}\,dx = -\frac{3(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.95

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*d)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(d x +c \right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \cos \left(c + d x\right)\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\cos(c+d\,x))^{2/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(2/3),x)
[Out] int((b*cos(c + d*x))^(2/3), x)
```

3.213 $\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx$

Optimal. Leaf size=55

$$-\frac{3(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x],x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{2/3}\sec(c+dx)\,dx &= b\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx\\ &= -\frac{3(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.02

$$-\frac{3b\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x],x]
[Out] (-3*b*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\sec(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*sec(d*x+c),x)
[Out] int((b*cos(d*x+c))^(2/3)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^{\frac{2}{3}}\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**(2/3)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(2/3)/cos(c + d*x),x)
```

```
[Out] int((b*cos(c + d*x))^(2/3)/cos(c + d*x), x)
```

3.214 $\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx$

Optimal. Leaf size=54

$$\frac{3b\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*b*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(
1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{2/3}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\,\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 54, normalized size = 1.00

$$\frac{3b\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^2,x]
```

```
[Out] (3*b*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(\sec^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c)**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^2,x)
```

```
[Out] int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^2, x)
```

3.215 $\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*b^2*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d
*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{2/3}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.09

$$\frac{3(b\cos(c+dx))^{2/3}\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*Sec[c + d*x]^3,x]
[Out] (3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Co
s[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{2}{3}}\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x)
[Out] int((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*sec(d*x+c)**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{2/3}}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^3,x)
```

```
[Out] int((b*cos(c + d*x))^(2/3)/cos(c + d*x)^3, x)
```

3.216 $\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=83

$$-\frac{3b\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(7+3m),\frac{1}{6}(13+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*b*cos(d*x+c)^(2+m)*(b*cos(d*x+c))^(1/3)*hypergeom([1/2, 7/6+1/2*m],[13/6
+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(7+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,\cos^{m+2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+7);\frac{1}{6}(3m+13);\cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*b*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7
 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Si
n[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(b\cos(c+dx))^{4/3}\,dx = \frac{\left(b\sqrt[3]{b\cos(c+dx)}\right)\int \cos^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= -\frac{3b\cos^{2+m}(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\mathrm{co}\right.}{d(7+3m)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.99

$$-\frac{\cos^{1+m}(c+dx)(b\cos(c+dx))^{4/3}\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{7}{3}+m\right),\frac{1}{2}\left(\frac{13}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{7}{3}+m\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3),x]
[Out] -((Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*Hypergeometric2
F1[1/2, (7/3 + m)/2, (13/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d
*(7/3 + m)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (\cos^m(dx+c))\,(b\cos(dx+c))^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x)
[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^m*cos(d*x + c), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^m\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)^m*(b*cos(c + d*x))^(4/3), x)
```

3.217 $\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{13/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/13*(b*cos(d*x+c))^(13/3)*hypergeom([1/2, 13/6],[19/6],cos(d*x+c)^2)*sin(
d*x+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)}{13b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]
^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\cos(c+dx))^{4/3}\,dx &= \frac{\int (b\cos(c+dx))^{10/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{13/3}\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\cos^2(c+dx)\right)\sin(c+dx)}{13b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 1.09

$$-\frac{3\cos^2(c+dx)(b\cos(c+dx))^{4/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{13}{6},\frac{19}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{13d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/
2, 13/6, 19/6, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(13*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3), x)
```

3.218 $\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{10/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/10*(b*cos(d*x+c))^(10/3)*hypergeom([1/2, 5/3],[8/3],cos(d*x+c)^2)*sin(d*
x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2
]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^{4/3}\,dx &= \frac{\int (b\cos(c+dx))^{7/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{10/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.00

$$-\frac{3(b\cos(c+dx))^{7/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{3},\frac{8}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{10bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(10*b*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \cos(dx+c)(b\cos(dx+c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)^2, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^(4/3), x)
```

3.219 $\int (b\cos(c+dx))^{4/3}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x
+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^{4/3}\,dx = -\frac{3(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 0.95

$$-\frac{3(b\cos(c+dx))^{4/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*d)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos (dx + c))^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int (b\cos(c+d\,x))^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(4/3),x)
```

```
[Out] int((b*cos(c + d*x))^(4/3), x)
```

3.220 $\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx$

Optimal. Leaf size=55

$$-\frac{3(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+
c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x],x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\sec(c+dx)\,dx &= b\int \sqrt[3]{b\cos(c+dx)}\,dx\\ &= -\frac{3(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 56, normalized size = 1.02

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x],x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(4/3)/cos(c + d*x),x)
```

```
[Out] int((b*cos(c + d*x))^(4/3)/cos(c + d*x), x)
```

3.221 $\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx$

Optimal. Leaf size=54

$$-\frac{3b\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*b*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+
c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3b\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^
2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\sec^2(c+dx)\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx\\ &= -\frac{3b\sqrt[3]{b\cos(c+dx)}\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 56, normalized size = 1.04

$$-\frac{3b^2\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b^2*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}} \left(\sec^2(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c)^2, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^{4/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^2,x)
```

```
[Out] int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^2, x)
```

3.222 $\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*b^2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c
))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d
*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^{4/3}\sec^3(c+dx)\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.00

$$\frac{3b^2\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*Sec[c + d*x]^3,x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^{\frac{4}{3}}\left(\sec^3(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*b*cos(d*x + c)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(b\,\cos(c+d\,x))^{4/3}}{\cos(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^3,x)
```

```
[Out] int((b*cos(c + d*x))^(4/3)/cos(c + d*x)^3, x)
```

3.223 $\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\cos^{1+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(2+3m),\frac{1}{6}(8+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/3+1/2*m],[4/3+1/2*m],cos(d*x+c)^2)*si
n(d*x+c)/d/(2+3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+2);\frac{1}{6}(3m+8);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c
 + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\cos(c+dx)}}$$
$$= -\frac{3\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(2+3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 1.00

$$-\frac{\cos^{1+m}(c+dx)\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{2}{3}+m\right),\frac{1}{2}\left(\frac{8}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{2}{3}+m\right)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (2/3 + m)/2, (8
/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(2/3 + m)*(b*Cos[c + d*
x])^(1/3)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{m}\left(c+dx\right)}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^m}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)^m/(b*cos(c + d*x))^(1/3), x)
```

3.224 $\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{8/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/8*(b*cos(d*x+c))^(8/3)*hypergeom([1/2, 4/3],[7/3],cos(d*x+c)^2)*sin(d*x+
c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{5/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{8/3}\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 63, normalized size = 1.09

$$-\frac{3\cos^2(c+dx)\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{4}{3},\frac{7}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{8d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*
x]^2]*Sqrt[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
[Out] int(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)**2/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^2}{\left(b\,\cos\left(c+d\,x\right)\right)^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)^2/(b*cos(c + d*x))^(1/3), x)
```

3.225 $\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x
+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= \frac{\int (b\cos(c+dx))^{2/3}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 1.00

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*b*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(cos(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b*cos(c + d*x))^(1/3),x)
```

```
[Out] int(cos(c + d*x)/(b*cos(c + d*x))^(1/3), x)
```

3.226 $\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx = -\frac{3(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.00, size = 55, normalized size = 0.95

$$-\frac{3\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-1/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\cos\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(1/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{b\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(-1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(-1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(1/3),x)
[Out] int(1/(b*cos(c + d*x))^(1/3), x)
```

3.227 $\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=53

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(1/
3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Co
s[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 1.02

$$\frac{3b\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(1/3),x]
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
[Out] int(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(1/3)), x)
```

3.228 $\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=56

$$\frac{3b\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*b*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))
^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = $\{16, 2722\}$

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 1.04

$$\frac{3b^2\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(1/3)), x)
```

3.229 $\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*b^2*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.00

$$\frac{3b^2 \csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(1/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{\sqrt[3]{b\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(1/3)), x)
```

3.230 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=82

$$-\frac{3\cos^{1+m}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(1+3m),\frac{1}{6}(7+3m),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*cos(d*x+c)^(1+m)*hypergeom([1/2, 1/6+1/2*m],[7/6+1/2*m],cos(d*x+c)^2)*si
n(d*x+c)/d/(1+3*m)/(b*cos(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{3\sin(c+dx)\cos^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m+1);\frac{1}{6}(3m+7);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, C
os[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c
 + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx = \frac{\cos^{\frac{2}{3}}(c+dx)\int \cos^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\cos(c+dx))^{2/3}}$$
$$= -\frac{3\cos^{1+m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\cos^2(c+dx)\right)\sin(c+dx)}{d(1+3m)(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 1.00

$$-\frac{\cos^{1+m}(c+dx)\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{3}+m\right),\frac{1}{2}\left(\frac{7}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{1}{3}+m\right)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(2/3),x]
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (1/3 + m)/2, (7
/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1/3 + m)*(b*Cos[c + d*
x])^(2/3)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x)
[Out] int(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{m}\left(c + dx\right)}{\left(b\cos\left(c + dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^m}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^m/(b*cos(c + d*x))^(2/3), x)
```

3.231 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{7/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*(b*cos(d*x+c))^(7/3)*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x
+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(7*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= \frac{\int (b\cos(c+dx))^{4/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{7/3}\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 63, normalized size = 1.09

$$-\frac{3\cos^2(c+dx)\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{13}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d
*x]^2]*Sqrt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*cos(d*x + c)/b, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)^2/(b*cos(c + d*x))^(2/3), x)
```

3.232 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{4/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*(b*cos(d*x+c))^(4/3)*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(4*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= \frac{\int \sqrt[3]{b\cos(c+dx)}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{4/3}\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 1.00

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\,\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2}{3},\frac{5}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Co
s[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(4*b*d)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x)
[Out] int(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(cos(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b*cos(c + d*x))^(2/3),x)
```

```
[Out] int(cos(c + d*x)/(b*cos(c + d*x))^(2/3), x)
```

3.233 $\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$-\frac{3\sqrt[3]{b\cos(c+dx)}\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*(b*cos(d*x+c))^(1/3)*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)
/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2722}

$$-\frac{3\sin(c+dx)\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-2/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{2/3}}\,dx = -\frac{3\sqrt[3]{b\cos(c+dx)}\ {}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 0.95

$$-\frac{3\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-2/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(d*(b*Cos[c + d*x])^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos \left(dx+c\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(1/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos \left(c+dx\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(-2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(-2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(2/3),x)
[Out] int(1/(b*cos(c + d*x))^(2/3), x)
```

3.234 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=55

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(
2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*
Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{5/3}}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\cos(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 1.02

$$\frac{3b\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{1}{2},\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2d(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(2*d*(b*Cos[c + d*x])^(5/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*cos(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(2/3)), x)
```

3.235 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3b\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*b*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))
^(5/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*(
b*Cos[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{8/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\cos(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 58, normalized size = 1.04

$$\frac{3b^2\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{6},\frac{1}{2},\frac{1}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(2/3),x]
[Out] (3*b^2*Cot[c + d*x]*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*Sqrt[
Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(8/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(2/3)), x)
```

3.236 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/8*b^2*hypergeom([-4/3, 1/2],[-1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+
c))^(8/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(2/3),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*
d*(b*Cos[c + d*x])^(8/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{2/3}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{11/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{8d(b\cos(c+dx))^{8/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 58, normalized size = 1.00

$$\frac{3b^2\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{4}{3},\frac{1}{2},-\frac{1}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{8d(b\cos(c+dx))^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(2/3),x]
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-4/3, 1/2, -1/3, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(8/3))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
[Out] int(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(2/3)), x)
```

3.237 $\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=83

$$\frac{3\cos^m(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{6}(-1+3m),\frac{1}{6}(5+3m),\cos^2(c+dx)\right)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*cos(d*x+c)^m*hypergeom([1/2, -1/6+1/2*m],[5/6+1/2*m],cos(d*x+c)^2)*sin(d*
x+c)/b/d/(1-3*m)/(b*cos(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$\frac{3\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(3m-1);\frac{1}{6}(3m+5);\cos^2(c+dx)\right)}{bd(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c +
 d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d
*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\cos(c+dx)}\int \cos^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\cos(c+dx)}}$$
$$= \frac{3\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\cos^2(c+dx)\right)\sin(c+dx)}{bd(1-3m)\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 82, normalized size = 0.99

$$-\frac{\cos^{1+m}(c+dx)\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{3}+m\right),\frac{1}{2}\left(\frac{5}{3}+m\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{1}{3}+m\right)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(b*Cos[c + d*x])^(4/3),x]
[Out] -((Cos[c + d*x]^(1 + m)*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/3 + m)/2, (
5/3 + m)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1/3 + m)*(b*Cos[c +
d*x])^(4/3)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x)
[Out] int(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{m}(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral(cos(c + d*x)**m/(b*cos(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^m}{(b\,\cos(c+d\,x))^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)^m/(b*cos(c + d*x))^(4/3), x)
```

3.238 $\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{5/3}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*(b*cos(d*x+c))^(5/3)*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x
+c)/b^3/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2
]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\int (b\cos(c+dx))^{2/3}\,dx}{b^2}\\ &= -\frac{3(b\cos(c+dx))^{5/3}\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5b^3d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 58, normalized size = 1.00

$$-\frac{3(b\cos(c+dx))^{2/3}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{5b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, C
os[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(5*b^2*d)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)^2/(b*cos(c + d*x))^(4/3), x)
```

3.239 $\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$-\frac{3(b\cos(c+dx))^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*(b*cos(d*x+c))^(2/3)*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+
c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{3\sin(c+dx)(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]
*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= \frac{\int \frac{1}{\sqrt[3]{b\cos(c+dx)}}\,dx}{b}\\ &= -\frac{3(b\cos(c+dx))^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 58, normalized size = 1.00

$$-\frac{3\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{2bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (-3*Cot[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(2*b*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral(cos(c + d*x)/(b*cos(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(c+d\,x\right)}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(b*cos(c + d*x))^(4/3),x)
```

```
[Out] int(cos(c + d*x)/(b*cos(c + d*x))^(4/3), x)
```

3.240 $\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*cos(d*x+c))^(
1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{2722\}$

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*
Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{1}{(b\cos(c+dx))^{4/3}}\,dx = \frac{3\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.00, size = 53, normalized size = 0.95

$$\frac{3\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{6},\frac{1}{2},\frac{5}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(-4/3),x]
```

```
[Out] (3*Cot[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sqrt[Sin[
c + d*x]^2])/(d*(b*Cos[c + d*x])^(4/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos \left(dx+c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(1/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^(-4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b \cos \left(c+dx\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral((b*cos(c + d*x))**(-4/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^(-4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(c + d*x))^(4/3),x)
[Out] int(1/(b*cos(c + d*x))^(4/3), x)
```

3.241 $\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=55

$$\frac{3\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c))^(
4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$\frac{3\sin(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*
Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b\int \frac{1}{(b\cos(c+dx))^{7/3}}\,dx\\ &= \frac{3\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\cos(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.02

$$\frac{3b\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{1}{2},\frac{1}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{4d(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Cot[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sqrt[Si
n[c + d*x]^2])/(4*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)}{\left(b\cos\left(dx+c\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(c+dx\right)}{\left(b\cos\left(c+dx\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)/(b*cos(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int(1/(cos(c + d*x)*(b*cos(c + d*x))^(4/3)), x)
```

3.242 $\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=56

$$\frac{3b\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*b*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x+c)
)^(7/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b\sin(c+dx)\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*
(b*Cos[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b^2\int \frac{1}{(b\cos(c+dx))^{10/3}}\,dx\\ &= \frac{3b\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\cos(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 58, normalized size = 1.04

$$\frac{3b^2\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{6},\frac{1}{2},-\frac{1}{6},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{7d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Cot[c + d*x]*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(b*cos(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^2*(b*cos(c + d*x))^(4/3)), x)
```

3.243 $\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=58

$$\frac{3b^2\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/10*b^2*hypergeom([-5/3, 1/2],[-2/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*cos(d*x
+c))^(10/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {16, 2722}

$$\frac{3b^2\sin(c+dx)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10d\sqrt{\sin^2(c+dx)}\,(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10
*d*(b*Cos[c + d*x])^(10/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{4/3}}\,dx &= b^3\int \frac{1}{(b\cos(c+dx))^{13/3}}\,dx\\ &= \frac{3b^2\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)\sin(c+dx)}{10d(b\cos(c+dx))^{10/3}\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 58, normalized size = 1.00

$$\frac{3b^2\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{3},\frac{1}{2},-\frac{2}{3},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{10d(b\cos(c+dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(b*Cos[c + d*x])^(4/3),x]
```

```
[Out] (3*b^2*Csc[c + d*x]*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2]*Sqrt
[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(b\cos(c+dx))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(b*cos(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(c+d\,x\right)^3\,{\left(b\,\cos\left(c+d\,x\right)\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)),x)
```

```
[Out] int(1/(cos(c + d*x)^3*(b*cos(c + d*x))^(4/3)), x)
```

3.244 $\int (a\cos(e+fx))^m (b\cos(e+fx))^n \, dx$

Optimal. Leaf size=82

$$-\frac{(a\cos(e+fx))^{1+m}(b\cos(e+fx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\cos^2(e+fx)\right)\sin(e+fx)}{af(1+m+n)\sqrt{\sin^2(e+fx)}}$$

[Out] -(a*cos(f*x+e))^(1+m)*(b*cos(f*x+e))^n*hypergeom([1/2, 1/2+1/2*m+1/2*n],[3/2+1/2*m+1/2*n],cos(f*x+e)^2)*sin(f*x+e)/a/f/(1+m+n)/(sin(f*x+e)^2)^(1/2)

Rubi [A]

time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\cos(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\cos^2(e+fx)\right)}{af(m+n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n,x]

[Out] -(((a*Cos[e + f*x])^(1 + m)*(b*Cos[e + f*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m + n)*Sqrt[Sin[e + f*x]^2]))

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m(b\cos(e+fx))^n\,dx &= \left((a\cos(e+fx))^{-n}(b\cos(e+fx))^n\right)\int (a\cos(e+fx))^{m+n}\,dx\\ &= -\frac{(a\cos(e+fx))^{1+m}(b\cos(e+fx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\cos^2(e+fx)\right)\sin(e+fx)}{af(1+m+n)\sqrt{\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 77, normalized size = 0.94

$$-\frac{(a\cos(e+fx))^m(b\cos(e+fx))^n\cot(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m+n),\frac{1}{2}(3+m+n),\cos^2(e+fx)\right)\sqrt{\sin^2(e+fx)}}{f(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n,x]
[Out] -(((a*Cos[e + f*x])^m*(b*Cos[e + f*x])^n*Cot[e + f*x]*Hypergeometric2F1[1/2
, (1 + m + n)/2, (3 + m + n)/2, Cos[e + f*x]^2]*Sqrt[Sin[e + f*x]^2])/(f*(1
 + m + n)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m (b\cos(fx+e))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x)
[Out] int((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(e+fx))^m (b\cos(e+fx))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*cos(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*cos(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*cos(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*cos(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(e+f\,x\right))^m\,(b\,\cos\left(e+f\,x\right))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b*cos(e + f*x))^n,x)
```

```
[Out] int((a*cos(e + f*x))^m*(b*cos(e + f*x))^n, x)
```

3.245 $\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{(b\cos(c+dx))^{3+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(b*cos(d*x+c))^(3+n)*hypergeom([1/2, 3/2+1/2*n],[5/2+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/b^3/d/(3+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+3}\,{}_2F_1\left(\frac{1}{2},\frac{n+3}{2};\frac{n+5}{2};\cos^2(c+dx)\right)}{b^3d(n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(b\cos(c+dx))^n\,dx &= \frac{\int (b\cos(c+dx))^{2+n}\,dx}{b^2}\\ &= -\frac{(b\cos(c+dx))^{3+n}\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^3d(3+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 72, normalized size = 1.04

$$-\frac{\cos^2(c+dx)(b\cos(c+dx))^n\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3+n}{2},\frac{5+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n,x]
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (3
 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(3 + n)))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n,x)
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^2*(b*cos(c + d*x))^n, x)
```

3.246 $\int \cos(c+dx)(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{(b\cos(c+dx))^{2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(b*cos(d*x+c))^(2+n)*hypergeom([1/2, 1+1/2*n],[2+1/2*n],cos(d*x+c)^2)*sin(
d*x+c)/b^2/d/(2+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {16, 2722}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{n+2}{2};\frac{n+4}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\cos(c+dx))^n\,dx &= \frac{\int (b\cos(c+dx))^{1+n}\,dx}{b}\\ &= -\frac{(b\cos(c+dx))^{2+n}\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{b^2d(2+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 1.01

$$-\frac{\cos(c+dx)(b\cos(c+dx))^n\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+n}{2},\frac{4+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n,x]
[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (2 +
 n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(2 + n)))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n,x)
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)*(b*cos(c + d*x))^n, x)
```

3.247 $\int (b\cos(c+dx))^n\,dx$

Optimal. Leaf size=69

$$-\frac{(b\cos(c+dx))^{1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(b*cos(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cos(d*x+c)^2)*
sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = $\{2722\}$

$$-\frac{\sin(c+dx)(b\cos(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Co
s[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]))
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (b\cos(c+dx))^n\,dx = -\frac{(b\cos(c+dx))^{1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 64, normalized size = 0.93

$$-\frac{(b\cos(c+dx))^n\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n
)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1 + n)))
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (b\cos (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n,x)
```

```
[Out] int((b*cos(d*x+c))^n,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos (c+dx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (b \cos(c + d x))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n,x)
[Out] int((b*cos(c + d*x))^n, x)
```

3.248 $\int (b\cos(c+dx))^n \sec(c+dx)\,dx$

Optimal. Leaf size=60

$$-\frac{(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] -(b*cos(d*x+c))^n*hypergeom([1/2, 1/2*n],[1+1/2*n],cos(d*x+c)^2)*sin(d*x+c)
/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {16, 2722}

$$-\frac{\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{n+2}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x],x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2
]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n \sec(c+dx)\,dx &= b\int (b\cos(c+dx))^{-1+n}\,dx\\ &= -\frac{(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 63, normalized size = 1.05

$$-\frac{b(b\cos(c+dx))^{-1+n}\cot(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{2+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x],x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*Hypergeometric2F1[1/2, n/2, (2
+ n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*n))
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \sec(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c),x)
```

```
[Out] int((b*cos(d*x+c))^n*sec(d*x+c),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \sec(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c),x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^n}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x),x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x), x)
```

3.249 $\int (b\cos(c+dx))^n \sec^2(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{b(b\cos(c+dx))^{-1+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*(b*cos(d*x+c))^(-1+n)*hypergeom([1/2, -1/2+1/2*n],[1/2+1/2*n],cos(d*x+c)^
2)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$\frac{b\sin(c+dx)(b\cos(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{n-1}{2};\frac{n+1}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^2,x]
```

```
[Out] (b*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2,
Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n\sec^2(c+dx)\,dx &= b^2\int (b\cos(c+dx))^{-2+n}\,dx\\ &= \frac{b(b\cos(c+dx))^{-1+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 67, normalized size = 0.99

$$-\frac{b(b\cos(c+dx))^{-1+n}\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-1+n),\frac{1+n}{2},\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^2,x]
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)
/2, (1 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(\sec^2(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c)^2,x)
[Out] int((b*cos(d*x+c))^n*sec(d*x+c)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \sec^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c)**2,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^2,x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^2, x)
```

3.250 $\int (b\cos(c+dx))^n \sec^3(c+dx)\,dx$

Optimal. Leaf size=68

$$\frac{b^2(b\cos(c+dx))^{-2+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] b^2*(b*cos(d*x+c))^(-2+n)*hypergeom([1/2, -1+1/2*n],[1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$\frac{b^2\sin(c+dx)(b\cos(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{n-2}{2};\frac{n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^3,x]
```

```
[Out] (b^2*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n\sec^3(c+dx)\,dx &= b^3\int (b\cos(c+dx))^{-3+n}\,dx\\ &= \frac{b^2(b\cos(c+dx))^{-2+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\cos^2(c+dx)\right)\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 70, normalized size = 1.03

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2+n),\frac{n}{2},\cos^2(c+dx)\right)\sec^2(c+dx)\sqrt{\sin^2(c+dx)}}{d(-2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^3,x]
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2,
Cos[c + d*x]^2]*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 + n)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n \left(\sec^3(dx+c)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c)^3,x)
[Out] int((b*cos(d*x+c))^n*sec(d*x+c)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \sec^3(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c)**3,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sec(c + d*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n}{{\cos\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^3,x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^3, x)
```

3.251 $\int (b\cos(c+dx))^n \sec^4(c+dx)\,dx$

Optimal. Leaf size=70

$$\frac{b^3(b\cos(c+dx))^{-3+n}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] b^3*(b*cos(d*x+c))^(-3+n)*hypergeom([1/2, -3/2+1/2*n],[-1/2+1/2*n],cos(d*x+
c)^2)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 2722}

$$\frac{b^3\sin(c+dx)(b\cos(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{n-3}{2};\frac{n-1}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n*Sec[c + d*x]^4,x]
```

```
[Out] (b^3*(b*Cos[c + d*x])^(-3 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/
2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (b\cos(c+dx))^n\sec^4(c+dx)\,dx &= b^4\int (b\cos(c+dx))^{-4+n}\,dx\\ &= \frac{b^3(b\cos(c+dx))^{-3+n}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-3+n);\frac{1}{2}(-1+n);\cos^2(c+dx)\right)\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 72, normalized size = 1.03

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-3+n),\frac{1}{2}(-1+n),\cos^2(c+dx)\right)\sec^3(c+dx)\sqrt{\sin^2(c+dx)}}{d(-3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*Sec[c + d*x]^4,x]
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 +
 n)/2, Cos[c + d*x]^2]*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(d*(-3 + n)))
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (b\cos(dx+c))^n\left(\sec^4(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*sec(d*x+c)^4,x)
[Out] int((b*cos(d*x+c))^n*sec(d*x+c)^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*sec(d*x+c)**4,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n}{{\cos\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^4,x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^4, x)
```

3.252 $\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(7+2n),\frac{1}{4}(11+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*cos(d*x+c)^(7/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 7/4+1/2*n],[11/4+1/2*n
],cos(d*x+c)^2)*sin(d*x+c)/d/(7+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{2\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+7);\frac{1}{4}(2n+11);\cos^2(c+dx)\right)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n,x]
```

```
[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/
4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{5}{2}+n}(c+dx)\,dx$$
$$= -\frac{2\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(7+2n);\frac{1}{4}(11+2n);\cos^2(c+\right.}{d(7+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.08, size = 80, normalized size = 1.00

$$-\frac{\cos^{\frac{7}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{7}{2}+n\right),\frac{1}{2}\left(\frac{11}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{7}{2}+n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n,x]
[Out] -((Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (7/2 + n)/2, (11/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(7/2
+ n)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{5}{2}}(dx+c)\right)(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x)
[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,{\left(b\,\cos\left(c+d\,x\right)\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^(5/2)*(b*cos(c + d*x))^n, x)
```

3.253 $\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n \, dx$

Optimal. Leaf size=80

$$-\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(5+2n),\frac{1}{4}(9+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 5/4+1/2*n],[9/4+1/2*n]
,cos(d*x+c)^2)*sin(d*x+c)/d/(5+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{2\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n \, {}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+5);\frac{1}{4}(2n+9);\cos^2(c+dx)\right)}{d(2n+5)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n,x]
```

```
[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/
4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{3}{2}+n}(c+dx)\,dx$$
$$= -\frac{2\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\cos^2(c+\right.}{d(5+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.07, size = 80, normalized size = 1.00

$$-\frac{\cos^{\frac{5}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{5}{2}+n\right),\frac{1}{2}\left(\frac{9}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{5}{2}+n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n,x]
[Out] -((Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (5/2 + n)/2, (9/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(5/2 +
 n)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\cos^{\frac{3}{2}}(dx+c)\right)(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x)
[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \cos^{\frac{3}{2}}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^{3/2}\,(b\,\cos(c+d\,x))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(b*cos(c + d*x))^n, x)
```

3.254 $\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,dx$

Optimal. Leaf size=80

$$-\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(3+2n),\frac{1}{4}(7+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*hypergeom([1/2, 3/4+1/2*n],[7/4+1/2*n]
,cos(d*x+c)^2)*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$-\frac{2\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+3);\frac{1}{4}(2n+7);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n,x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/
4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{\frac{1}{2}+n}(c+dx)\,dx$$
$$= -\frac{2\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\cos^2(c+\right.}{d(3+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 1.00

$$-\frac{\cos^{\frac{3}{2}}(c+dx)(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{3}{2}+n\right),\frac{1}{2}\left(\frac{7}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{3}{2}+n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n,x]
```

```
[Out] -((Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (3/2 + n)/2, (7/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(3/2 +
 n)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \left(\sqrt{\cos}\,(dx+c)\right)(b\cos(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x)
```

```
[Out] int(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cos(c+dx))^n \sqrt{\cos(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(b*cos(d*x+c))**n,x)
```

```
[Out] Integral((b*cos(c + d*x))**n*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(b*cos(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos(c+d\,x)}\,(b\,\cos(c+d\,x))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n,x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(b*cos(c + d*x))^n, x)
```

3.255 $\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=80

$$-\frac{2\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(1+2n),\frac{1}{4}(5+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -2*(b*cos(d*x+c))^n*hypergeom([1/2, 1/4+1/2*n],[5/4+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)*cos(d*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {20, 2722}

$$-\frac{2\sin(c+dx)\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n+1);\frac{1}{4}(2n+5);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/
4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{1}{2}+n}(c+dx)\,dx$$
$$= -\frac{2\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\cos^2(c+dx)\right)\sin(c}{d(1+2n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 1.00

$$-\frac{\sqrt{\cos(c+dx)}\,(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(\frac{1}{2}+n\right),\frac{1}{2}\left(\frac{5}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(\frac{1}{2}+n\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Sqrt[Cos[c + d*x]],x]
```

```
[Out] -((Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2
, (1/2 + n)/2, (5/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(1/2 +
 n)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n}{\sqrt{\cos(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n}{\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*cos(c + d*x))**n/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n}{\sqrt{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^(1/2),x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^(1/2), x)
```

3.256 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-1+2n),\frac{1}{4}(3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*(b*cos(d*x+c))^n*hypergeom([1/2, -1/4+1/2*n],[3/4+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/d/(1-2*n)/cos(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-1);\frac{1}{4}(2n+3);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(3/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{3}{2}+n}(c+dx)\,dx$$
$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(1-2n)\sqrt{\cos(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 1.00

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{1}{2}+n\right),\frac{1}{2}\left(\frac{3}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{1}{2}+n\right)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(3/2),x]
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-1/2 + n)/2, (3/
2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-1/2 + n)*Sqrt[Cos[c +
d*x]]))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x)
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(3/2),x)
[Out] Integral((b*cos(c + d*x))**n/cos(c + d*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos(c+d\,x))^n}{\cos(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^(3/2),x)
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^(3/2), x)
```

3.257 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-3+2n),\frac{1}{4}(1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*(b*cos(d*x+c))^n*hypergeom([1/2, -3/4+1/2*n],[1/4+1/2*n],cos(d*x+c)^2)*si
n(d*x+c)/d/(3-2*n)/cos(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-3);\frac{1}{4}(2n+1);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(5/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos
[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x
]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{5}{2}+n}(c+dx)\,dx$$
$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(3-2n)\cos^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 1.00

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{3}{2}+n\right),\frac{1}{2}\left(\frac{1}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{3}{2}+n\right)\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(5/2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-3/2 + n)/2, (1/
2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-3/2 + n)*Cos[c + d*x]^
(3/2)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="fricas")
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\cos{(c+dx)})^n}{\cos^{\frac{5}{2}}{(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(5/2),x)
[Out] Integral((b*cos(c + d*x))**n/cos(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,\cos\left(c+d\,x\right)\right)}^n}{{\cos\left(c+d\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^(5/2),x)
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^(5/2), x)
```

3.258 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2(b\cos(c+dx))^n \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-5+2n),\frac{1}{4}(-1+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*(b*cos(d*x+c))^n*hypergeom([1/2, -5/4+1/2*n],[-1/4+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(5-2*n)/cos(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-5);\frac{1}{4}(2n-1);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(7/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{7}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{7}{2}+n}(c+dx)\,dx$$
$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-5+2n);\frac{1}{4}(-1+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(5-2n)\cos^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.05, size = 80, normalized size = 1.00

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{5}{2}+n\right),\frac{1}{2}\left(-\frac{1}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{5}{2}+n\right)\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(7/2),x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-5/2 + n)/2, (-1
/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-5/2 + n)*Cos[c + d*x]
^(5/2)))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x)
```

```
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b\,\cos\left(c+d\,x\right))^n}{\cos\left(c+d\,x\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^(7/2),x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^(7/2), x)
```

3.259 $\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{2(b\cos(c+dx))^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{4}(-7+2n),\frac{1}{4}(-3+2n),\cos^2(c+dx)\right)\sin(c+dx)}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

```
[Out] 2*(b*cos(d*x+c))^n*hypergeom([1/2, -7/4+1/2*n],[-3/4+1/2*n],cos(d*x+c)^2)*s
in(d*x+c)/d/(7-2*n)/cos(d*x+c)^(7/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {20, 2722}

$$\frac{2\sin(c+dx)(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2n-7);\frac{1}{4}(2n-3);\cos^2(c+dx)\right)}{d(7-2n)\sqrt{\sin^2(c+dx)}\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^n/Cos[c + d*x]^(9/2),x]
```

```
[Out] (2*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Co
s[c + d*x]^2]*Sin[c + d*x])/(d*(7 - 2*n)*Cos[c + d*x]^(7/2)*Sqrt[Sin[c + d*
x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(b\cos(c+dx))^n}{\cos^{\frac{9}{2}}(c+dx)}\,dx = \left(\cos^{-n}(c+dx)(b\cos(c+dx))^n\right)\int \cos^{-\frac{9}{2}+n}(c+dx)\,dx$$
$$= \frac{2(b\cos(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-7+2n);\frac{1}{4}(-3+2n);\cos^2(c+dx)\right)\sin(c+dx)}{d(7-2n)\cos^{\frac{7}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.06, size = 80, normalized size = 1.00

$$-\frac{(b\cos(c+dx))^n\csc(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(-\frac{7}{2}+n\right),\frac{1}{2}\left(-\frac{3}{2}+n\right),\cos^2(c+dx)\right)\sqrt{\sin^2(c+dx)}}{d\left(-\frac{7}{2}+n\right)\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n/Cos[c + d*x]^(9/2),x]
[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*Hypergeometric2F1[1/2, (-7/2 + n)/2, (-3
/2 + n)/2, Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]^2])/(d*(-7/2 + n)*Cos[c + d*x]
^(7/2)))
```

Maple [F]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(b\cos(dx+c))^n}{\cos(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x)
[Out] int((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\,\cos\left(c+d\,x\right)\right)^n}{{\cos\left(c+d\,x\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(c + d*x))^n/cos(c + d*x)^(9/2),x)
```

```
[Out] int((b*cos(c + d*x))^n/cos(c + d*x)^(9/2), x)
```

3.260 $\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx$

Optimal. Leaf size=88

$$-\frac{(a\cos(e+fx))^{1+m}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\cos^2(e+fx)\right)(b\sec(e+fx))^n\sin}{af(1+m-n)\sqrt{\sin^2(e+fx)}}$$

```
[Out] -(a*cos(f*x+e))^(1+m)*hypergeom([1/2, 1/2+1/2*m-1/2*n],[3/2+1/2*m-1/2*n],co
s(f*x+e)^2)*(b*sec(f*x+e))^n*sin(f*x+e)/a/f/(1+m-n)/(sin(f*x+e)^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2668, 2722}

$$-\frac{\sin(e+fx)(a\cos(e+fx))^{m+1}(b\sec(e+fx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(m-n+1);\frac{1}{2}(m-n+3);\cos^2(e+fx)\right)}{af(m-n+1)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Sec[e + f*x])^n,x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m - n)/2, (3 + m -
n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(a*f*(1 + m - n)*Sqr
t[Sin[e + f*x]^2]))
```

Rule 2668

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[(a*b)^IntPart[n]*(a*Sin[e + f*x])^FracPart[n]*(b*Csc
[e + f*x])^FracPart[n], Int[(a*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b
, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\sec(e+fx))^n\,dx = \left((a\cos(e+fx))^n(b\sec(e+fx))^n\right)\int (a\cos(e+fx))^{m-n}\,dx$$

$$= -\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m-n);\frac{1}{2}(3+m-n);\cos^2(e+\right.}{af(1+m-n)\sqrt{\sin^2(e+fx)}}$$

Mathematica [A]
time = 6.76, size = 89, normalized size = 1.01

$$-\frac{\cos(e+fx)(a\cos(e+fx))^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(1+m-n),\frac{1}{2}(3+m-n),\cos^2(e+fx)\right)(b\sec(e+fx))^n\sin(e+fx)}{f(1+m-n)\sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Sec[e + f*x])^n,x]
[Out] -((Cos[e + f*x]*(a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, (1 + m - n)/2, (3
 + m - n)/2, Cos[e + f*x]^2]*(b*Sec[e + f*x])^n*Sin[e + f*x])/(f*(1 + m - n
)*Sqrt[Sin[e + f*x]^2]))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m (b\sec(fx+e))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x)
[Out] int((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="fricas")
[Out] integral((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(e+fx))^m (b\sec(e+fx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*sec(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*sec(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*sec(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*sec(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(e+f\,x\right))^m\,{\left(\frac{b}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b/cos(e + f*x))^n,x)
```

```
[Out] int((a*cos(e + f*x))^m*(b/cos(e + f*x))^n, x)
```

3.261 $\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=15

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

```
[Out] 2/b/csc(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2701, 30}

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] 2/(b*Sqrt[Csc[a + b*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(a+bx)\sqrt{\csc(a+bx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \csc(a+bx)\right)}{b}\\ &= \frac{2}{b\sqrt{\csc(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$\frac{2}{b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] 2/(b*Sqrt[Csc[a + b*x]])
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{2}{b\sqrt{\csc\left(bx+a\right)}}$	14
default	$\frac{2}{b\sqrt{\csc\left(bx+a\right)}}$	14
risch	$\frac{2\sqrt{2}\,\sqrt{\frac{i e^{i(bx+a)}}{e^{2i(bx+a)}-1}}\,\sin(bx+a)}{b}$	42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/b/csc(b*x+a)^(1/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.87

$$\frac{2\,\sqrt{\sin\left(bx+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sin(b*x + a))/b
```

Fricas [A]
time = 0.36, size = 13, normalized size = 0.87

$$\frac{2\,\sqrt{\sin\left(bx+a\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(sin(b*x + a))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

Giac [A]
time = 0.43, size = 13, normalized size = 0.87

$$\frac{2\sqrt{\sin(bx+a)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(sin(b*x + a))/b
```

Mupad [B]
time = 0.30, size = 15, normalized size = 1.00

$$\frac{2}{b\sqrt{\frac{1}{\sin(a+b\,x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
```

```
[Out] 2/(b*(1/sin(a + b*x))^(1/2))
```

3.262 $\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=17

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3/b/csc(b*x+a)^(3/2)
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2701, 30}

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] 2/(3*b*Csc[a + b*x]^(3/2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, \csc(a+bx)\right)}{b}\\ &= \frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] 2/(3*b*Csc[a + b*x]^(3/2))
```

Maple [A]
time = 0.04, size = 14, normalized size = 0.82

method	result	size
derivativedivides	$\frac{2}{3b\csc(bx+a)^{\frac{3}{2}}}$	14
default	$\frac{2}{3b\csc(bx+a)^{\frac{3}{2}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/b/csc(b*x+a)^(3/2)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 0.76

$$\frac{2\sin\left(bx+a\right)^{\frac{3}{2}}}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sin(b*x + a)^(3/2)/b
```

Fricas [A]
time = 0.36, size = 23, normalized size = 1.35

$$-\frac{2\left(\cos\left(bx+a\right)^{2}-1\right)}{3\,b\sqrt{\sin\left(bx+a\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(b*x + a)^2 - 1)/(b*sqrt(sin(b*x + a)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

Giac [A]
time = 0.47, size = 13, normalized size = 0.76

$$\frac{2 \sin\left(bx+a\right)^{\frac{3}{2}}}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*sin(b*x + a)^(3/2)/b
```

Mupad [B]
time = 0.21, size = 15, normalized size = 0.88

$$\frac{2}{3 b \left(\frac{1}{\sin(a+b x)}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(1/sin(a + b*x))^(1/2),x)
```

```
[Out] 2/(3*b*(1/sin(a + b*x))^(3/2))
```

3.263 $\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=67

$$\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)\sqrt{\sin(a+bx)}}{3b}$$

```
[Out] 2/3*cos(b*x+a)/b/csc(b*x+a)^(1/2)-4/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/s
in(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b
*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2708, 3856, 2720}

$$\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b*x]]) + (4*Sqrt[Csc[a + b*x]]*EllipticF
[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b)
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{2}{3}\int \sqrt{\csc(a+bx)}\,dx$$
$$= \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{1}{3}\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx}}$$
$$= \frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx}}{3b}$$

Mathematica [A]
time = 0.07, size = 53, normalized size = 0.79

$$\frac{\sqrt{\csc(a+bx)}\left(-4F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+\sin(2(a+bx))\right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(-4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*
x]] + Sin[2*(a + b*x)]))/(3*b)
```

Maple [A]
time = 0.10, size = 88, normalized size = 1.31

method	result
default	$\frac{\frac{2\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\,\mathrm{EllipticF}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)}{3}+\frac{2(}{}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellip
ticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2/3*cos(b*x+a)^2*sin(b*x+a))/cos(b*x
+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 67, normalized size = 1.00

$$\frac{2\left(\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}-i\sqrt{2i}\,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right))+i\sqrt{-2i}\,\text{weierstrassPInverse}(4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right))\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(cos(b*x + a)*sqrt(sin(b*x + a)) - I*sqrt(2*I)*weierstrassPInverse(4, 0
, cos(b*x + a) + I*sin(b*x + a)) + I*sqrt(-2*I)*weierstrassPInverse(4, 0, c
os(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos^2\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2*sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2*sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(a+b\,x\right)^2\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2), x)
```

3.264 $\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}} dx$

Optimal. Leaf size=67

$$\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)\sqrt{\sin(a+bx)}}{5b}$$

```
[Out] 2/5*cos(b*x+a)/b/csc(b*x+a)^(3/2)-4/5*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/s
in(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b
*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2708, 3856, 2719}

$$\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Cos[a + b*x])/(5*b*Csc[a + b*x]^(3/2)) + (4*Sqrt[Csc[a + b*x]]*EllipticE
[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(5*b)
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{2}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx \\
&= \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{1}{5}\left(2\sqrt{\csc(a+bx)}\,\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx \\
&= \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)\sqrt{\sin(a+bx)}}{5b}
\end{aligned}
$$

Mathematica [**A**]
time = 0.09, size = 61, normalized size = 0.91

$$
-\frac{2\sqrt{\csc(a+bx)}\left(2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|\,2\right)\sqrt{\sin(a+bx)}-\cos(a+bx)\sin^2(a+bx)\right)}{5b}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a +
b*x]] - Cos[a + b*x]*Sin[a + b*x]^2))/(5*b)
```

Maple [**A**]
time = 0.09, size = 142, normalized size = 2.12

method	result
default	$\frac{-\frac{2\left(\sin^4(bx+a)\right)}{5}+\frac{2\left(\sin^2(bx+a)\right)}{5}-\frac{4\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{\sin\right.}{5}}{\cos(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5*sin(b*x+a)^4+2/5*sin(b*x+a)^2-4/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)
+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2
/5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*Ellipti
cF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 83, normalized size = 1.24

$$\frac{2\left(\sqrt{2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))+\sqrt{-2i}\ \mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)-i\sin(bx+a)))-\frac{\cos(bx+a)^3-\cos(bx+a)}{\sqrt{\sin(bx+a)}}\right)}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a)
 + I*sin(b*x + a))) + sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassPInverse(
4, 0, cos(b*x + a) - I*sin(b*x + a))) - (cos(b*x + a)^3 - cos(b*x + a))/sqr
t(sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**2/sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2/(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2/(1/sin(a + b*x))^(1/2), x)
```

3.265 $\int \cos^3(x)\csc^{\frac{9}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)$$

```
[Out] 2/3*csc(x)^(3/2)-2/7*csc(x)^(7/2)
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}=0.182$, Rules used = {2701, 14}

$$\frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Csc[x]^(9/2),x]
```

```
[Out] (2*Csc[x]^(3/2))/3 - (2*Csc[x]^(7/2))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\csc^{\frac{9}{2}}(x)\,dx&=-\text{Subst}\left(\int \sqrt{x}\,\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\&=-\text{Subst}\left(\int \left(-\sqrt{x}+x^{5/2}\right)\,dx,x,\csc(x)\right)\\&=\frac{2}{3}\csc^{\frac{3}{2}}(x)-\frac{2}{7}\csc^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.86

$$\frac{2}{21}\csc^{\frac{3}{2}}(x)\left(7-3\csc^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Csc[x]^(9/2),x]
[Out] (2*Csc[x]^(3/2)*(7 - 3*Csc[x]^2))/21
```

Maple [A]
time = 0.09, size = 14, normalized size = 0.67

method	result	size
default	$\frac{2}{3\sin(x)^{\frac{3}{2}}}-\frac{2}{7\sin(x)^{\frac{7}{2}}}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*csc(x)^(9/2),x,method=_RETURNVERBOSE)
[Out] 2/3/sin(x)^(3/2)-2/7/sin(x)^(7/2)
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.62

$$\frac{2}{3\sin(x)^{\frac{3}{2}}}-\frac{2}{7\sin(x)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="maxima")
[Out] 2/3/sin(x)^(3/2) - 2/7/sin(x)^(7/2)
```

Fricas [A]
time = 0.35, size = 22, normalized size = 1.05

$$\frac{2\left(7\cos(x)^2-4\right)}{21\left(\cos(x)^2-1\right)\sin(x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="fricas")
[Out] 2/21*(7*cos(x)^2 - 4)/((cos(x)^2 - 1)*sin(x)^(3/2))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*csc(x)**(9/2),x)
[Out] Timed out
```

Giac [A]
time = 0.46, size = 14, normalized size = 0.67

$$\frac{2\left(7\sin\left(x\right)^{2}-3\right)}{21\sin\left(x\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*csc(x)^(9/2),x, algorithm="giac")
[Out] 2/21*(7*sin(x)^2 - 3)/sin(x)^(7/2)
```

Mupad [B]
time = 0.33, size = 16, normalized size = 0.76

$$\frac{2\left(7\sin\left(x\right)^{2}-3\right)\left(\frac{1}{\sin(x)}\right)^{7/2}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(1/sin(x))^(9/2),x)
[Out] (2*(7*sin(x)^2 - 3)*(1/sin(x))^(7/2))/21
```

3.266 $\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=33

$$-\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{2}{b\sqrt{\csc(a+bx)}}$$

```
[Out] -2/5/b/csc(b*x+a)^(5/2)+2/b/csc(b*x+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2701, 14}

$$\frac{2}{b\sqrt{\csc(a+bx)}}-\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3*Sqrt[Csc[a + b*x]],x]
```

```
[Out] -2/(5*b*Csc[a + b*x]^(5/2)) + 2/(b*Sqrt[Csc[a + b*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\sqrt{\csc(a+bx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{-1+x^2}{x^{7/2}}\,dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{x^{7/2}}+\frac{1}{x^{3/2}}\right)\,dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{2}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{2}{b\sqrt{\csc(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 0.82

$$\frac{9+\cos(2(a+bx))}{5b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (9 + Cos[2*(a + b*x)])/(5*b*Sqrt[Csc[a + b*x]])
```

Maple [A]
time = 0.10, size = 26, normalized size = 0.79

method	result	size
default	$\frac{-\frac{2\left(\sin^{\frac{5}{2}}(bx+a)\right)}{5}+2\left(\sqrt{\sin}\,(bx+a)\right)}{b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-2/5*sin(b*x+a)^(5/2)+2*sin(b*x+a)^(1/2))/b
```

Maxima [A]
time = 0.28, size = 25, normalized size = 0.76

$$\frac{2\left(\frac{5}{\sin(bx+a)^2}-1\right)\sin\left(bx+a\right)^{\frac{5}{2}}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(5/sin(b*x + a)^2 - 1)*sin(b*x + a)^(5/2)/b
```

Fricas [A]
time = 0.36, size = 23, normalized size = 0.70

$$\frac{2\left(\cos\left(bx+a\right)^2+4\right)\sqrt{\sin\left(bx+a\right)}}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(cos(b*x + a)^2 + 4)*sqrt(sin(b*x + a))/b
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*csc(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 24, normalized size = 0.73

$$-\frac{2\left(\sin\left(bx+a\right)^{\frac{5}{2}}-5\sqrt{\sin\left(bx+a\right)}\right)}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/5*(sin(b*x + a)^(5/2) - 5*sqrt(sin(b*x + a)))/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int \cos\left(a+b\,x\right)^3\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2), x)
```

3.267 $\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=35

$$-\frac{2}{7b\csc^{\frac{7}{2}}(a+bx)}+\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}$$

[Out] -2/7/b/csc(b*x+a)^(7/2)+2/3/b/csc(b*x+a)^(3/2)

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2701, 14}

$$\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2}{7b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[Cos[a + b*x]^3/Sqrt[Csc[a + b*x]],x]

[Out] -2/(7*b*Csc[a + b*x]^(7/2)) + 2/(3*b*Csc[a + b*x]^(3/2))

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{-1+x^2}{x^{9/2}}\,dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{x^{9/2}}+\frac{1}{x^{5/2}}\right)\,dx,x,\csc(a+bx)\right)}{b}\\ &= -\frac{2}{7b\csc^{\frac{7}{2}}(a+bx)}+\frac{2}{3b\csc^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 0.83

$$\frac{2(-3+7\csc^2(a+bx))}{21b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
[Out] (2*(-3 + 7*Csc[a + b*x]^2))/(21*b*Csc[a + b*x]^(7/2))
```

Maple [A]
time = 0.07, size = 26, normalized size = 0.74

method	result	size
default	$\frac{-\frac{2\left(\sin^{\frac{7}{2}}(bx+a)\right)}{7}+\frac{2\left(\sin^{\frac{3}{2}}(bx+a)\right)}{3}}{b}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-2/7*sin(b*x+a)^(7/2)+2/3*sin(b*x+a)^(3/2))/b
```

Maxima [A]
time = 0.29, size = 25, normalized size = 0.71

$$\frac{2\left(\frac{7}{\sin(bx+a)^2}-3\right)\sin(bx+a)^{\frac{7}{2}}}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="maxima")
[Out] 2/21*(7/sin(b*x + a)^2 - 3)*sin(b*x + a)^(7/2)/b
```

Fricas [A]
time = 0.38, size = 33, normalized size = 0.94

$$-\frac{2\left(3\cos(bx+a)^4+\cos(bx+a)^2-4\right)}{21\,b\sqrt{\sin(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="fricas")
[Out] -2/21*(3*cos(b*x + a)^4 + cos(b*x + a)^2 - 4)/(b*sqrt(sin(b*x + a)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3/csc(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 26, normalized size = 0.74

$$-\frac{2\left(3\sin\left(bx+a\right)^{\frac{7}{2}}-7\sin\left(bx+a\right)^{\frac{3}{2}}\right)}{21\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -2/21*(3*sin(b*x + a)^(7/2) - 7*sin(b*x + a)^(3/2))/b
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.03

$$\int\frac{\cos\left(a+b\,x\right)^3}{\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3/(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3/(1/sin(a + b*x))^(1/2), x)
```

3.268 $\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=92

$$\frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{8\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{7b}$$

```
[Out] 4/7*cos(b*x+a)/b/csc(b*x+a)^(1/2)+2/7*cos(b*x+a)^3/b/csc(b*x+a)^(1/2)-8/7*(
sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(
1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2708, 3856, 2720}

$$\frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}}+\frac{8\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[a + b*x])/(7*b*Sqrt[Csc[a + b*x]]) + (2*Cos[a + b*x]^3)/(7*b*Sqrt[Cs
c[a + b*x]]) + (8*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[
Sin[a + b*x]])/(7*b)
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \cos^4(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{6}{7}\int \cos^2(a+bx)\sqrt{\csc(a+bx)}\,dx$$
$$= \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{4}{7}\int \sqrt{\csc(a+bx)}\,dx$$
$$= \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{1}{7}\Big(4\sqrt{\csc(a+bx)}\,\sqrt{\sin(a+bx}$$
$$= \frac{4\cos(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{2\cos^3(a+bx)}{7b\sqrt{\csc(a+bx)}} + \frac{8\sqrt{\csc(a+bx)}\,F\big(\frac{1}{2}\big(a-\frac{\pi}{2}+bx}{7b}$$

Mathematica [A]
time = 0.09, size = 63, normalized size = 0.68

$$\frac{\sqrt{\csc(a+bx)}\left(-32F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+10\sin(2(a+bx))+\sin(4(a+bx))\right)}{28b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(-32*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + 10*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]))/(28*b)
```

Maple [A]
time = 0.10, size = 100, normalized size = 1.09

method	result
default	$\frac{\frac{2\left(\sin^5(bx+a)\right)}{7}-\frac{8\left(\sin^3(bx+a)\right)}{7}+\frac{6\sin(bx+a)}{7}+\frac{4\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\mathrm{Ellipti}}{7}}{\cos(bx+a)\sqrt{\sin(bx+a)}\,b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (2/7*sin(b*x+a)^5-8/7*sin(b*x+a)^3+6/7*sin(b*x+a)+4/7*(sin(b*x+a)+1)^(1/2)*
(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),
1/2*2^(1/2)))/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 78, normalized size = 0.85

$$\frac{2\left(\left(\cos\left(bx+a\right)^{3}+2\cos\left(bx+a\right)\right)\sqrt{\sin\left(bx+a\right)}-2i\sqrt{2i}\ \mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right))+2i\sqrt{-2i}\ \mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right))\right)}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/7*((cos(b*x + a)^3 + 2*cos(b*x + a))*sqrt(sin(b*x + a)) - 2*I*sqrt(2*I)*w
eierstrassPInverse(4, 0, cos(b*x + a) + I*sin(b*x + a)) + 2*I*sqrt(-2*I)*we
ierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\cos^{4}\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**4*sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^4*sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int{\cos\left(a+b\,x\right)}^4\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2), x)
```

3.269 $\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=92

$$\frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)}+\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}+\frac{8\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|\,2\right)\sqrt{\sin(a+bx)}}{15b}$$

```
[Out] 4/15*cos(b*x+a)/b/csc(b*x+a)^(3/2)+2/9*cos(b*x+a)^3/b/csc(b*x+a)^(3/2)-8/15
*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(co
s(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2708, 3856, 2719}

$$\frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)}+\frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)}+\frac{8\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|\,2\right)}{15b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[a + b*x])/(15*b*Csc[a + b*x]^(3/2)) + (2*Cos[a + b*x]^3)/(9*b*Csc[a
+ b*x]^(3/2)) + (8*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt
[Sin[a + b*x]])/(15*b)
```

Rule 2708

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(-a)*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n +
 1)/(b*f*(m + n))), x] + Dist[(n + 1)/(b^2*(m + n)), Int[(a*Csc[e + f*x])^m
*(b*Sec[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && LtQ[n, -1
] && NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{2}{3}\int \frac{\cos^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)} + \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{4}{15}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)} + \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{1}{15}\left(4\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin}$$
$$= \frac{4\cos(a+bx)}{15b\csc^{\frac{3}{2}}(a+bx)} + \frac{2\cos^3(a+bx)}{9b\csc^{\frac{3}{2}}(a+bx)} + \frac{8\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(}}{15b}$$

Mathematica [**A**]
time = 0.26, size = 63, normalized size = 0.68

$$\frac{39\cos(a+bx)+5\cos(3(a+bx))-\frac{48E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)}{\sin^{\frac{3}{2}}(a+bx)}}{90b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
[Out] (39*Cos[a + b*x] + 5*Cos[3*(a + b*x)] - (48*EllipticE[(-2*a + Pi - 2*b*x)/4
, 2])/Sin[a + b*x]^(3/2))/(90*b*Csc[a + b*x]^(3/2))
```

Maple [**A**]
time = 0.10, size = 152, normalized size = 1.65

method	result
default	$-\frac{2(\cos^6(bx+a))}{9}-\frac{8\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)}\right.}{15}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^4/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] (-2/9*cos(b*x+a)^6-8/15*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(
b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+4/15*(sin(b*x+a)+
1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+
1)^(1/2),1/2*2^(1/2))-2/45*cos(b*x+a)^4+4/15*cos(b*x+a)^2)/cos(b*x+a)/sin(b
*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 95, normalized size = 1.03

$$\frac{2\left(6\sqrt{2i}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)))+6\sqrt{-2i}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)-i\sin(bx+a)))-\frac{5\cos(bx+a)^5+\cos(bx+a)^3-6\cos(bx+a)}{\sqrt{\sin(bx+a)}}\right)}{45b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*(6*sqrt(2*I)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x +
 a) + I*sin(b*x + a))) + 6*sqrt(-2*I)*weierstrassZeta(4, 0, weierstrassPInv
erse(4, 0, cos(b*x + a) - I*sin(b*x + a))) - (5*cos(b*x + a)^5 + cos(b*x +
a)^3 - 6*cos(b*x + a))/sqrt(sin(b*x + a)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**4/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(cos(a + b*x)**4/sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^4}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^4/(1/sin(a + b*x))^(1/2),x)
[Out] int(cos(a + b*x)^4/(1/sin(a + b*x))^(1/2), x)
```

3.270 $\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx$

Optimal. Leaf size=10

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

```
[Out] -3/4*csc(x)^(4/3)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2701, 30}

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[x]^(7/3),x]
```

```
[Out] (-3*Csc[x]^(4/3))/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc^{\frac{7}{3}}(x)\,dx &= -\text{Subst}\left(\int \sqrt[3]{x}\,dx, x, \csc(x)\right)\\ &= -\frac{3}{4}\csc^{\frac{4}{3}}(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-\frac{3}{4}\csc^{\frac{4}{3}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[x]^(7/3),x]
```

```
[Out] (-3*Csc[x]^(4/3))/4
```

Maple [A]
time = 0.03, size = 7, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{3\left(\csc^{\frac{4}{3}}(x)\right)}{4}$	7
default	$-\frac{3\left(\csc^{\frac{4}{3}}(x)\right)}{4}$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(x)^(7/3),x,method=_RETURNVERBOSE)
```

```
[Out] -3/4*csc(x)^(4/3)
```

Maxima [A]
time = 0.28, size = 6, normalized size = 0.60

$$-\frac{3}{4\sin(x)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^(7/3),x, algorithm="maxima")
```

```
[Out] -3/4/sin(x)^(4/3)
```

Fricas [A]
time = 0.36, size = 6, normalized size = 0.60

$$-\frac{3}{4\sin(x)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^(7/3),x, algorithm="fricas")
```

```
[Out] -3/4/sin(x)^(4/3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)**(7/3),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 6, normalized size = 0.60

$$-\frac{3}{4 \sin{(x)}^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^(7/3),x, algorithm="giac")
```

```
[Out] -3/4/sin(x)^(4/3)
```

Mupad [B]
time = 0.20, size = 8, normalized size = 0.80

$$-\frac{3 \left(\frac{1}{\sin(x)}\right)^{4/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1/sin(x))^(7/3),x)
```

```
[Out] -(3*(1/sin(x))^(4/3))/4
```

3.271 $\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx$

Optimal. Leaf size=32

$$-\frac{\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

```
[Out] -arctan(csc(b*x+a)^(1/2))/b+arctanh(csc(b*x+a)^(1/2))/b
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2701, 335, 304, 209, 212}

$$\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}-\frac{\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x],x]
```

```
[Out] -(ArcTan[Sqrt[Csc[a + b*x]]]/b) + ArcTanh[Sqrt[Csc[a + b*x]]]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(a+bx)}\sec(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{2\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}\\ &= -\frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b} + \frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 47, normalized size = 1.47

$$\frac{\left(\text{ArcTan}\left(\sqrt{\sin(a+bx)}\right)+\tanh^{-1}\left(\sqrt{\sin(a+bx)}\right)\right)\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x],x]
```

```
[Out] ((ArcTan[Sqrt[Sin[a + b*x]]] + ArcTanh[Sqrt[Sin[a + b*x]]])*Sqrt[Csc[a + b*
x]]*Sqrt[Sin[a + b*x]])/b
```

Maple [A]
time = 0.07, size = 24, normalized size = 0.75

method	result	size
default	$\frac{\operatorname{arctanh}\left(\sqrt{\sin}\,(bx+a)\right)+\arctan\left(\sqrt{\sin}\,(bx+a)\right)}{b}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] (arctanh(sin(b*x+a)^(1/2))+arctan(sin(b*x+a)^(1/2)))/b
```

Maxima [A]
time = 0.54, size = 41, normalized size = 1.28

$$-\frac{2\arctan\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}\right)-\log\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}+1\right)+\log\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*arctan(1/sqrt(sin(b*x + a))) - log(1/sqrt(sin(b*x + a)) + 1) + log(
1/sqrt(sin(b*x + a)) - 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(28) = 56.
time = 0.43, size = 95, normalized size = 2.97

$$\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin\left(bx+a\right)}}\right)+\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin\left(bx+a\right)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a))) + log((cos(b*x + a
)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b*x
+ a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\csc\left(a+bx\right)}\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a),x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x), x)
```

Giac [A]
time = 0.44, size = 42, normalized size = 1.31

$$\frac{2\arctan\left(\sqrt{\sin\left(bx+a\right)}\right)+\log\left(\sqrt{\sin\left(bx+a\right)}+1\right)-\log\left(\left|\sqrt{\sin\left(bx+a\right)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*arctan(sqrt(sin(b*x + a))) + log(sqrt(sin(b*x + a)) + 1) - log(abs(s
qrt(sin(b*x + a)) - 1)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^(1/2)/cos(a + b*x),x)
```

```
[Out] int((1/sin(a + b*x))^(1/2)/cos(a + b*x), x)
```

3.272 $\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=31

$$\frac{\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

```
[Out] arctan(csc(b*x+a)^(1/2))/b+arctanh(csc(b*x+a)^(1/2))/b
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2701, 335, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]/Sqrt[Csc[a + b*x]],x]
```

```
[Out] ArcTan[Sqrt[Csc[a + b*x]]]/b + ArcTanh[Sqrt[Csc[a + b*x]]]/b
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
```

```
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx, x, \csc(a+bx)\right)}{b}\\ &= -\frac{2\text{Subst}\left(\int \frac{1}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}\\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}+\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{b}\\ &= \frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{b}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.61

$$-\frac{\left(\text{ArcTan}\left(\sqrt{\sin(a+bx)}\right)-\tanh^{-1}\left(\sqrt{\sin(a+bx)}\right)\right)\sqrt{\csc(a+bx)}\,\sqrt{\sin(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]/Sqrt[Csc[a + b*x]],x]
[Out] -(((ArcTan[Sqrt[Sin[a + b*x]]] - ArcTanh[Sqrt[Sin[a + b*x]]])*Sqrt[Csc[a +
b*x]]*Sqrt[Sin[a + b*x]])/b)
```

Maple [A]
time = 0.08, size = 43, normalized size = 1.39

method	result	size

default	$\frac{-\frac{\ln\left(\sqrt{\sin}\,(bx+a)-1\right)}{2}+\frac{\ln\left(\sqrt{\sin}\,(bx+a)+1\right)}{2}-\arctan\left(\sqrt{\sin}\,(bx+a)\right)}{b}$	43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*ln(sin(b*x+a)^(1/2)-1)+1/2*ln(sin(b*x+a)^(1/2)+1)-arctan(sin(b*x+a)^(
1/2)))/b
```

Maxima [A]
time = 0.54, size = 41, normalized size = 1.32

$$\frac{2\arctan\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}\right)+\log\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}+1\right)-\log\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arctan(1/sqrt(sin(b*x + a))) + log(1/sqrt(sin(b*x + a)) + 1) - log(1
/sqrt(sin(b*x + a)) - 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(27) = 54$.
time = 0.42, size = 97, normalized size = 3.13

$$-\frac{2\arctan\left(\frac{\sin\left(bx+a\right)-1}{2\sqrt{\sin\left(bx+a\right)}}\right)-\log\left(\frac{\cos\left(bx+a\right)^{2}+\frac{4\left(\cos\left(bx+a\right)^{2}-\sin\left(bx+a\right)-1\right)}{\sqrt{\sin\left(bx+a\right)}}-6\sin\left(bx+a\right)-2}{\cos\left(bx+a\right)^{2}+2\sin\left(bx+a\right)-2}\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a))) - log((cos(b*x +
a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b*x
 + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)/sqrt(csc(a + b*x)), x)
```

Giac [A]
time = 0.52, size = 42, normalized size = 1.35

$$-\frac{2\arctan\left(\sqrt{\sin\left(bx+a\right)}\right)-\log\left(\sqrt{\sin\left(bx+a\right)}+1\right)+\log\left(\left|\sqrt{\sin\left(bx+a\right)}-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(2*arctan(sqrt(sin(b*x + a))) - log(sqrt(sin(b*x + a)) + 1) + log(abs(
sqrt(sin(b*x + a)) - 1)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\cos\left(a+b\,x\right)\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*(1/sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(a + b*x)*(1/sin(a + b*x))^(1/2)), x)
```

3.273 $\int \sqrt{\csc(a+bx)}\,\sec^2(a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}$$

```
[Out] sec(b*x+a)/b/csc(b*x+a)^(1/2)-(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a
+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1
/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2706, 3856, 2720}

$$\frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}}+\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^2,x]
```

```
[Out] Sec[a + b*x]/(b*Sqrt[Csc[a + b*x]]) + (Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi
/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx = \frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}} + \frac{1}{2}\int \sqrt{\csc(a+bx)}\,dx$$
$$= \frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}} + \frac{1}{2}\left(\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,d$$
$$= \frac{\sec(a+bx)}{b\sqrt{\csc(a+bx)}} + \frac{\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}$$

Mathematica [A]
time = 0.09, size = 49, normalized size = 0.80

$$\frac{\sec(a+bx)+\frac{F\left(\frac{1}{4}(2a-\pi+2bx)\middle|2\right)}{\sqrt{\sin(a+bx)}}}{b\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x] + EllipticF[(2*a - Pi + 2*b*x)/4, 2]/Sqrt[Sin[a + b*x]])/(b*S
qrt[Csc[a + b*x]])
```

Maple [A]
time = 0.12, size = 123, normalized size = 2.02

method	result
default	$\frac{\sqrt{(\cos^2(bx+a))\sin(bx+a)}\left(\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\ \mathrm{Elliptic}\right.}{2\sqrt{-\sin(bx+a)(\sin(bx+a)-1)(\sin(bx+a)+1)}\ \cos(bx+a)\sqrt{\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*((sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)
^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))+2*si
n(b*x+a))/(-sin(b*x+a)*(sin(b*x+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(
b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 82, normalized size = 1.34

$$\frac{-i\sqrt{2i}\cos(bx+a)\,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a))+i\sqrt{-2i}\cos(bx+a)\,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)-i\sin(bx+a))+2\sqrt{\sin(bx+a)}}{2b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*sqrt(2*I)*cos(b*x + a)*weierstrassPInverse(4, 0, cos(b*x + a) + I*s
in(b*x + a)) + I*sqrt(-2*I)*cos(b*x + a)*weierstrassPInverse(4, 0, cos(b*x
+ a) - I*sin(b*x + a)) + 2*sqrt(sin(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\csc(a+bx)}\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^2,x)
```

```
[Out] int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^2, x)
```

3.274 $\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}$$

```
[Out] sec(b*x+a)/b/csc(b*x+a)^(3/2)+(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a
+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1
/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2706, 3856, 2719}

$$\frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] Sec[a + b*x]/(b*Csc[a + b*x]^(3/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi
/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= \frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{2}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx\\ &= \frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{2}\left(\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a+bx)}\,dx\\ &= \frac{\sec(a+bx)}{b\csc^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.87

$$\frac{\sqrt{\csc(a+bx)}\left(E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+\sin(a+bx)\tan(a+bx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]
 + Sin[a + b*x]*Tan[a + b*x]))/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(85) = 170$.
time = 0.10, size = 177, normalized size = 2.85

method	result
default	$\frac{\sqrt{(\cos^2(bx+a))\sin(bx+a)}\left(2\sqrt{\sin(bx+a)+1}\sqrt{-2\sin(bx+a)+2}\sqrt{-\sin(bx+a)}\,\text{Ellip}\right.}{2\sqrt{-\sin(bx+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*(2*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+
2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))-(s
in(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((s
in(b*x+a)+1)^(1/2),1/2*2^(1/2))-2*cos(b*x+a)^2+2)/(-sin(b*x+a)*(sin(b*x+a)-
1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(b*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 96, normalized size = 1.55

$$-\frac{\sqrt{2i}\cos\left(bx+a\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\sin\left(bx+a\right)))+\sqrt{-2i}\cos\left(bx+a\right)\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)-i\sin\left(bx+a\right)))+\frac{2\left(\cos(bx+a)^2-1\right)}{\sqrt{\sin\left(bx+a\right)}}}{2b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2*I)*cos(b*x + a)*weierstrassZeta(4, 0, weierstrassPInverse(4, 0
, cos(b*x + a) + I*sin(b*x + a))) + sqrt(-2*I)*cos(b*x + a)*weierstrassZeta
(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) + 2*(cos(b
*x + a)^2 - 1)/sqrt(sin(b*x + a)))/(b*cos(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)**2/sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^2/sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{{\cos\left(a+b\,x\right)}^2\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*(1/sin(a + b*x))^(1/2)), x)
```

3.275 $\int \sqrt{\csc(a+bx)} \sec^3(a+bx)\,dx$

Optimal. Leaf size=62

$$-\frac{3\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{3\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}}$$

```
[Out] -3/4*arctan(csc(b*x+a)^(1/2))/b+3/4*arctanh(csc(b*x+a)^(1/2))/b+1/2*sec(b*x
+a)^2/b/csc(b*x+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2701, 294, 335, 304, 209, 212}

$$-\frac{3\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}}+\frac{3\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^3,x]
```

```
[Out] (-3*ArcTan[Sqrt[Csc[a + b*x]]])/(4*b) + (3*ArcTanh[Sqrt[Csc[a + b*x]]])/(4*
b) + Sec[a + b*x]^2/(2*b*Sqrt[Csc[a + b*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\csc(a+bx)}\,\sec^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \frac{x^{5/2}}{(-1+x^2)^2}\,dx, x, \csc(a+bx)\right)}{b}\\ &= \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}} - \frac{3\text{Subst}\left(\int \frac{\sqrt{x}}{-1+x^2}\,dx, x, \csc(a+bx)\right)}{4b}\\ &= \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}} - \frac{3\text{Subst}\left(\int \frac{x^2}{-1+x^4}\,dx, x, \sqrt{\csc(a+bx)}\right)}{2b}\\ &= \frac{\sec^2(a+bx)}{2b\sqrt{\csc(a+bx)}} + \frac{3\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\csc(a+bx)}\right)}{4b} - \frac{3\text{Subst}\left(\int \frac{1}{\dots}\right.}{}\\ &= -\frac{3\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{3\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b} + \frac{\sec^2(a+\dots)}{2b\sqrt{\csc(a+\dots}}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 1.18

$$\frac{\sqrt{\csc(a+bx)}\left(3\left(\text{ArcTan}\left(\sqrt{\sin(a+bx)}\right)+\tanh^{-1}\left(\sqrt{\sin(a+bx)}\right)\right)+2\sec^2(a+bx)\sqrt{\sin(a+bx)}\right)\sqrt{\sin(a+bx)}}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^3,x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(3*(ArcTan[Sqrt[Sin[a + b*x]]] + ArcTanh[Sqrt[Sin[a + b
*x]]]) + 2*Sec[a + b*x]^2*Sqrt[Sin[a + b*x]])*Sqrt[Sin[a + b*x]])/(4*b)
```

Maple [A]
time = 0.10, size = 73, normalized size = 1.18

method	result	size
default	$\frac{-\left(-3\ln\left(\sqrt{\sin}\,(bx+a)+1\right)+3\ln\left(\sqrt{\sin}\,(bx+a)-1\right)-6\arctan\left(\sqrt{\sin}\,(bx+a)\right)\right)(\cos^2(bx+a))+4\left(\sqrt{\sin}\,(bx+a)\right)}{8\cos(bx+a)^2 b}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(-(-3*ln(sin(b*x+a)^(1/2)+1)+3*ln(sin(b*x+a)^(1/2)-1)-6*arctan(sin(b*x+
a)^(1/2)))*cos(b*x+a)^2+4*sin(b*x+a)^(1/2))/cos(b*x+a)^2/b
```

Maxima [A]
time = 0.51, size = 65, normalized size = 1.05

$$\frac{\frac{4}{\left(\frac{1}{\sin(bx+a)^2}-1\right)\sin(bx+a)^{\frac{3}{2}}}-6\arctan\left(\frac{1}{\sqrt{\sin(bx+a)}}\right)+3\log\left(\frac{1}{\sqrt{\sin(bx+a)}}+1\right)-3\log\left(\frac{1}{\sqrt{\sin(bx+a)}}-1\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4/((1/sin(b*x + a)^2 - 1)*sin(b*x + a)^(3/2)) - 6*arctan(1/sqrt(sin(b*
x + a))) + 3*log(1/sqrt(sin(b*x + a)) + 1) - 3*log(1/sqrt(sin(b*x + a)) - 1
))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(50) = 100$.
time = 0.40, size = 131, normalized size = 2.11

$$\frac{6\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin(bx+a)}}\right)\cos(bx+a)^2+3\cos(bx+a)^2\log\left(\frac{\cos(bx+a)^2+\frac{4\left(\cos(bx+a)^2-\sin(bx+a)-1\right)}{\sqrt{\sin(bx+a)}}-6\sin(bx+a)-2}{\cos(bx+a)^2+2\sin(bx+a)-2}\right)+8\sqrt{\sin(bx+a)}}{16b\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a)))*cos(b*x + a)^2 +
3*cos(b*x + a)^2*log((cos(b*x + a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1
```

```
)/sqrt(sin(b*x + a)) - 6*sin(b*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a)
 - 2)) + 8*sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(a+bx)}\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x)**3, x)
```

Giac [A]
time = 0.43, size = 66, normalized size = 1.06

$$-\frac{\frac{4\sqrt{\sin(bx+a)}}{\sin(bx+a)^2-1} - 6\arctan\left(\sqrt{\sin(bx+a)}\right) - 3\log\left(\sqrt{\sin(bx+a)}+1\right) + 3\log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*sqrt(sin(b*x + a))/(sin(b*x + a)^2 - 1) - 6*arctan(sqrt(sin(b*x + a
))) - 3*log(sqrt(sin(b*x + a)) + 1) + 3*log(abs(sqrt(sin(b*x + a)) - 1)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^3,x)
```

```
[Out] int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^3, x)
```

3.276 $\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}$$

```
[Out] 1/4*arctan(csc(b*x+a)^(1/2))/b+1/4*arctanh(csc(b*x+a)^(1/2))/b+1/2*sec(b*x+
a)^2/b/csc(b*x+a)^(3/2)
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2701, 294, 335, 218, 212, 209}

$$\frac{\text{ArcTan}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^3/Sqrt[Csc[a + b*x]],x]
```

```
[Out] ArcTan[Sqrt[Csc[a + b*x]]]/(4*b) + ArcTanh[Sqrt[Csc[a + b*x]]]/(4*b) + Sec[
a + b*x]^2/(2*b*Csc[a + b*x]^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2701

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> Dist[-(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^{3/2}}{(-1+x^2)^2}\,dx,x,\csc(a+bx)\right)}{b}\\
&= \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}-\frac{\operatorname{Subst}\left(\int \frac{1}{\sqrt{x}\,(-1+x^2)}\,dx,x,\csc(a+bx)\right)}{4b}\\
&= \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}-\frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^4}\,dx,x,\sqrt{\csc(a+bx)}\right)}{2b}\\
&= \frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}+\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{\csc(a+bx)}\right)}{4b}\\
&= \frac{\tan^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\tanh^{-1}\left(\sqrt{\csc(a+bx)}\right)}{4b}+\frac{\sec^2(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.02, size = 33, normalized size = 0.53

$$
\frac{2\text{Hypergeometric2F1}\left(\frac{3}{4},2,\frac{7}{4},\sin^2(a+bx)\right)}{3b\csc^{\frac{3}{2}}(a+bx)}
$$

Antiderivative was successfully verified.

[In] Integrate[Sec[a + b*x]^3/Sqrt[Csc[a + b*x]],x]

[Out] (2*Hypergeometric2F1[3/4, 2, 7/4, Sin[a + b*x]^2])/(3*b*Csc[a + b*x]^(3/2))

Maple [A]
time = 0.09, size = 71, normalized size = 1.15

method	result	size
default	$-\frac{\left(-\ln\left(\sqrt{\sin}\left(bx+a\right)+1\right)+\ln\left(\sqrt{\sin}\left(bx+a\right)-1\right)+2\arctan\left(\sqrt{\sin}\left(bx+a\right)\right)\right)\left(\cos^{2}\left(bx+a\right)\right)+4\left(\sin^{\frac{3}{2}}\left(bx+a\right)\right)}{8\cos\left(bx+a\right)^{2}b}$	71

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(b*x+a)^3/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8*(-(-ln(sin(b*x+a)^(1/2)+1)+ln(sin(b*x+a)^(1/2)-1)+2*arctan(sin(b*x+a)^(1/2)))*cos(b*x+a)^2+4*sin(b*x+a)^(3/2))/cos(b*x+a)^2/b

Maxima [A]
time = 0.50, size = 63, normalized size = 1.02

$$\frac{\frac{4}{\left(\frac{1}{\sin(bx+a)^2}-1\right)\sqrt{\sin\left(bx+a\right)}}+2\arctan\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}\right)+\log\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}+1\right)-\log\left(\frac{1}{\sqrt{\sin\left(bx+a\right)}}-1\right)}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="maxima")

[Out] 1/8*(4/((1/sin(b*x + a)^2 - 1)*sqrt(sin(b*x + a))) + 2*arctan(1/sqrt(sin(b*x + a))) + log(1/sqrt(sin(b*x + a)) + 1) - log(1/sqrt(sin(b*x + a)) - 1))/b

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.
time = 0.40, size = 141, normalized size = 2.27

$$-\frac{2\arctan\left(\frac{\sin(bx+a)-1}{2\sqrt{\sin\left(bx+a\right)}}\right)\cos\left(bx+a\right)^{2}-\cos\left(bx+a\right)^{2}\log\left(\frac{\cos(bx+a)^{2}+\frac{4\left(\cos(bx+a)^{2}-\sin(bx+a)-1\right)}{\sqrt{\sin\left(bx+a\right)}}-6\sin(bx+a)-2}{\cos(bx+a)^{2}+2\sin(bx+a)-2}\right)+\frac{8\left(\cos(bx+a)^{2}-1\right)}{\sqrt{\sin\left(bx+a\right)}}}{16\,b\cos\left(bx+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="fricas")

[Out] -1/16*(2*arctan(1/2*(sin(b*x + a) - 1)/sqrt(sin(b*x + a)))*cos(b*x + a)^2 - cos(b*x + a)^2*log((cos(b*x + a)^2 + 4*(cos(b*x + a)^2 - sin(b*x + a) - 1)/sqrt(sin(b*x + a)) - 6*sin(b*x + a) - 2)/(cos(b*x + a)^2 + 2*sin(b*x + a) - 2)) + 8*(cos(b*x + a)^2 - 1)/sqrt(sin(b*x + a)))/(b*cos(b*x + a)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**3/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)**3/sqrt(csc(a + b*x)), x)
```

Giac [A]
time = 0.47, size = 64, normalized size = 1.03

$$-\frac{\frac{4\sin(bx+a)^{\frac{3}{2}}}{\sin(bx+a)^2-1}+2\arctan\left(\sqrt{\sin(bx+a)}\right)-\log\left(\sqrt{\sin(bx+a)}+1\right)+\log\left(\left|\sqrt{\sin(bx+a)}-1\right|\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^3/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*sin(b*x + a)^(3/2)/(sin(b*x + a)^2 - 1) + 2*arctan(sqrt(sin(b*x + a
))) - log(sqrt(sin(b*x + a)) + 1) + log(abs(sqrt(sin(b*x + a)) - 1)))/b
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos(a+b\,x)^3\,\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*(1/sin(a + b*x))^(1/2)), x)
```

3.277 $\int \sqrt{\csc(a+bx)} \sec^4(a+bx)\,dx$

Optimal. Leaf size=92

$$\frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}}+\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{5\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{6b}$$

```
[Out] 5/6*sec(b*x+a)/b/csc(b*x+a)^(1/2)+1/3*sec(b*x+a)^3/b/csc(b*x+a)^(1/2)-5/6*(
sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(
1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2706, 3856, 2720}

$$\frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}}+\frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}}+\frac{5\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^4,x]
```

```
[Out] (5*Sec[a + b*x])/(6*b*Sqrt[Csc[a + b*x]]) + Sec[a + b*x]^3/(3*b*Sqrt[Csc[a
+ b*x]]) + (5*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[
a + b*x]])/(6*b)
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \sqrt{\csc(a+bx)}\,\sec^4(a+bx)\,dx = \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{5}{6}\int \sqrt{\csc(a+bx)}\,\sec^2(a+bx)\,dx$$
$$= \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}} + \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{5}{12}\int \sqrt{\csc(a+bx)}\,dx$$
$$= \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}} + \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{1}{12}\Big(5\sqrt{\csc(a+bx)}\,\sqrt{\sin(a+b$$
$$= \frac{5\sec(a+bx)}{6b\sqrt{\csc(a+bx)}} + \frac{\sec^3(a+bx)}{3b\sqrt{\csc(a+bx)}} + \frac{5\sqrt{\csc(a+bx)}\,F\big(\frac{1}{2}(a-\frac{\pi}{2}+b}{6b}$$

Mathematica [A]
time = 0.24, size = 64, normalized size = 0.70

$$\frac{\sqrt{\csc(a+bx)}\left(-5F\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sqrt{\sin(a+bx)}+(5+2\sec^2(a+bx))\tan(a+bx)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Csc[a + b*x]]*Sec[a + b*x]^4,x]
[Out] (Sqrt[Csc[a + b*x]]*(-5*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*
x]] + (5 + 2*Sec[a + b*x]^2)*Tan[a + b*x]))/(6*b)
```

Maple [A]
time = 0.13, size = 168, normalized size = 1.83

method	result
default	$-\frac{\sqrt{(\cos^2(bx+a))\sin(bx+a)}\left(5\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\text{Elli}\right.}{12(\sin(bx+a)-1)(\sin(bx+a)+1)\sqrt{-\sin(bx+a)(\sin(bx+a)-1)(\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] -1/12*(cos(b*x+a)^2*sin(b*x+a))^(1/2)*(5*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a
)+2)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*
cos(b*x+a)^2+10*cos(b*x+a)^2*sin(b*x+a)+4*sin(b*x+a))/(sin(b*x+a)-1)/(sin(b
*x+a)+1)/(-sin(b*x+a)*(sin(b*x+a)-1)*(sin(b*x+a)+1))^(1/2)/cos(b*x+a)/sin(b
*x+a)^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 98, normalized size = 1.07

$$\frac{-5i\sqrt{2i}\cos(bx+a)^3 \,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)+i\sin(bx+a)) + 5i\sqrt{-2i}\cos(bx+a)^3 \,\mathrm{weierstrassPInverse}(4,0,\cos(bx+a)-i\sin(bx+a)) + 2\left(5\cos(bx+a)^2+2\right)\sqrt{\sin(bx+a)}}{12\,b\cos(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(-5*I*sqrt(2*I)*cos(b*x + a)^3*weierstrassPInverse(4, 0, cos(b*x + a)
+ I*sin(b*x + a)) + 5*I*sqrt(-2*I)*cos(b*x + a)^3*weierstrassPInverse(4, 0,
 cos(b*x + a) - I*sin(b*x + a)) + 2*(5*cos(b*x + a)^2 + 2)*sqrt(sin(b*x + a
)))/(b*cos(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\csc(a+bx)}\,\sec^4(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**(1/2)*sec(b*x+a)**4,x)
```

```
[Out] Integral(sqrt(csc(a + b*x))*sec(a + b*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^(1/2)*sec(b*x+a)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(csc(b*x + a))*sec(b*x + a)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{1}{\sin(a+b\,x)}}}{\cos(a+b\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^4,x)
```

```
[Out] int((1/sin(a + b*x))^(1/2)/cos(a + b*x)^4, x)
```

3.278 $\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}} \, dx$

Optimal. Leaf size=92

$$\frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}+\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{2b}$$

```
[Out] 1/2*sec(b*x+a)/b/csc(b*x+a)^(3/2)+1/3*sec(b*x+a)^3/b/csc(b*x+a)^(3/2)+1/2*(
sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(
1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {2706, 3856, 2719}

$$\frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)}+\frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)}-\frac{\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] Sec[a + b*x]/(2*b*Csc[a + b*x]^(3/2)) + Sec[a + b*x]^3/(3*b*Csc[a + b*x]^(3
/2)) - (Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(2*b)
```

Rule 2706

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*((b_.)*sec[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[a*b*(a*Csc[e + f*x])^(m - 1)*((b*Sec[e + f*x])^(n -
1)/(f*(n - 1))), x] + Dist[b^2*((m + n - 2)/(n - 1)), Int[(a*Csc[e + f*x])^
m*(b*Sec[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1
] && IntegersQ[2*m, 2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rubi steps

$$\int \frac{\sec^4(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{1}{2}\int \frac{\sec^2(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{4}\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx$$
$$= \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{1}{4}\left(\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \sqrt{\sin(a-$$
$$= \frac{\sec(a+bx)}{2b\csc^{\frac{3}{2}}(a+bx)} + \frac{\sec^3(a+bx)}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+}}{2b}$$

Mathematica [A]
time = 0.15, size = 76, normalized size = 0.83

$$\frac{\cos(a+bx)\sqrt{\csc(a+bx)}\left(-3+\sec^2(a+bx)+2\sec^4(a+bx)+3E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sec(a+bx)\sqrt{\sin(a+bx)}\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^4/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (Cos[a + b*x]*Sqrt[Csc[a + b*x]]*(-3 + Sec[a + b*x]^2 + 2*Sec[a + b*x]^4 +
3*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sec[a + b*x]*Sqrt[Sin[a + b*x]]))/(6*
b)
```

Maple [A]
time = 0.11, size = 160, normalized size = 1.74

method	result
default	$\frac{6\sqrt{\sin(bx+a)+1}\,\sqrt{-2\sin(bx+a)+2}\,\sqrt{-\sin(bx+a)}\,\mathrm{EllipticE}\left(\sqrt{\sin(bx+a)+1},\frac{\sqrt{2}}{2}\right)(\mathrm{c}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^4/csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/cos(b*x+a)^3/sin(b*x+a)^(1/2)*(6*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2
)^(1/2)*(-sin(b*x+a))^(1/2)*EllipticE((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos
(b*x+a)^2-3*(sin(b*x+a)+1)^(1/2)*(-2*sin(b*x+a)+2)^(1/2)*(-sin(b*x+a))^(1/2
)*EllipticF((sin(b*x+a)+1)^(1/2),1/2*2^(1/2))*cos(b*x+a)^2-6*cos(b*x+a)^4+2
*cos(b*x+a)^2+4)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 114, normalized size = 1.24

$$-\frac{3\,\sqrt{2i}\,\cos\left(bx+a\right)^{3}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)))+3\,\sqrt{-2i}\,\cos\left(bx+a\right)^{3}\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)))+\frac{2\left(3\,\cos(bx+a)^{4}-\cos(bx+a)^{2}-2\right)}{\sqrt{\sin\left(bx+a\right)}}}{12\,b\cos\left(bx+a\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(2*I)*cos(b*x + a)^3*weierstrassZeta(4, 0, weierstrassPInverse
(4, 0, cos(b*x + a) + I*sin(b*x + a))) + 3*sqrt(-2*I)*cos(b*x + a)^3*weiers
trassZeta(4, 0, weierstrassPInverse(4, 0, cos(b*x + a) - I*sin(b*x + a))) +
 2*(3*cos(b*x + a)^4 - cos(b*x + a)^2 - 2)/sqrt(sin(b*x + a)))/(b*cos(b*x +
 a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}\left(a+bx\right)}{\sqrt{\csc\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**4/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(sec(a + b*x)**4/sqrt(csc(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^4/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)^4/sqrt(csc(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos(a+b\,x)^4\,\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2)),x)
[Out] int(1/(cos(a + b*x)^4*(1/sin(a + b*x))^(1/2)), x)
```

3.279 $\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{d\sqrt{d\cos(a+bx)}\,\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}$$

```
[Out] d*csc(b*x+a)^(-1+p)*hypergeom([-1/4, 1/2-1/2*p],[3/2-1/2*p],sin(b*x+a)^2)*(
d*cos(b*x+a))^(1/2)/b/(1-p)/(cos(b*x+a)^2)^(1/4)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 2657}

$$\frac{d\sqrt{d\cos(a+bx)}\,\csc^{p-1}(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^p,x]
```

```
[Out] (d*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 -
p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(Cos[a + b*x]^2)^(1/4))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (d\cos(a+bx))^{3/2}\csc^p(a+bx)\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int (d\cos(a+bx))^{3/2}\sin^{-p}(a+bx)\,dx\\ &= \frac{d\sqrt{d\cos(a+bx)}\,\csc^{-1+p}(a+bx)\,{}_2F_1\left(-\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt[4]{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [**A**]
time = 41.82, size = 105, normalized size = 1.38

$$-\frac{2(d\cos(a+bx))^{5/2}\csc^{-1+p}(a+bx)\left(9\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1}{2}(-1+p),\frac{9}{4},\cos^2(a+bx)\right)+5\cos^2(a+bx)\text{Hypergeometric2F1}\left(\frac{9}{4},\frac{1+p}{2},\frac{13}{4},\cos^2(a+bx)\right)\right)\sin^2(a+bx)^{\frac{1}{2}(-1+p)}}{45bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^p,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(5/2)*Csc[a + b*x]^(-1 + p)*(9*Hypergeometric2F1[5/4,
(-1 + p)/2, 9/4, Cos[a + b*x]^2] + 5*Cos[a + b*x]^2*Hypergeometric2F1[9/4,
(1 + p)/2, 13/4, Cos[a + b*x]^2])*(Sin[a + b*x]^2)^((-1 + p)/2))/(45*b*d)
```

Maple [**F**]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}\left(\csc^{p}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x)
```

```
[Out] int((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^p, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*d*csc(b*x + a)^p*cos(b*x + a), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(3/2)*csc(b*x+a)**p,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(3/2)*csc(b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate((d*cos(b*x + a))^(3/2)*csc(b*x + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,\cos(a+b\,x))^{3/2}\,\left(\frac{1}{\sin(a+b\,x)}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(3/2)*(1/sin(a + b*x))^p,x)
```

```
[Out] int((d*cos(a + b*x))^(3/2)*(1/sin(a + b*x))^p, x)
```

3.280 $\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx$

Optimal. Leaf size=76

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}$$

```
[Out] d*(cos(b*x+a)^2)^(1/4)*csc(b*x+a)^(-1+p)*hypergeom([1/4, 1/2-1/2*p],[3/2-1/
2*p],sin(b*x+a)^2)/b/(1-p)/(d*cos(b*x+a))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 2657}

$$\frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^p,x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[1/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \sqrt{d\cos(a+bx)}\sin^{-p}(a+bx)\,dx\\ &= \frac{d\sqrt[4]{\cos^2(a+bx)}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{1}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 10.11, size = 70, normalized size = 0.92

$$-\frac{2(d\cos(a+bx))^{3/2}\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1+p}{2},\frac{7}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1}{2}(-1+p)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^p,x]
```

```
[Out] (-2*(d*Cos[a + b*x])^(3/2)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1
+ p)/2, 7/4, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(3*b*d)
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(bx+a)}\,(\csc^p(bx+a))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x)
```

```
[Out] int((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d\cos(a+bx)}\csc^p(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))**(1/2)*csc(b*x+a)**p,x)
```

```
[Out] Integral(sqrt(d*cos(a + b*x))*csc(a + b*x)**p, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(b*x+a))^(1/2)*csc(b*x+a)^p,x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*cos(b*x + a))*csc(b*x + a)^p, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,\cos\left(a+b\,x\right)}\,{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(a + b*x))^(1/2)*(1/sin(a + b*x))^p,x)
```

```
[Out] int((d*cos(a + b*x))^(1/2)*(1/sin(a + b*x))^p, x)
```

3.281 $\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx$

Optimal. Leaf size=76

$$\frac{d\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}$$

```
[Out] d*(cos(b*x+a)^2)^(3/4)*csc(b*x+a)^(-1+p)*hypergeom([3/4, 1/2-1/2*p],[3/2-1/
2*p],sin(b*x+a)^2)/b/(1-p)/(d*cos(b*x+a))^(3/2)
```

Rubi [A]

time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 2657}

$$\frac{d\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^p/Sqrt[d*Cos[a + b*x]],x]
```

```
[Out] (d*(Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[3/4, (1 -
 p)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^p(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \frac{\sin^{-p}(a+bx)}{\sqrt{d\cos(a+bx)}}\,dx\\ &= \frac{d\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{3}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{b(1-p)(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 10.14, size = 68, normalized size = 0.89

$$-\frac{2\sqrt{d\cos(a+bx)}\,\csc^{1+p}(a+bx)\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1+p}{2},\frac{5}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1+p}{2}}}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^p/Sqrt[d*Cos[a + b*x]],x]
[Out] (-2*Sqrt[d*Cos[a + b*x]]*Csc[a + b*x]^(1 + p)*Hypergeometric2F1[1/4, (1 + p
)/2, 5/4, Cos[a + b*x]^2]*(Sin[a + b*x]^2)^((1 + p)/2))/(b*d)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\csc^{p}\left(bx+a\right)}{\sqrt{d\cos\left(bx+a\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x)
[Out] int(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^p/sqrt(d*cos(b*x + a)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d*cos(b*x + a)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{p}\left(a+bx\right)}{\sqrt{d\cos\left(a+bx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(1/2),x)
```

```
[Out] Integral(csc(a + b*x)**p/sqrt(d*cos(a + b*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^p/sqrt(d*cos(b*x + a)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^p}{\sqrt{d\,\cos\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(1/2),x)
```

```
[Out] int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(1/2), x)
```

3.282 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(\frac{5}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}$$

```
[Out] (cos(b*x+a)^2)^(1/4)*csc(b*x+a)^(-1+p)*hypergeom([5/4, 1/2-1/2*p],[3/2-1/2*
p],sin(b*x+a)^2)/b/d/(1-p)/(d*cos(b*x+a))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2667, 2657}

$$\frac{\sqrt[4]{\cos^2(a+bx)}\,\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{5}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^p/(d*Cos[a + b*x])^(3/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(1/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[5/4, (1 - p
)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*Sqrt[d*Cos[a + b*x]])
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \frac{\sin^{-p}(a+bx)}{(d\cos(a+bx))^{3/2}}\,dx\\ &= \frac{\sqrt[4]{\cos^2(a+bx)}\,\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{5}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)\sqrt{d\cos(a+bx)}}\end{aligned}$$

Mathematica [A]
time = 10.17, size = 68, normalized size = 0.87

$$\frac{2\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1+p}{2},\frac{3}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1}{2}(-1+p)}}{bd\sqrt{d\cos(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^p/(d*Cos[a + b*x])^(3/2),x]
[Out] (2*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-1/4, (1 + p)/2, 3/4, Cos[a + b*
x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(b*d*Sqrt[d*Cos[a + b*x]])
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\csc^{p}\left(bx+a\right)}{\left(d\cos\left(bx+a\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x)
[Out] int(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d^2*cos(b*x + a)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{p}\left(a+bx\right)}{\left(d\cos\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(3/2),x)
```

```
[Out] Integral(csc(a + b*x)**p/(d*cos(a + b*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(\frac{1}{\sin(a+b\,x)}\right)}^p}{{\left(d\,\cos\left(a+b\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(3/2),x)
```

```
[Out] int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(3/2), x)
```

3.283 $\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$\frac{\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(\frac{7}{4},\frac{1-p}{2},\frac{3-p}{2},\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}$$

```
[Out] (cos(b*x+a)^2)^(3/4)*csc(b*x+a)^(-1+p)*hypergeom([7/4, 1/2-1/2*p],[3/2-1/2*
p],sin(b*x+a)^2)/b/d/(1-p)/(d*cos(b*x+a))^(3/2)
```

Rubi [A]

time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 2657}

$$\frac{\cos^2(a+bx)^{3/4}\csc^{p-1}(a+bx)\,{}_2F_1\left(\frac{7}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x]^p/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] ((Cos[a + b*x]^2)^(3/4)*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[7/4, (1 - p
)/2, (3 - p)/2, Sin[a + b*x]^2])/(b*d*(1 - p)*(d*Cos[a + b*x])^(3/2))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^p(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx &= \left(\csc^p(a+bx)\sin^p(a+bx)\right)\int \frac{\sin^{-p}(a+bx)}{(d\cos(a+bx))^{5/2}}\,dx\\ &= \frac{\cos^2(a+bx)^{3/4}\csc^{-1+p}(a+bx)\,{}_2F_1\left(\frac{7}{4},\frac{1-p}{2};\frac{3-p}{2};\sin^2(a+bx)\right)}{bd(1-p)(d\cos(a+bx))^{3/2}}\end{aligned}$$

Mathematica [A]
time = 10.19, size = 70, normalized size = 0.90

$$\frac{2\csc^{-1+p}(a+bx)\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{1+p}{2},\frac{1}{4},\cos^2(a+bx)\right)\sin^2(a+bx)^{\frac{1}{2}(-1+p)}}{3bd(d\cos(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x]^p/(d*Cos[a + b*x])^(5/2),x]
```

```
[Out] (2*Csc[a + b*x]^(-1 + p)*Hypergeometric2F1[-3/4, (1 + p)/2, 1/4, Cos[a + b*
x]^2]*(Sin[a + b*x]^2)^((-1 + p)/2))/(3*b*d*(d*Cos[a + b*x])^(3/2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\csc^{p}\left(bx+a\right)}{\left(d\cos\left(bx+a\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x)
```

```
[Out] int(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*cos(b*x + a))*csc(b*x + a)^p/(d^3*cos(b*x + a)^3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**p/(d*cos(b*x+a))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^p/(d*cos(b*x+a))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^p/(d*cos(b*x + a))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{1}{\sin(a+b\,x)}\right)^p}{\left(d\,\cos\left(a+b\,x\right)\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(5/2),x)
```

```
[Out] int((1/sin(a + b*x))^p/(d*cos(a + b*x))^(5/2), x)
```

3.284 $\int \cos^m(e+fx)\csc^n(e+fx)\,dx$

Optimal. Leaf size=85

$$\frac{\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] cos(f*x+e)^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*csc(f*x+e)^(-1+n)*hypergeom([1
/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

Rubi [A]
time = 0.06, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2667, 2657}

$$\frac{\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^m*Csc[e + f*x]^n,x]
```

```
[Out] (Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1 + n)*H
ypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f*(1 -
n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx = \left(\csc^n(e+fx)\sin^n(e+fx)\right)\int \cos^m(e+fx)\sin^{-n}(e+fx)\,dx$$
$$= \frac{\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 1.21, size = 312, normalized size = 3.67

$$-\frac{2(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\cos^m(e+fx)\csc^n(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)\left((-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,1+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(1+m-n)F_1\left(\frac{3}{2}-\frac{n}{2};-m,2+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^m*Csc[e + f*x]^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Cos[e + f*x]^m*Csc[e + f*x]^n*S
in[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3
/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 + 2*(
m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2, -Tan
[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2 - n/2
, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cos^m(fx+e)\right)\left(\csc^n(fx+e)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*csc(f*x+e)^n,x)
```

```
[Out] int(cos(f*x+e)^m*csc(f*x+e)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \cos^m(e+fx)\csc^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*csc(f*x+e)**n,x)
[Out] Integral(cos(e + f*x)**m*csc(e + f*x)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*csc(f*x+e)^n,x, algorithm="giac")
[Out] integrate(cos(f*x + e)^m*csc(f*x + e)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^m\,\left(\frac{1}{\sin(e+f\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^m*(1/sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^m*(1/sin(e + f*x))^n, x)
```

3.285 $\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx$

Optimal. Leaf size=88

$$\frac{a(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] a*(a*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*csc(f*x+e)^(-1+n)*hyperg
eom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

Rubi [A]

time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 2657}

$$\frac{a\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{n-1}(e+fx)(a\cos(e+fx))^{m-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*Csc[e + f*x]^n,x]
```

```
[Out] (a*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Csc[e + f*x]^(-1
+ n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f
*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m\csc^n(e+fx)\,dx &= (\csc^n(e+fx)\sin^n(e+fx))\int (a\cos(e+fx))^m\sin^{-n}(e+fx)\,dx\\ &= \frac{a(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}\csc^{-1+n}(e+fx)\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\right.}{f(1-n)}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.18, size = 314, normalized size = 3.57

$$-\frac{2(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx))^m\csc^n(e+fx)\sin\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)\left((-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,1+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(1+m-n)F_1\left(\frac{3}{2}-\frac{n}{2};-m,2+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[e + f*x])^m*Csc[e + f*x]^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(a*Cos[e + f*x])^m*Csc[e + f*x]
^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m -
n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 +
 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2 -
 n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m (\csc^n(fx+e))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*csc(f*x+e)^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*csc(f*x+e)^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(e+fx))^m \csc^n(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*csc(f*x+e)**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*csc(e + f*x)**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*csc(f*x+e)^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*csc(f*x + e)^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos(e+f\,x))^m\,{\left(\frac{1}{\sin(e+f\,x)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(1/sin(e + f*x))^n,x)
```

```
[Out] int((a*cos(e + f*x))^m*(1/sin(e + f*x))^n, x)
```

3.286 $\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=88

$$\frac{b\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] b*cos(f*x+e)^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*(b*csc(f*x+e))^(-1+n)*hyperg
eom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

Rubi [A]
time = 0.06, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2667, 2657}

$$\frac{b\cos^{m-1}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (b*Cos[e + f*x]^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x])^(-1
+ n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^2])/(f
*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int \cos^m(e+fx)(b\csc(e+fx))^n\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int \cos^m(e+fx)(b\sin(e+$$

$$= \frac{b\cos^{-1+m}(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2}\right.}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.17, size = 314, normalized size = 3.57

$$-\frac{2(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)\cos^m(e+fx)(b\csc(e+fx))^n\sin\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)\left((-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,1+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(1+m-n)F_1\left(\frac{3}{2}-\frac{n}{2};-m,2+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*Cos[e + f*x]^m*(b*Csc[e + f*x])
^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m -
n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^2 +
 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]^2,
-Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5/2 -
 n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \left(\cos^m\left(fx+e\right)\right)\left(b\csc\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(b*csc(f*x+e))^n,x)
```

```
[Out] int(cos(f*x+e)^m*(b*csc(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\csc(e+fx))^n \cos^m(e+fx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(b*csc(f*x+e))**n,x)
[Out] Integral((b*csc(e + f*x))**n*cos(e + f*x)**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(b*csc(f*x+e))^n,x, algorithm="giac")
[Out] integrate((b*csc(f*x + e))^n*cos(f*x + e)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(e+f\,x)^m\left(\frac{b}{\sin(e+f\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^m*(b/sin(e + f*x))^n,x)
[Out] int(cos(e + f*x)^m*(b/sin(e + f*x))^n, x)
```

3.287 $\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx$

Optimal. Leaf size=91

$$\frac{ab(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

```
[Out] a*b*(a*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*(b*csc(f*x+e))^(-1+n)*
hypergeom([1/2-1/2*n, 1/2-1/2*m],[3/2-1/2*n],sin(f*x+e)^2)/f/(1-n)
```

Rubi [A]
time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2667, 2657}

$$\frac{ab\cos^2(e+fx)^{\frac{1-m}{2}}(a\cos(e+fx))^{m-1}(b\csc(e+fx))^{n-1}\,{}_2F_1\left(\frac{1-m}{2},\frac{1-n}{2};\frac{3-n}{2};\sin^2(e+fx)\right)}{f(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (a*b*(a*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f*x]
)^(-1 + n)*Hypergeometric2F1[(1 - m)/2, (1 - n)/2, (3 - n)/2, Sin[e + f*x]^
2])/(f*(1 - n))
```

Rule 2657

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Frac
Part[(n - 1)/2])*((a*Sin[e + f*x])^(m + 1)/(a*f*(m + 1)*(Cos[e + f*x]^2)^Fr
acPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, Sin[
e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^n\,dx = \left(b^2(b\csc(e+fx))^{-1+n}(b\sin(e+fx))^{-1+n}\right)\int (a\cos(e+fx))^m(b\sin$$

$$= \frac{ab(a\cos(e+fx))^{-1+m}\cos^2(e+fx)^{\frac{1-m}{2}}(b\csc(e+fx))^{-1+n}\,{}_2F_1\left(\frac{1-m}{2}\right.}{f(1-n)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.17, size = 316, normalized size = 3.47

$$-\frac{2(-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^3\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx))^m(b\csc(e+fx))^n\sin\left(\frac{1}{2}(e+fx)\right)}{f(-1+n)\left((-3+n)F_1\left(\frac{1}{2}-\frac{n}{2};-m,1+m-n;\frac{3}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\cos^2\left(\frac{1}{2}(e+fx)\right)+2\left(mF_1\left(\frac{3}{2}-\frac{n}{2};1-m,1+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(1+m-n)F_1\left(\frac{3}{2}-\frac{n}{2};-m,2+m-n;\frac{5}{2}-\frac{n}{2};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\sin^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^n,x]
```

```
[Out] (-2*(-3 + n)*AppellF1[1/2 - n/2, -m, 1 + m - n, 3/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]^3*(a*Cos[e + f*x])^m*(b*Csc[e + f
*x])^n*Sin[(e + f*x)/2])/(f*(-1 + n)*((-3 + n)*AppellF1[1/2 - n/2, -m, 1 +
m - n, 3/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[(e + f*x)/2]
^2 + 2*(m*AppellF1[3/2 - n/2, 1 - m, 1 + m - n, 5/2 - n/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2] + (1 + m - n)*AppellF1[3/2 - n/2, -m, 2 + m - n, 5
/2 - n/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Sin[(e + f*x)/2]^2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m (b\csc(fx+e))^n\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="fricas")
```

```
[Out] integral((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos (e+fx))^m (b\csc (e+fx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**n,x)
```

```
[Out] Integral((a*cos(e + f*x))**m*(b*csc(e + f*x))**n, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m*(b*csc(f*x + e))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos (e+f\,x))^m\,{\left(\frac{b}{\sin (e+f\,x)}\right)}^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^n,x)
```

```
[Out] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^n, x)
```

3.288 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx$

Optimal. Leaf size=78

$$-\frac{b^3(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{9}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{af(1+m)}$$

```
[Out] -b^3*(a*cos(f*x+e))^(1+m)*hypergeom([9/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)
^2)*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/f/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2667, 2656}

$$-\frac{b\sin^2(e+fx)^{5/4}(b\csc(e+fx))^{5/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{9}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(7/2),x]
```

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(5/2)*Hypergeometric2F1[9/4,
 (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(5/4))/(a*f*(1 + m)
))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n]
```

Rubi steps

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{7/2}\,dx = \left(b^2(b\csc(e+fx))^{5/2}(b\sin(e+fx))^{5/2}\right)\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{7/2}}\,dx$$

$$= -\frac{b(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{5/2}\,{}_2F_1\left(\frac{9}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e\right.}{af(1+m)}$$

Mathematica [A]
time = 9.83, size = 94, normalized size = 1.21

$$\frac{2ab(a\cos(e+fx))^{-1+m}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(b\csc(e+fx))^{5/2}\,{}_2F_1\left(\frac{1}{4}(7-2m),\frac{1-m}{2};\frac{1}{4}(11-2m);\csc^2(e+fx)\right)}{f(-7+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(7/2),x]
[Out] (2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f
*x])^(5/2)*Hypergeometric2F1[(7 - 2*m)/4, (1 - m)/2, (11 - 2*m)/4, Csc[e +
f*x]^2])/(f*(-7 + 2*m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a \cos \left(f x+e\right)\right)^{m} \left(b \csc \left(f x+e\right)\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x)
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^(7/2)*(a*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b^3*csc(f*x + e)^3, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^(7/2)*(a*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cos(e+fx))^m \left(\frac{b}{\sin(e+fx)}\right)^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(7/2),x)
```

```
[Out] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(7/2), x)
```

3.289 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{5/2}\,dx$

Optimal. Leaf size=76

$$-\frac{b(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{3/4}}{af(1+m)}$$

```
[Out] -b*(a*cos(f*x+e))^(1+m)*(b*csc(f*x+e))^(3/2)*hypergeom([7/4, 1/2+1/2*m],[3/
2+1/2*m],cos(f*x+e)^2)*(sin(f*x+e)^2)^(3/4)/a/f/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$,
Rules used = {2667, 2656}

$$-\frac{b\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{7}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[7/4,
 (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*f*(1 + m)
))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (a\cos(e+fx))^m(b\csc(e+fx))^{5/2}\,dx = \left(b^2(b\csc(e+fx))^{3/2}(b\sin(e+fx))^{3/2}\right)\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{5/2}}\,dx$$

$$= -\frac{b(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{7}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+\right.}{af(1+m)}$$

Mathematica [A]
time = 6.16, size = 94, normalized size = 1.24

$$\frac{2ab(a\cos(e+fx))^{-1+m}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{1}{4}(5-2m),\frac{1-m}{2};\frac{1}{4}(9-2m);\csc^2(e+fx)\right)}{f(-5+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(5/2),x]
[Out] (2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*(b*Csc[e + f
*x])^(3/2)*Hypergeometric2F1[(5 - 2*m)/4, (1 - m)/2, (9 - 2*m)/4, Csc[e + f
*x]^2])/(f*(-5 + 2*m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int (a\cos(fx+e))^m (b\csc(fx+e))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x)
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^(5/2)*(a*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b^2*csc(f*x + e)^2, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^(5/2)*(a*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\,\cos\left(e+f\,x\right))^m\,\left(\frac{b}{\sin\left(e+f\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(5/2),x)
```

```
[Out] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(5/2), x)
```

3.290 $\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx$

Optimal. Leaf size=76

$$-\frac{b(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{af(1+m)}$$

```
[Out] -b*(a*cos(f*x+e))^(1+m)*hypergeom([5/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2
)*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/f/(1+m)
```

Rubi [A]
time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2667, 2656}

$$-\frac{b\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}\,(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{5}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{af(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(3/2),x]
```

```
[Out] -((b*(a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[5/4, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*f*(1 + m)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2667

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[b^2*(b*Cos[e + f*x])^(n - 1)*(b*Sec[e + f*x])^(n - 1)
, Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e, f, m
, n}, x] && !IntegerQ[m] && !IntegerQ[n]
```

Rubi steps

$$\int (a\cos(e+fx))^m (b\csc(e+fx))^{3/2}\,dx = \left(b^2\sqrt{b\csc(e+fx)}\sqrt{b\sin(e+fx)}\right)\int \frac{(a\cos(e+fx))^m}{(b\sin(e+fx))^{3/2}}\,dx$$

$$= -\frac{b(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{5}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+\right.}{af(1+m)}$$

Mathematica [A]
time = 5.43, size = 94, normalized size = 1.24

$$\frac{2ab(a\cos(e+fx))^{-1+m}\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{1}{4}(3-2m),\frac{1-m}{2};\frac{1}{4}(7-2m);\csc^2(e+fx)\right)}{f(-3+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*(b*Csc[e + f*x])^(3/2),x]
[Out] (2*a*b*(a*Cos[e + f*x])^(-1 + m)*(-Cot[e + f*x]^2)^((1 - m)/2)*Sqrt[b*Csc[e
 + f*x]]*Hypergeometric2F1[(3 - 2*m)/4, (1 - m)/2, (7 - 2*m)/4, Csc[e + f*x
]^2])/(f*(-3 + 2*m))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\left(b\csc\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x)
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((b*csc(f*x + e))^(3/2)*(a*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m*b*csc(f*x + e), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*csc(f*x + e))^(3/2)*(a*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cos(e+fx))^m \left(\frac{b}{\sin(e+fx)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(3/2),x)
```

```
[Out] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(3/2), x)
```

3.291 $\int (a\cos(e+fx))^m \sqrt{b\csc(e+fx)}\, dx$

Optimal. Leaf size=78

$$-\frac{(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sin^2(e+fx)^{3/4}}{abf(1+m)}$$

```
[Out] -(a*cos(f*x+e))^(1+m)*(b*csc(f*x+e))^(3/2)*hypergeom([3/4, 1/2+1/2*m],[3/2+
1/2*m],cos(f*x+e)^2)*(sin(f*x+e)^2)^(3/4)/a/b/f/(1+m)
```

Rubi [A]

time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2666, 2656}

$$-\frac{\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m*Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*(b*Csc[e + f*x])^(3/2)*Hypergeometric2F1[3/4, (
1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(3/4))/(a*b*f*(1 + m)
))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cos(e+fx))^m\sqrt{b\csc(e+fx)}\,dx &= \frac{\left((b\csc(e+fx))^{3/2}(b\sin(e+fx))^{3/2}\right)\int\frac{(a\cos(e+fx))^m}{\sqrt{b\sin(e+fx)}}\,dx}{b^2}\\ &= -\frac{(a\cos(e+fx))^{1+m}(b\csc(e+fx))^{3/2}\,{}_2F_1\left(\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+f\right.}{abf(1+m)}\end{aligned}$$

Mathematica [A]
time = 5.15, size = 96, normalized size = 1.23

$$\frac{2(a\cos(e+fx))^m\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{1}{4}(1-2m),\frac{1-m}{2};\frac{1}{4}(5-2m);\csc^2(e+fx)\right)\tan(e+fx)}{f(-1+2m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m*Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] (2*(a*Cos[e + f*x])^m*(-Cot[e + f*x]^2)^((1 - m)/2)*Sqrt[b*Csc[e + f*x]]*Hy
pergeometric2F1[(1 - 2*m)/4, (1 - m)/2, (5 - 2*m)/4, Csc[e + f*x]^2]*Tan[e
+ f*x])/(f*(-1 + 2*m))
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(fx+e\right)\right)^{m}\sqrt{b\csc\left(fx+e\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x)
```

```
[Out] int((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\cos\left(e+fx\right)\right)^{m}\sqrt{b\csc\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(b*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m*sqrt(b*csc(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(b*csc(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a\cos(e+f\,x))^m\,\sqrt{\frac{b}{\sin(e+f\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(1/2),x)
```

```
[Out] int((a*cos(e + f*x))^m*(b/sin(e + f*x))^(1/2), x)
```

3.292 $\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx$

Optimal. Leaf size=78

$$-\frac{(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{abf(1+m)}$$

```
[Out] -(a*cos(f*x+e))^(1+m)*hypergeom([1/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)*
(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/b/f/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2666, 2656}

$$-\frac{\sqrt[4]{\sin^2(e+fx)}\sqrt{b\csc(e+fx)}\,(a\cos(e+fx))^{m+1}\,{}_2F_1\left(\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m/Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Sqrt[b*Csc[e + f*x]]*Hypergeometric2F1[1/4, (1
+ m)/2, (3 + m)/2, Cos[e + f*x]^2]*(Sin[e + f*x]^2)^(1/4))/(a*b*f*(1 + m)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx = \frac{\left(\sqrt{b\csc(e+fx)}\,\sqrt{b\sin(e+fx)}\right)\int (a\cos(e+fx))^m\sqrt{b\sin(e+fx)}\,dx}{b^2}$$

$$= -\frac{(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\;{}_2F_1\left(\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+f}}{abf(1+m)}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 17.42, size = 225, normalized size = 2.88

$$\frac{14bF_1\left(\frac{3}{4};-m,\frac{3}{2}+m;\frac{7}{4};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)(a\cos(e+fx))^m}{3f(b\csc(e+fx))^{3/2}\left(7F_1\left(\frac{3}{4};-m,\frac{3}{2}+m;\frac{7}{4};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)-2\left(2mF_1\left(\frac{7}{4};1-m,\frac{3}{2}+m;\frac{11}{4};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+(3+2m)F_1\left(\frac{7}{4};-m,\frac{5}{2}+m;\frac{11}{4};\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)\right)\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[e + f*x])^m/Sqrt[b*Csc[e + f*x]],x]
```

```
[Out] (14*b*AppellF1[3/4, -m, 3/2 + m, 7/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(a*Cos[e + f*x])^m)/(3*f*(b*Csc[e + f*x])^(3/2)*(7*AppellF1[3/4, -m, 3/
2 + m, 7/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 2*(2*m*AppellF1[7/4,
 1 - m, 3/2 + m, 11/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (3 + 2*m)
*AppellF1[7/4, -m, 5/2 + m, 11/4, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])
*Tan[(e + f*x)/2]^2))
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(fx+e))^m}{\sqrt{b\csc(fx+e)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x)
```

```
[Out] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m/sqrt(b*csc(f*x + e)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b*csc(f*x + e)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(e+fx))^m}{\sqrt{b\csc(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(1/2),x)
[Out] Integral((a*cos(e + f*x))**m/sqrt(b*csc(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate((a*cos(f*x + e))^m/sqrt(b*csc(f*x + e)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\cos(e+f\,x))^m}{\sqrt{\frac{b}{\sin(e+f\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(1/2),x)
[Out] int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(1/2), x)
```

3.293 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{abf(1+m)\sqrt{b\csc(e+fx)}\,\sqrt[4]{\sin^2(e+fx)}}$$

```
[Out] -(a*cos(f*x+e))^(1+m)*hypergeom([-1/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)
/a/b/f/(1+m)/(sin(f*x+e)^2)^(1/4)/(b*csc(f*x+e))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2666, 2656}

$$-\frac{(a\cos(e+fx))^{m+1}\,{}_2F_1\left(-\frac{1}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)\sqrt[4]{\sin^2(e+fx)}\,\sqrt{b\csc(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(3/2),x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-1/4, (1 + m)/2, (3 + m)/2, C
os[e + f*x]^2])/(a*b*f*(1 + m)*Sqrt[b*Csc[e + f*x]]*(Sin[e + f*x]^2)^(1/4))
)
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{3/2}}\,dx = \frac{\int (a\cos(e+fx))^m (b\sin(e+fx))^{3/2}\,dx}{b^2\sqrt{b\csc(e+fx)}\,\sqrt{b\sin(e+fx)}}$$
$$= -\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(-\frac{1}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{abf(1+m)\sqrt{b\csc(e+fx)}\,\sqrt[4]{\sin^2(e+fx)}}$$

Mathematica [A]
time = 8.69, size = 116, normalized size = 1.49

$$\frac{2a(a\cos(e+fx))^{-1+m}\cos(2(e+fx))\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1}{4}(-3-2m),\frac{1-m}{2};\frac{1}{4}(1-2m);\csc^2(e+fx)\right)}{bf(3+2m)\sqrt{b\csc(e+fx)}\,\left(-2+\csc^2(e+fx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(3/2),x]
[Out] (2*a*(a*Cos[e + f*x])^(-1 + m)*Cos[2*(e + f*x)]*(-Cot[e + f*x]^2)^((1 - m)/
2)*Hypergeometric2F1[(-3 - 2*m)/4, (1 - m)/2, (1 - 2*m)/4, Csc[e + f*x]^2])
/(b*f*(3 + 2*m)*Sqrt[b*Csc[e + f*x]]*(-2 + Csc[e + f*x]^2))
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x)
[Out] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="maxima")
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b^2*csc(f*x + e)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\cos{(e+fx)})^m}{(b\csc{(e+fx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(3/2),x)
```

```
[Out] Integral((a*cos(e + f*x))**m/(b*csc(e + f*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a\,\cos\left(e+f\,x\right)\right)}^m}{{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(3/2),x)
```

```
[Out] int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(3/2), x)
```

3.294 $\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx$

Optimal. Leaf size=78

$$-\frac{(a\cos(e+fx))^{1+m}\sqrt{b\csc(e+fx)}\,{}_2F_1\left(-\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)\sqrt[4]{\sin^2(e+fx)}}{ab^3f(1+m)}$$

```
[Out] -(a*cos(f*x+e))^(1+m)*hypergeom([-3/4, 1/2+1/2*m],[3/2+1/2*m],cos(f*x+e)^2)
*(sin(f*x+e)^2)^(1/4)*(b*csc(f*x+e))^(1/2)/a/b^3/f/(1+m)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2666, 2656}

$$-\frac{(a\cos(e+fx))^{m+1}\,{}_2F_1\left(-\frac{3}{4},\frac{m+1}{2};\frac{m+3}{2};\cos^2(e+fx)\right)}{abf(m+1)\sin^2(e+fx)^{3/4}(b\csc(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] -(((a*Cos[e + f*x])^(1 + m)*Hypergeometric2F1[-3/4, (1 + m)/2, (3 + m)/2, C
os[e + f*x]^2])/(a*b*f*(1 + m)*(b*Csc[e + f*x])^(3/2)*(Sin[e + f*x]^2)^(3/4
)))
```

Rule 2656

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(-b^(2*IntPart[(n - 1)/2] + 1))*(b*Sin[e + f*x])^(2*F
racPart[(n - 1)/2])*((a*Cos[e + f*x])^(m + 1)/(a*f*(m + 1)*(Sin[e + f*x]^2)
^FracPart[(n - 1)/2]))*Hypergeometric2F1[(1 + m)/2, (1 - n)/2, (3 + m)/2, C
os[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] && SimplerQ[n, m]
```

Rule 2666

```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Dist[(1/b^2)*(b*Cos[e + f*x])^(n + 1)*(b*Sec[e + f*x])^(n
+ 1), Int[(a*Sin[e + f*x])^m/(b*Cos[e + f*x])^n, x], x] /; FreeQ[{a, b, e,
f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && LtQ[n, 1]
```

Rubi steps

$$\int \frac{(a\cos(e+fx))^m}{(b\csc(e+fx))^{5/2}}\,dx = \frac{\int (a\cos(e+fx))^m(b\sin(e+fx))^{5/2}\,dx}{b^2(b\csc(e+fx))^{3/2}(b\sin(e+fx))^{3/2}}$$
$$= -\frac{(a\cos(e+fx))^{1+m}\,{}_2F_1\left(-\frac{3}{4},\frac{1+m}{2};\frac{3+m}{2};\cos^2(e+fx)\right)}{abf(1+m)(b\csc(e+fx))^{3/2}\sin^2(e+fx)^{3/4}}$$

Mathematica [A]
time = 6.70, size = 125, normalized size = 1.60

$$\frac{2(a\cos(e+fx))^m(1+2\cos(2(e+fx)))\left(-\cot^2(e+fx)\right)^{\frac{1-m}{2}}\,{}_2F_1\left(\frac{1}{4}(-5-2m),\frac{1-m}{2};\frac{1}{4}(-1-2m);\csc^2(e+fx)\right)\tan(e+fx)}{b^2f(5+2m)\sqrt{b\csc(e+fx)}\,(-4+3\csc^2(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[e + f*x])^m/(b*Csc[e + f*x])^(5/2),x]
```

```
[Out] (2*(a*Cos[e + f*x])^m*(1 + 2*Cos[2*(e + f*x)])*(-Cot[e + f*x]^2)^((1 - m)/2
)*Hypergeometric2F1[(-5 - 2*m)/4, (1 - m)/2, (-1 - 2*m)/4, Csc[e + f*x]^2]*
Tan[e + f*x])/(b^2*f*(5 + 2*m)*Sqrt[b*Csc[e + f*x]]*(-4 + 3*Csc[e + f*x]^2)
)
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(a\cos(fx+e))^m}{(b\csc(fx+e))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x)
```

```
[Out] int((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*csc(f*x + e))*(a*cos(f*x + e))^m/(b^3*csc(f*x + e)^3), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m/(b*csc(f*x+e))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m/(b*csc(f*x+e))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*cos(f*x + e))^m/(b*csc(f*x + e))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a\,\cos\left(e+f\,x\right))^m}{{\left(\frac{b}{\sin\left(e+f\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(5/2),x)
```

```
[Out] int((a*cos(e + f*x))^m/(b/sin(e + f*x))^(5/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section 1092
4.2 Listing of Grading functions 1092

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))     #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```